Fractional Timing Offset and Channel estimation for MIMO OFDM Systems over Flat fading channels

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***Abstract*—This paper addresses the problem of fractional timing offset and channel estimation in Multiple input Multiple output orthogonal frequency division multiplexing (MIMO OFDM) systems. The estimators have been derived assuming a flat fading channel and using the maximum likelihood criterion. Closed form Cramer Rao bound (CRB) expressions for fractional timing offset and channel response are also derived. Simulation results have been used to cross-check the accuracy of the proposed estimation algorithm.**

***Index Terms*—Timing offset, multiple input multiple output, orthogonal frequency division multiplexing, Cramer Rao bound.**

I. Introduction

OFDM systems are receiving significant interests because of their robustness against frequency selective fading channel, efficient use of spectrum, low equalization complexity and reduced cost of implementation using FFT techniques. Multiple input multiple output systems provide improved spectral efficiency and capacity over single antenna systems. Hence, MIMO OFDM systems offering the combined benefits have been considered the promising technique for the present demand of high data rates. However, MIMO OFDM systems are sensitive to the presence of a timing offset.Timing offset is caused by a mismatch in sampling time at the receiver. Receiver knows the sampling rate, but does not know the exact sampling time instant to maximize the signal to noise ratio. Timing offset reduces the effective cyclic prefix length and also induces inter symbol interference. For reliable transmission, timing offset has to be estimated and compensated before detection. Moreover, for detection of symbols, knowledge of channel response is inevitable at the receiver.

Integer timing offset estimation for OFDM systems is considered thoroughly in the literature ([1]-[4]). However, estimation of fractional timing offset is not considered in any of the above works. In [5]-[9], estimation of fractional timing offset in single carrier systems over flat fading channels is considered. We extended the system model given in the above works for single carrier systems to OFDM systems in [10]. Here, we considered data aided estimation of fractional timing offset and channel for MIMO OFDM systems over flat fading channels.

Notations: $(\mathbf{A})^{-1}, (\mathbf{A})^T, (\bar{\mathbf{A}}), (\mathbf{A})^H, \mathbf{A}(:, 1:L), [\mathbf{A}]_{mn}$ denotes the inverse, transpose, conjugate, hermitian, first L columns and the (m,n)th element of a matrix $\mathbf{A}$ respectively. $\mathbf{A}^H(1:L,:)$ denotes the first L rows of $\mathbf{A}^H$.

The remaining sections of the paper are organized as follows. Section II describes the system model for MIMO OFDM systems with fractional timing offset and the ML estimator for timing offset and channel response. Section III gives the derived closed form CRB expressions. Section IV presents simulation results to validate the proposed estimator. The paper is concluded in Section V.

II. System Model

We consider a MIMO OFDM system with N_t transmit and N_r receive antennas. Let the frequency domain and the corresponding time domain symbols of an OFDM block at the m^{th} transmit antenna be denoted as $\mathbf{d}_m$ and $\mathbf{s}_m$ respectively. The relation between them is given by

$$\mathbf{s}_m = \mathbf{F}^H \mathbf{d}_m \tag{1}$$

where

$$\mathbf{d}_m = [d_m(0)\ d_m(1)\ \dots\ d_m(N-1)]^T$$
$$\mathbf{s}_m = [s_m(0)\ s_m(1)\ \dots\ s_m(N-1)]^T$$
$$\mathbf{F}(k,l) = \exp\left(-j2\pi kl/N\right)/\sqrt{N} \quad \text{for} \quad 0 \le k, l \le N-1$$

$\mathbf{F}$ is the unitary FFT matrix and N is the number of subcarriers. Suitable Cyclic prefix (L_{cp}) is added to the time domain OFDM block to eliminate inter block interference. A suffix of L_g symbols are also added to the $N + L_{cp}$ symbols and then passed through a transmit filter $g(t)$. The necessity of extra L_g symbols is explained shortly. The OFDM symbol transmitted is as shown in Fig. 1.

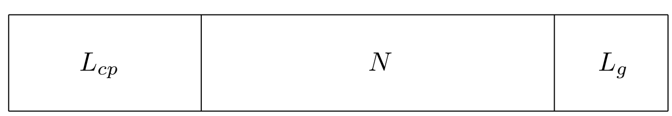

Fig. 1. Structure of the training sequence

The baseband pulse shaped OFDM signal at the m^{th} transmit antenna is given by

$$c_m(t) = \sum_{k=-\infty}^{\infty} s_m(k) g(t - kT) \tag{2}$$

where $(N + L_{cp} + L_g)T$ is the OFDM symbol period. The individual contributions of $s_m(k)$ in the signal $c_m(t)$ are shown in Fig. 2. It can be noted that the response of $g(t)$ is nonzero for the adjacent 4 or 5 symbol periods only. These number of symbols depend on the filter characteristics. We denote these symbols as L_g. Hence, at any time instant t, $c_m(t)$ has contributions only from the present transmitted symbol, past L_g symbols and future L_g symbols. Hence, the summation in (2) need not be from $-\infty$ to ∞ and contains $2L_g + 1$ significant terms only. It can be noted that at any time $t = nT$, $c_m(t)$ has contribution from only the present transmitted symbol and all the components from other symbols are zero.

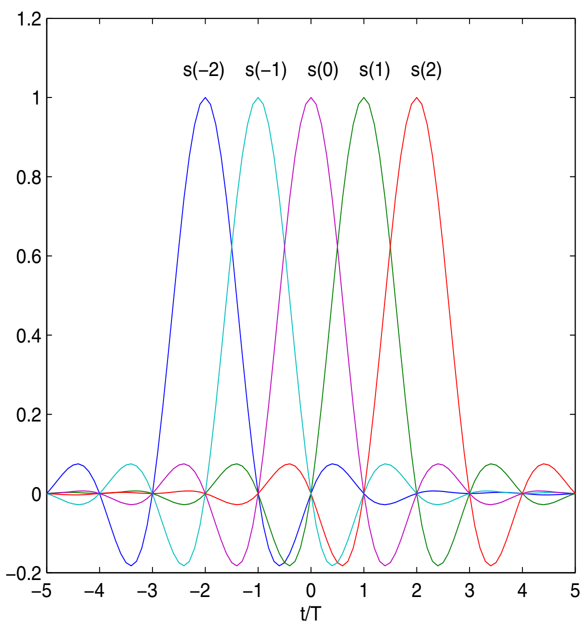

Fig. 2. Individual contributions of s(n) on the transmitted signal c(t)

The extra L_g symbols are required to be used as we are considering the presence of a fractional timing offset. The presence of a timing offset implies that we are sampling at an instant other than $t = nT$. This causes the last L_g symbols of the received OFDM symbol $s_m(N - L_g : N - 1)$ to be dependent on the next transmitted OFDM symbol also. This requires the next OFDM symbol to be used in estimation process and to be used as a training OFDM symbol. Hence, the extra L_g symbols are used as the postfix to the OFDM symbol. We considered these L_g symbols to be $\mathbf{s}_m(0 : L_g - 1)$ in this paper. The overhead used here (L_g) is negligible when compared to the larger values of the number of subcarriers and cyclic prefix. This cost can be paid in return to estimation of fractional timing offset thereby improving the performance of the system. Usually, the value of L_{cp} is larger than L_g. If it is not the case, then the OFDM symbol should be preceded by $(L_g - L_{cp})$ symbols as $s(0 : L_g - 1)$ depend on the past symbols. Hence, the baseband pulse shaped OFDM signal at the m^{th} transmit antenna can be rewritten as

$$\mathrm{c}_m(t) = \sum_{k=-L_g}^{N+L_g-1} s_m(k) g(t - kT) \tag{3}$$

The range of k is choosen such that for any time instant t in the interval $0 \leq t \leq NT$, $c_m(t)$ contains the contributions from all significant $(2L_g + 1)$ symbols.

The complex envelope of the signal received at n^{th} receive antenna (within $0 \leq t \leq NT$) from m^{th} transmit antenna can be expressed as [7]

$$\mathrm{x}_{mn}(t) = h_{mn} \sum_{k=-L_g}^{N+L_g-1} s_m(k) g(t - kT - \epsilon T) \tag{4}$$

where h_{mn} is the complex channel coefficient between the m^{th} transmit antenna and the n^{th} receive antenna and are assumed to be statistically independent; ϵ is the normalized timing offset; We assume a quasi static channel and also that an integer timing offset is already known. Hence the range of ϵ is [-0.5, 0.5]. A superposition of faded signals from all the N_t transmit antennas plus noise is received at each receive antenna. Upon reception, the signal is sampled at a rate $f_s = 1/T_s$, where $T_s = T/\alpha$, α being the oversampling ratio. Removing the samples corresponding to cyclic prefix and the extra L_g symbols, the received and useful discrete time $N\alpha$ consecutive samples at the n^{th} receive antenna (within $0 \leq t \leq NT$) from m^{th} transmit antenna is given by

$$\mathrm{x}_{mn}(lT_s) = h_{mn} \sum_{k=-L_g}^{N+L_g-1} s_m(k) g(lT_s - kT - \epsilon T)$$
$$for \; l \; = \; 0, \; 1, \; \ldots, \; N\alpha - 1 \tag{5}$$

Stacking all the above samples gives

$$\mathbf{x}_{mn} = h_{mn} \mathbf{G}(\epsilon) \mathbf{A}_m \tag{6}$$

where

$$\mathbf{x}_{mn} = [\mathrm{x}_{mn}(0) \; \mathrm{x}_{mn}(1) \; \ldots \; \mathrm{x}_{mn}(N\alpha - 1)]^T \tag{7}$$
$$[\mathbf{G}]_{lr} = g(lT/\alpha - (r - L_g)T - \epsilon T) \tag{8}$$
$$[\mathbf{A}_m]_r = s_m(-L_g + r) \tag{9}$$

with $l = 0, 1, \ldots, N\alpha - 1$ and $r = 0, 1, \ldots, N + 2L_g - 1$. Vector $\mathbf{A}_m$ can also be represented as

$$\mathbf{A}_m = \tilde{\mathbf{F}} \mathbf{d}_m \tag{10}$$
$$\tilde{\mathbf{F}} = [\mathbf{F}^H(N - \mathrm{L_g} + 1 : N, :); \mathbf{F}^H; \mathbf{F}^H(1 : \mathrm{L_g}, :)] \tag{11}$$

This definition of $\mathbf{A}_m$ comes from assuming the L_g suffix symbols as $\mathbf{s}_m(0 : L_g - 1)$. Since the received signal at the n^{th} receive antenna is the sum of signals from all N_T transmit antennas and noise, the received signal at this antenna is given by

$$\mathbf{x}_n = \sum_{m=1}^{N_t} \mathbf{x}_{mn} + \mathbf{w}_n = \sum_{m=1}^{N_t} h_{mn} \mathbf{G}(\epsilon) \mathbf{A}_m + \mathbf{w}_n \tag{12}$$

where $\mathbf{w}_n = [\mathrm{w}_n(0)\ \mathrm{w}_n(1)\ ...\ \mathrm{w}_n(N\alpha-1)]^T$. $\mathbf{w}_n$ is complex white Gaussian noise with zero mean and covariance matrix $\sigma^2\mathbf{I}_{N\alpha\times N\alpha}$.

Denoting $\mathbf{x} = [\mathbf{x}_1^T\ \mathbf{x}_2^T\ \ldots\ \mathbf{x}_{N_r}^T]^T$, $\mathbf{w} = [\mathbf{w}_1^T\ \mathbf{w}_2^T\ \ldots\ \mathbf{w}_{N_r}^T]^T$, $\mathbf{h} = [\mathbf{h}_1^T\ \mathbf{h}_2^T\ \ldots\ \mathbf{h}_{N_r}^T]^T$ with $\mathbf{h}_n = [h_{1n}\ h_{2n}\ \ldots\ h_{N_t n}]^T$, the signal model from (12) can be rewritten as

$$\mathbf{x} = \mathbf{Q}(\epsilon)\mathbf{h} + \mathbf{w} \tag{13}$$

where $\mathbf{Q}(\epsilon) = \mathbf{I}_{N_r\times N_r} \otimes [\mathbf{G}(\epsilon)\mathbf{A}_1 \ldots \mathbf{G}(\epsilon)\mathbf{A}_{N_t}]$.

Based on the signal model in (13), ML estimates of parameters $\theta = \{\epsilon, \mathbf{h}\}$ are obtained by maximizing [11]

$$\Lambda(\mathbf{x}; \tilde{\epsilon}, \tilde{\mathbf{h}}) = \frac{1}{(\pi\sigma^2)^{N_r}} \exp\left\{ -\frac{\left[\mathbf{x} - \mathbf{Q}(\tilde{\epsilon})\tilde{\mathbf{h}}\right]^H \left[\mathbf{x} - \mathbf{Q}(\tilde{\epsilon})\tilde{\mathbf{h}}\right]}{\sigma^2} \right\} \tag{14}$$

or equivalently minimizing

$$\psi(\mathbf{x}; \tilde{\epsilon}, \tilde{\mathbf{h}}) = \left[\mathbf{x} - \mathbf{Q}(\tilde{\epsilon})\tilde{\mathbf{h}}\right]^H \left[\mathbf{x} - \mathbf{Q}(\tilde{\epsilon})\tilde{\mathbf{h}}\right] \tag{15}$$

where $\tilde{\epsilon}, \tilde{\mathbf{h}}$ are trial values of ϵ and $\mathbf{h}$ respectively. Due to linearity of the parameter $\mathbf{h}$, ML estimate of $\mathbf{h}$ (when $\tilde{\epsilon}$ is fixed) is given by

$$\hat{\mathbf{h}} = (\mathbf{Q}^H(\tilde{\epsilon})\mathbf{Q}(\tilde{\epsilon}))^{-1}\mathbf{Q}^H(\tilde{\epsilon})\mathbf{x} \tag{16}$$

substituting $\hat{\mathbf{h}}$ back into (15) and upon some manipulations gives the ML estimate for ϵ as

$$\hat{\epsilon} \quad = \quad \arg\max_{\tilde{\epsilon}} \|\mathbf{Q}(\mathbf{Q}^H\mathbf{Q})^{-1}\mathbf{Q}^H\mathbf{x}\|^2 \tag{17}$$

$\mathbf{Q}(\epsilon)$ is represented by $\mathbf{Q}$ for expression simplicity.

III. CRB Analysis

We derive the closed form CRB expressions for ϵ, $\mathbf{h}$ in this section. From (13), $\mathbf{x}$ is complex gaussian with mean $\boldsymbol{\mu} = \mathbf{Qh}$ and covariance matrix $\sigma^2\mathbf{I}_{NN_r\alpha\times NN_r\alpha}$. Let the parameters of interest be $\boldsymbol{\eta} = [\epsilon\ \ \mathbf{h}_R\ \mathbf{h}_I\ ]^T$, where $\mathbf{h}_R$ and $\mathbf{h}_I$ denote the real and imaginary parts of $\mathbf{h}$. The Fisher Information Matrix is given by [11]

$$\mathbf{FIM} = \frac{2}{\sigma^2}\Re\left[\frac{\partial\boldsymbol{\mu}^H}{\partial\boldsymbol{\eta}}\frac{\partial\boldsymbol{\mu}}{\partial\boldsymbol{\eta}^T}\right] \tag{18}$$

Following results are used to get the **FIM** matrix

$$\frac{\partial\boldsymbol{\mu}}{\partial\epsilon} = \mathbf{Y} \tag{19}$$

$$\frac{\partial\boldsymbol{\mu}}{\partial\mathbf{h}_R} = \mathbf{Q} \tag{20}$$

$$\frac{\partial\boldsymbol{\mu}}{\partial\mathbf{h}_I} = j\mathbf{Q} \tag{21}$$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_{11} + \mathbf{Y}_{21} + \ldots + \mathbf{Y}_{N_t 1} \\ \mathbf{Y}_{12} + \mathbf{Y}_{22} + \ldots + \mathbf{Y}_{N_t 2} \\ \vdots \\ \mathbf{Y}_{1N_r} + \mathbf{Y}_{2N_r} + \ldots + \mathbf{Y}_{N_t N_r} \end{bmatrix} \tag{22}$$

$$\mathbf{Y}_{mn} = h_{mn}\mathbf{G}_d(\epsilon)\mathbf{A}_m \tag{23}$$

with $\mathbf{G}_d(\epsilon) = d\mathbf{G}(\epsilon)/d\epsilon$.

Substituting these results into (18) gives the Fisher Information Matrix as given below.

$$\mathbf{FIM} = \frac{2}{\sigma^2}\begin{bmatrix} \Re(\mathbf{Y}^H\mathbf{Y}) & \Re(\mathbf{Y}^H\mathbf{Q}) & -\Im(\mathbf{Y}^H\mathbf{Q}) \\ \Re(\mathbf{Q}^H\mathbf{Y}) & \Re(\mathbf{Q}^H\mathbf{Q}) & -\Im(\mathbf{Q}^H\mathbf{Q}) \\ \Im(\mathbf{Q}^H\mathbf{Y}) & \Im(\mathbf{Q}^H\mathbf{Q}) & \Re(\mathbf{Q}^H\mathbf{Q}) \end{bmatrix} \tag{24}$$

The **CRB** matrix can be obtained by inverting the Fisher information matrix. Using the above definitions, **CRB** matrix will be given by the expression in [12] as shown at the bottom of the page, where

$$\boldsymbol{\beta} = (\mathbf{Q}^H\mathbf{Q})^{-1}\mathbf{Q}^H\mathbf{Y} \tag{26}$$

$$\gamma = \left(\Re\left\{\mathbf{Y}^H\boldsymbol{\Pi}_Q^{\perp}\mathbf{Y}\right\}\right) \tag{27}$$

$$\boldsymbol{\Pi}_Q^{\perp} = \mathbf{I}_{NN_r\alpha\times NN_r\alpha} - \mathbf{Q}(\mathbf{Q}^H\mathbf{Q})^{-1}\mathbf{Q} \tag{28}$$

Following an approach shown in [12], we obtain the CRB for ϵ and $\mathbf{h}$ as below

$$\mathrm{CRB}(\epsilon) = \frac{\sigma^2}{2}\left(\Re\left\{\mathbf{Y}^H\boldsymbol{\Pi}_Q^{\perp}\mathbf{Y}\right\}\right)^{-1} \tag{29}$$

$$\mathrm{CRB}(\mathbf{h}) = \frac{\sigma^2}{2}(2(\mathbf{Q}^H\mathbf{Q})^{-1} + \boldsymbol{\beta}\gamma^{-1}\boldsymbol{\beta}^H) \tag{30}$$

IV. Simulation Results

In this section, simulation results are presented to demonstrate the effectiveness of the estimation algorithm. The considered OFDM system has the following parameters: $N = 64$, $N_t = 2$, $N_r = 2$, $L_g = 4$, $\alpha = 2$. Channel coefficients h_{mn} are generated as complex Gaussian random variable with zero mean and a variance of 0.5 per dimension. Raised cosine pulse with a roll off factor 0.5 is taken as $g(t)$. ϵ is assumed to be a random varaible uniformly distributed in [-0.5,0.5]. Signal to Noise ratio used in the simulations is

$$\mathrm{SNR} = \frac{E(|\mathrm{x}(m) - w(m)|^2)}{E(|w(m)|^2)} \tag{31}$$

$$\mathbf{CRB} = \frac{\sigma^2}{2}\begin{bmatrix} \gamma^{-1} & -\gamma^{-1}\Re(\boldsymbol{\beta}^T) & \gamma^{-1}\Im(\boldsymbol{\beta}^H) \\ -\Re(\boldsymbol{\beta})\gamma^{-1} & \Re[(\mathbf{Q}^H\mathbf{Q})^{-1}] + \Re(\boldsymbol{\beta})\gamma^{-1}\Re(\boldsymbol{\beta}^T) & -\Im[(\mathbf{Q}^H\mathbf{Q})^{-1}] - \Re(\boldsymbol{\beta})\gamma^{-1}\Im(\boldsymbol{\beta}^H) \\ -\Im(\boldsymbol{\beta})\gamma^{-1} & \Im[(\mathbf{Q}^H\mathbf{Q})^{-1}] + \gamma^{-1}\Im(\boldsymbol{\beta})\Re(\boldsymbol{\beta}^T) & \Re[(\mathbf{Q}^H\mathbf{Q})^{-1}] - \gamma^{-1}\Im(\boldsymbol{\beta})\Im(\boldsymbol{\beta}^H) \end{bmatrix} \tag{25}$$

We considered code division multiplexing (CDM) sequences

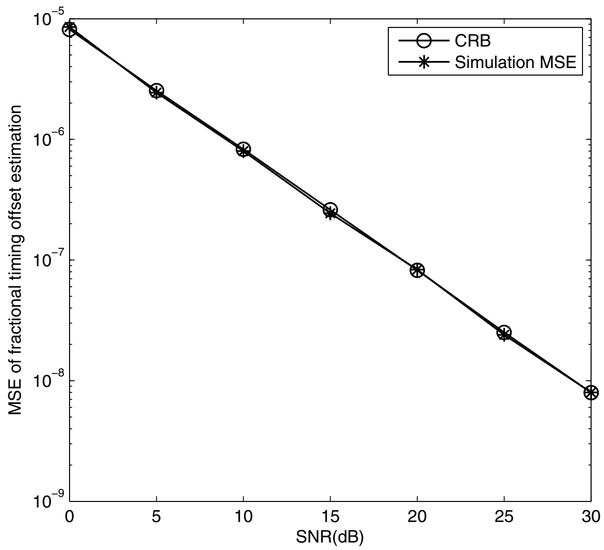

Fig. 3. The MSE performance of the fractional timing offset estimator

proposed in [13] as the training sequence. All the simulation results of the proposed algorithm are averaged over 1000 Monte Carlo runs. Timing offset is obtained by plugging different values of $\tilde{\epsilon}$ in (17). Using this estimated timing, channel response is estimated by (16). CRBs of timing and channel are plotted by averaging through many channel realizations. The Mean square error (MSE) performance of the timing and

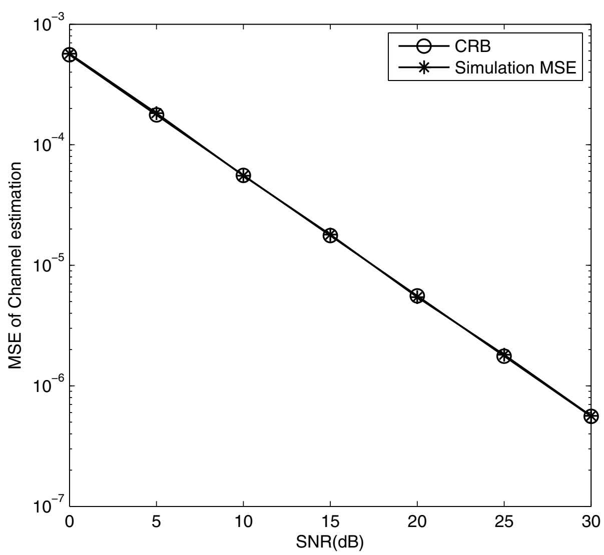

Fig. 4. The MSE performance of the channel estimator

channel estimates are plotted versus SNR in Figs. 3 and 4 respectively. Their corresponding CRBs are also shown in the figures as references. It can be noticed that the MSE performance of both parameters closely match with its CRB over all ranges of SNR.

V. Conclusions

The problem of fractional timing offset and channel estimation in MIMO OFDM systems is considered. An ML estimator for timing offset and channel is given. Closed form Cramer Rao bound expressions for fractional timing offset and channel in MIMO OFDM systems are derived. It can be seen that the mean square errors of the Maximum Likelihood estimates of both parameters coincide with the respective Cramer Rao bounds.

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