The packing factor

Animesh Das

One of the commonly used materials in Civil Engineering is stone aggregates. For a specific type of construction, the aggregates are required to follow certain recommended size distribution. This size distribution is also known as aggregate gradation. The aggregate gradation is adjusted in such a way that it results in minimum void or, it achieves some desirable level of void space. Theoretical estimation of void ratio (volume of void divided by volume of solid) or, packing ratio (volume of solid divided by total volume) for a given gradation is a difficult task (see Figure 1); yet it is quite easy to obtain the value experimentally.

Figure 1: How much is the packing ratio of aggregates for a given gradation?

Even if the aggregates are assumed to be all spherical in size and also of equal diameter, theoretical estimation of void ratio still remains difficult. Spherical particles are dealt in various areas in science and engineering, and the researchers remain curious to know what arrangement of spherical equal sized particles (refer Figure 2) makes it densest (Laso et al. 2009). Way back in 1611, Kepler proposed a conjecture that face centered cubic close packing and hexagonal close packing of spherical equal sized particles give rise to the densest packing, and the packing ratio is $\frac{\pi}{\sqrt{18}}$ (Kepler 1966). Since then, a number of mathematicians have worked on this problem, and this conjecture has subsequently been proved in recent times (Hales 1998, 2008).

Figure 2: What is the densest arrangement?

Simulation studies have been carried out with spherical particles, sphero-cylinders, ellipsoids etc (Delaney et al. 2005, Reyes and Iglesia 1991). Deformable objects (for example foam) can generate even denser packing than spherical equal sized particles, and so also objects having a size distribution (refer Figure 3). Fuller and Thompson (1907) performed series of experiments on aggregates and they found that the aggregate gradation following Equation (1) achieves maximum density.

$$\text{Percentage passing for particle size } d = \left(\frac{d}{D}\right)^n \times 100 \qquad (1)$$

where, D is the maximum size of aggregate being considered, n = an empirical constant ranging between 0.45 to 0.70. Studies have been done using mixing rules to predict the resultant void ratio, when the void ratios for individual single sized aggregates are known (Stovali et al. 1986). Attempts have also been made to statistically predict the void ratio of aggregates for a given gradation (Åberg 1996a, 1996b, Alshibli and El-Saidany 2001).

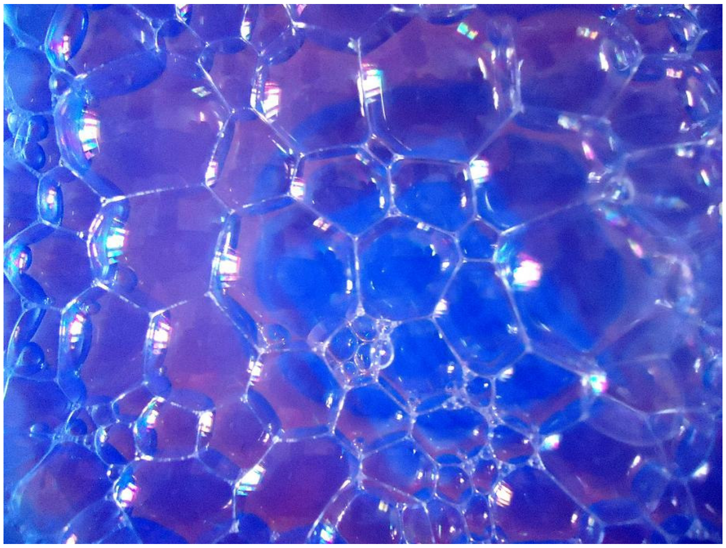

Figure 3: Foams can generate even denser packing.

Consumption of binder material for mix preparation and the performance of the mix are significantly influenced by the void content (Larrad et al. 1994, Sánchez-Leal 2007, Gopalkrishnan and Shashidhar 2006, TRC 2002). In an aggregate-binder mixture, not all aggregate particles are in contact with each other. Further, aggregates show significant variations in their shape (i.e. form, angularity and texture), and this makes prediction of void ratio further difficult. Thus, the recommendations on aggregate gradation for various mixes have been primarily developed through experience and/or experimental parametric studies.

References

Åberg, B., Grain-size distribution for smallest possible void ratio, *Journal of Geotechnical Engineering*, ASCE, Vol. 122 (1), 1996a, pp.74-77.

Åberg, B., Void sizes of granular solids, *Journal of Geotechnical Engineering*, ASCE, Vol. 122 (3), 1996b, pp.236-239.

Alshibli, K. A., and El-Saidany, H. A., Quantifying void ratio in granular materials using Voronoi Tessellation, *Journal of Computing in Civil Engineering*, ASCE, 2001, pp.232-238.

Delaney, G., Weaire, D., Hutzler, S., Murphy, S., Random packing of elliptical disks, *Philosophical Magazine Letters*, Vol.85(2), 2005, pp.89-96.

Fuller, W. B., and Thompson, S. E., The laws of proportioning concrete, *Transactions of ASCE*, ASCE, Vol. 59, 1907, pp.67-143.

Gopalkrishnan, K., and Shashidhar, N., Structural characteristics of three-dimensional random packing of aggregates with wide size distribution, *International Journal of Information Technology*, Vol.(3), 2006, pp.201-208.

Hales, T. C., *An overview of the Kepler conjecture*, 1998, http://www.math.pitt.edu/~thales/kepler98/sphere0.ps

Hales, T. C., *A blueprint for the formal proof of the Kepler conjecture*, 2008, http://flyspeck.googlecode.com/svn/trunk/kepler_tex/

Kepler, J., *On the six-cornered snowflakes*, Oxford University Press, 1966.

Laso, M., Karayiannis, N. Ch., Foteinopoulou, K., Mansfield, M. L., and Kröger, M., Random packing of model polymers: local structure, topological hindrance and universal scaling, *Soft Matter*, 2009, Vol.5, pp.1762-1770.

Larrad, F. de, and Sedran, T., Optimization of ultra-high-performance concrete by the use if a parking model, Cement and Concrete Research, Vol.24(6), 1994, pp.997-1009.

Sánchez-Leal, F. L., Gradation chart for asphalt mixes: development, *Journal of Materials in Civil Engineering*, ASCE, 19(2), 2007, pp.185-197.

Stovali, T., Larrard, F. De, and Buil, M., Linear packing density model of grain mixtures, *Powder Technology*, 48, 1986, pp.1-12.

Reyes, S. C., and Iglesia, E., Monte Carlo simulations for structural properties of packed beds, *Chemical Engineering Science*, 46(4), 1991, pp.1089-1099.

Transportation Research Circular (TRC), E-C044, *Bailey method for gradation selection in hot-mix asphalt mixture design*, Transportation Research Board, 2002.