

# Weighted Sum of Spectral Efficiency and Energy Efficiency in Spatial Modulation–MIMO Systems

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**Abstract**—In this paper, we propose a novel method for selecting the transmission configuration in Spatial Modulation (SM) based multiple-input–multiple-output (MIMO) systems that provides the desired relative contribution of Spectral Efficiency (SE) and Energy Efficiency (EE) in the overall performance. This is done on the basis of a function proposed here called Weighted Efficiency Function (WEF) which takes into account both SE and EE simultaneously in the optimization. Here, we have considered the generalized model of the SM (GSM) to construct a set of operating modes incorporating the different system parameters. The goal of this work is to select the most appropriate modulation scheme and number of RF chains that maximizes the WEF. This selection is bound by certain constraints on power consumption, data rate and Symbol Error Rate (SER). In the end, we use simulation results to evaluate the performance of the proposed methodology and compare it with the state of the art.

**Index Terms**—Spatial Modulation (SM)–MIMO System, Spectral Efficiency (SE), Energy Efficiency (EE), Symbol Error Rate (SER), Weighted Efficiency Function (WEF).

## I. INTRODUCTION

WITH the increasing demands for high data rate, lesser power consumption and better bandwidth utilization in a wireless communication system, the SM-based MIMO systems have gained high popularity in recent years. As the name suggests, they employ an extra spatial dimension by encoding information in the index of the transmitting antenna (TA) in addition to the conventional modulated symbol [1], [2]. The generalized model of SM-MIMO (GSM-MIMO) used here, allows more than one simultaneously active TA [3]. The performance of any such wireless system design is generally evaluated in terms of two key metrics – SE and EE which are defined as data rate per unit bandwidth and data rate per unit total power respectively. The major benefit of SM-MIMO over conventional MIMO is that it allows for a reduction in the number of RF chains in order to enhance EE by sacrificing slightly on SE [4], [5]. These performance benefits of SM-MIMO over state-of-the-art MIMO schemes are well experimented and clearly highlighted by Di Renzo *et al.* in [1], [6].

To improve the SE and EE performance of SM-based MIMO systems, approaches involving adaptive designs have

been proposed in the past [1], [7]. Also, there has been a lot of effort to (a) improve the error rate or (b) maximize SE for a given SER or (c) maximize EE by switching between different modulation designs for MIMO & SM-MIMO [8], [9]. In this work, we look to design a system which adaptively operates in a transmission configuration that optimizes a linear combination of SE and EE. Thus, instead of maximizing either of the two metrics as in [10], we look to achieve optimal performance along any desired axis in the plane of SE and EE. For this purpose, we use a fixed candidate set of transmission modes (Section II-B) defined by a unique pair of attributes – the modulation scheme and the number of active RF chains. With this mode definition, both SE and EE can be treated as fixed constants within each mode, determined using the system and power models described ahead in Sections II-A and II-C respectively.

We then propose a Weighted Efficiency Function (WEF) as defined in Section III-B, to form the basis for selecting a transmission mode from the candidate set. WEF considers a weighted sum of SE and EE in different proportions as per the application. This ensures that both SE and EE are together taken into account in the mode selection which involves maximization of this function over the set of all modes. The optimization problem is also bound by upper constraints on average transmission power and allowed SER while a lower constraint on the required bit rate. Assuming the channel characteristic to be Rayleigh fading with a-priori knowledge of the non-zero large-scale fading loss and spatial correlations, we use a closed form approximation of the SER as described in Section II-D to simplify the error constraint in terms of transmitted power. A mode satisfying all the above constraints qualifies to be selected for transmission. The optimization then reduces to a simple exhaustive search among all the qualifying modes with precomputed values of WEF.

## II. FRAMEWORK FOR MODE SELECTION

### A. System and Signal Models

In this paper, we have considered point-to-point communication in MIMO system with number of transmit antennas as  $N_t$ , number of receive antennas as  $N_r$  and number of simultaneously active RF chains as  $N_{a,m}$  in a particular mode  $m$  of the predefined candidate set  $\Phi$ . Moreover, the number

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of RF chains is chosen such that  $N_{a,m} \leq \min(N_t, N_r)$  where  $\min(N_t, N_r)$  is the minimum of  $N_t$  and  $N_r$ . Using the given system parameters, the best mode of operation is selected and it is used to map the information bits to the transmitted SM-based signal which is detected by the receiver. The signal model for the baseband MIMO system [10] with flat fading is given by

$$\mathbf{y} = \sqrt{\frac{G_a P_{tr,m}}{d_{loss}}} \mathbf{R}_r^{1/2} \mathbf{H}_w \mathbf{R}_t^{1/2} \mathbf{x}_{i,m} + \mathbf{n} \quad (1)$$

where  $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$  is the received signal;  $\mathbf{R}_t \in \mathbb{C}^{N_t \times N_t}$  and  $\mathbf{R}_r \in \mathbb{C}^{N_r \times N_r}$  are the transmit correlation and receive correlation matrices;  $\mathbf{H}_w \in \mathbb{C}^{N_r \times N_t}$  is the channel matrix of the uncorrelated Rayleigh fading channel;  $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$  is the zero mean, complex white Gaussian noise with variance  $\sigma_n^2$ ;  $\mathbf{x}_{i,m} \in \mathbb{C}^{N_t \times 1}$  is the  $i^{th}$  complex SM-based signal transmitted via mode  $m$ ;  $P_{tr,m}$  is the average transmit power of mode  $m$  in any current configuration;  $d_{loss}$  is the large-scale fading loss and  $G_a$  is the power gain of the directive antennas.

The SM-based signal [10] can be written as

$$\mathbf{x}_{i,m} = (1/\sqrt{N_{a,m}}) \left( \sum_{k=1}^{N_{a,m}} \mathbf{v}_{l_{i,m}^{(k)}} b_{k,i,m} \right) \quad (2)$$

where  $l_{i,m}^{(k)}$  represents the index of the  $k^{th}$  antenna used in  $\mathbf{x}_{i,m}$ ,  $\mathbf{v}_i$  is the  $N_t \times 1$  vector with only  $i^{th}$  entry 1 and others 0, and  $b_{k,i,m}$  is the  $k^{th}$  string of bits multiplexed using a modulation of order  $N_m$ . The number of spatial constellation points active at a given  $N_{a,m}$  in mode  $m$  is

$$N_{l,m} = 2^{\lfloor \log_2 \binom{N_t}{N_{a,m}} \rfloor} \quad (3)$$

while the total number of points in the spatial constellation diagram is

$$N_{c,m} = (N_m)^{N_{a,m}} N_{l,m}. \quad (4)$$

The rate of transmission is given by [10]

$$S_m = \log_2(N_{l,m}) + N_{a,m} \log_2(N_m) \quad (5)$$

where  $S_m$  is also the SE of the system in bpcu (bits per channel use) or bits/s/Hz.

### B. Transmission Mode Selection

The transmission modes in the candidate set  $\Phi$  are defined by the ordered pair  $(N_m, N_{a,m})$ . Thus, the set  $\Phi$  is obtained by exhaustively pairing each entry in the set of modulation schemes with all the permissible values of the number of RF chains. Each modulation used in  $\Phi$  has a different order, as can also be noted from Table I. This mode definition is sufficient to completely determine the corresponding transmission configuration for each mode  $m$ . Given the predefined candidate set as above, we then adopt the mode selection approach to choose the one mode that yields the maximum value of the proposed function WEF.

### C. Power Consumption Model

Taking into consideration both the transmission power and the circuit power consumption, the model for the power consumed on the transmitter side when operating in mode  $m$  is given by [10]

$$P_{tot,m} = P_{cc,m} + \eta^{-1} \kappa_m P_{tr,m} \quad (6)$$

where  $P_{cc,m}$  is the total circuit power consumption of mode  $m$ ,  $\kappa_m$  is the Peak-to-Average Power Ratio (PAPR) of the adopted modulation in mode  $m$  and  $\eta$  is the power amplifier efficiency. The total power consumption for mode  $m$  is formulated as [11], [12]:

$$P_{tot,m} = R_m P_c + N_{a,m} P_b + P_f + N_{a,m} \eta^{-1} \kappa_m P_{T,m} \quad (7)$$

where  $P_c$  is the power consumed in the process of channel coding,  $P_b$  is the factor responsible for circuit power consumption,  $P_f$  is the fixed power consumption in the circuit,  $P_{T,m} = P_{tr,m}/N_{a,m}$  is the average transmit power per antenna,  $R_m = B S_m$  is the rate of transmission in bits/s for mode  $m$  and  $B$  is the bandwidth. The EE of the system [4], [5] can therefore be obtained using

$$E_m = \frac{R_m}{P_{tot,m}}. \quad (8)$$

### D. SER Approximation for SM-Based MIMO Systems

Here, we discuss a closed-form approximation of SER in SM-based MIMO systems for transmission via Rayleigh fading channel. Considering that  $\mathbf{R}_r$  has distinct nonzero eigenvalues, the pairwise error probability can be expressed as [10]

$$P(x_{i,m} \rightarrow x_{j,m}) = \frac{(2P-1)!}{P!(P-1)!} \left( \frac{1}{\rho_m \psi_{ij,m}} \right)^P \prod_{k=1}^P \xi_k \quad (9)$$

where  $\psi_{ij,m} = (\mathbf{x}_{i,m} - \mathbf{x}_{j,m})^H \mathbf{R}_t (\mathbf{x}_{i,m} - \mathbf{x}_{j,m})$ ,  $\xi_1, \dots, \xi_P$  are the distinct nonzero eigenvalues of  $\mathbf{R}_r$ ,  $P \leq N_r$  denotes the rank of  $\mathbf{R}_r$  and  $\rho_m = (G_a P_{tr,m}) / (\sigma_n^2 d_{loss})$  is the average SNR.

Using the approximation (9) and the SER expression in [13], the bound for SER in mode  $m$  is constructed in [10] as:

$$p_{e,m} \lesssim \frac{(2P-1)!}{P!(P-1)!} \frac{\rho_m^{-P} \psi_{c,m}}{N_{c,m} \prod_{k=1}^P \xi_k} \quad (10)$$

where

$$\psi_{c,m} = \sum_{i=1}^{N_{c,m}} \sum_{j=1, i \neq j}^{N_{c,m}} (\psi_{ij,m})^{-P}. \quad (11)$$

The performance of the above approximation is also evaluated in [10] with the conclusion that it gives accurate result for SER less than  $10^{-5}$ .

### III. PROPOSED METHODOLOGY

In this section, we discuss the motivation to switch from the SE- and EE-based methods to the proposed WEF-based mode selection. Next, we formally define the WEF and describe how it can be modified to serve different applications. Lastly, we define our problem statement and state the mechanism for selecting the transmission configuration on the basis of the proposed function.

#### A. Motivation for WEF-based Mode Selection

Existing techniques in the literature treat SE and EE maximization as two independent problems and try to find the best mode in each case. As stated in [10], SE-based mode selection obtains the optimal value of SE by maximizing the transmission power to the upper limit of its constraint and thus minimizing the SER. The EE-based approach, on the other hand, looks to meet the SER constraint with equality in order to minimize the transmit power and hence maximize the EE. The two solutions therefore lie at the opposite extrema of power and error. This is because the two metrics SE and EE follow completely opposite trends with respect to both power and error. Thus, maximizing either one degrades the other inevitably. Considering this fact from the practical point of view, it is not desirable to compromise one for the other.

The WEF-based method can be useful in eliminating this limitation. It considers both SE and EE together as a single optimization problem rather than maximizing them independently as two separate problems. By virtue of the definition of WEF, the SE-based and EE-based mode selections are just two special cases of the more generalized WEF-based mode selection.

#### B. WEF : Definition

Now that it is clear that we need to come up with a single optimization problem for mode selection, we look to combine SE and EE using a weighted sum of the two efficiencies normalized by their respective maximum values. This ensures that the two quantities are unit-less and within the same scale of 0 to 1. Thus, we define WEF  $W_m$  as

$$W_m = \mu \hat{S}_m + (1 - \mu) \hat{E}_m \quad (12)$$

where  $\hat{S}_m$  and  $\hat{E}_m$  are the normalized values of SE  $S_m$  and EE  $E_m$  respectively for a mode  $m$ , and  $0 \leq \mu \leq 1$  is the relative weight of SE with respect to EE in the WEF. The normalization also ensures that the final value of  $W_m$  also lies on the same efficiency scale, i.e.  $[0, 1]$ . Moreover, the value of  $\mu$  is also an important factor in  $W_m$  and can be varied between 0 and 1 depending upon the application. High data rate requirement implies that we select  $\mu$  greater than 0.5 whereas selecting  $\mu$  less than 0.5 is suitable for low power consumption models.

#### C. WEF-based Mode Selection

Given the maximum allowed average transmit power  $P_{tr,max}$  and SER  $p_{e,max}$ , the minimum required bit rate  $R_{min}$

and the mode definitions as in Section II-B, the problem for WEF-based mode selection can be formulated as:

$$\begin{aligned} m_{opt} &= \arg \max_{m \in \Phi} W_m \\ \text{s.t. } &P_{tr,m} \leq P_{tr,max}, p_{e,m} \leq p_{e,max}, R_m \geq R_{min} \end{aligned} \quad (13)$$

where  $P_{tr,m}$ ,  $p_{e,m}$  and  $R_m$  are the average transmission power, the SER and the bit rate respectively for mode  $m$ .

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#### Algorithm 1 Construction of $\Phi_q$ via mode elimination

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m ← 1
M ← |Φ|
Φq ← null
while m ≤ M do
    Ptr,m ← PT,mNa,m                                ▷ II-A
    if Ptr,m > Ptr,max then
        goto next
    pe,m ←  $\frac{(2P-1)!}{P!(P-1)!} \frac{G_a^{-P} \psi_{c,m}}{(\sigma_n^2 d_{loss})^{-P} N_{c,m} \prod_{k=1}^P \xi_k} P_{tr,m}^{-P}$     ▷ II-D
    if pe,m > pe,max then
        goto next
    Rm ← B(log2(Nl,m) + Na,m log2(Nm))    ▷ II-A&II-C
    if Rm < Rmin then
        goto next
    Add mode m to Φq
next:
m ← m + 1
end while

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The first step of the solution to the above problem is the elimination of modes that do not satisfy one or more constraints. The procedure for the same is described in Algorithm 1, following which, we arrive at a reduced set of qualifying modes  $\Phi_q$ . The expressions and parameters used there in calculating  $P_{tr,m}$ ,  $p_{e,m}$  and  $R_m$  are given in Section II as is also referred in Algorithm 1. Using the  $\Phi_q$  so obtained, (13) simplifies to

$$m_{opt} = \arg \max_{m \in \Phi_q} W_m \quad (14)$$

The next step involves calculating  $S_m$  &  $E_m$  for each qualifying mode using (5) & (8), normalizing them by their respective maximums to obtain  $\hat{S}_m$  &  $\hat{E}_m$  and finally computing  $W_m$  using (12). Once we have the value of WEF for each mode, finding the mode index that maximizes it is as trivial as a simple exhaustive search in  $\Phi_q$ .

### IV. RESULTS AND DISCUSSION

Here, we analyze the simulation results obtained using the proposed WEF-based mode selection technique. We also draw comparisons of the proposed method with the SE-based and EE-based approaches. The values of different system parameters used for this purpose are [11], [12]:  $G_a = 1$ ,  $N_0 = -174$  (dBm/Hz),  $B = 1$  (MHz),  $\eta = 0.1$ ,  $d_{loss} = 102$  (dB),  $P_b = 0.0571$ ,  $P_c = 10^{-8}$ ,  $P_f = 0.1$ ,  $N_t = 8$ ,  $N_r = 4$ ,  $\mathbf{R}_t = \mathbf{I}_{N_t}$  and  $\mathbf{R}_r = \mathbf{I}_{N_r}$  where  $\mathbf{I}_N = N \times N$  identity matrix.

The modes in  $\Phi$  used for generating the results are defined in Table I. Since  $N_r = 4$ ,  $N_{a,m}$  takes values 1–4, thereby yielding 4 modes against each modulation used. Table I also contains the per antenna average transmission power (in dBm) for each constellation which is computed keeping the bit energy and the symbol duration constant. 7 out of these 32 modes are ruled out by the constraints which are chosen as:  $P_{tr,max} = 24$  (dBm),  $p_{e,max} = 10^{-5}$  and  $R_{min} = 8$  (Mbps). The reduced set  $\Phi_q$  therefore contains only 25 modes. Moreover, the constraint on  $p_{e,m}$  ensures that it is at least as good as  $10^{-5}$  in all the mode selection results ahead.

TABLE I: Definition of modes used in  $\Phi$  with  $N_{a,m} = 1-4$ .

Modes	Modulation	$P_{T,m}$	Modes	Modulation	$P_{T,m}$
1-4	BPSK	10.00	17-20	32-QAM	16.99
5-8	QPSK	13.01	21-24	64-QAM	17.78
9-12	8-QAM	14.77	25-28	128-QAM	18.45
13-16	16-QAM	16.02	29-32	256-QAM	19.03

Table II shows a quantitative comparison of the three mode selection techniques using the above constraints at  $\mu = 0.5$ . The first two approaches look to maximize SE and EE respectively whereas the proposed method maximizes WEF. The peak values of SE and EE obtained, which are also used for normalization here, are 38 bits/s/Hz and 13.464 Mbits/J respectively. It is clearly evident from the table that in the existing SE- and EE-based techniques, the other metric degrades so drastically that they fail to reach even one-half of their maximums. On the other hand, the WEF-based approach not only yields a greater weighted sum of the 2 efficiencies, but also ensures that they are comparatively closer to each other and reasonably high enough - both being above 0.5.

TABLE II: Comparison of SE-based, EE-based and WEF-based mode selection results at  $\mu = 0.5$

Mode Selection	Selected Mode			Resultant Efficiencies		
	$m$	$N_m$	$N_{a,m}$	SE	EE	WEF
SE-based	24	64	4	0.7895	0.3865	0.588
EE-based	3	2	3	0.2105	0.9123	0.5614
WEF-based	20	32	4	0.6842	0.5179	0.6011

In Fig. 1, we study the variation of the above efficiency results with the weight factor,  $\mu$ . Fig. 1(a) shows the comparison of the WEF obtained using the 3 selection techniques. It clearly demonstrates that the proposed approach always performs better than or same as the other 2 methods in terms of WEF at all values of  $\mu$ . The range of  $\mu$  from 0.3 to 0.6 is critically important where the WEF-based curve is strictly above the SE-based and EE-based curves. In other intervals, it merges with either of them. Fig. 1(b) shows the WEF-based SE, EE and WEF as they vary with  $\mu$ . As expected, at lower values of  $\mu$ , the proposed method selects a mode with greater EE & lesser SE and vice-versa at higher values of  $\mu$ . Also, the WEF curve bends towards SE at higher  $\mu$  while at lower  $\mu$ , it bends towards EE because of the relative weightage.

Therefore, the two curves collectively suggest that the best choice for value of  $\mu$  is somewhere around 0.45, where we outperform the other techniques by greatest margin in terms of WEF while still performing reasonably well along both SE and EE.

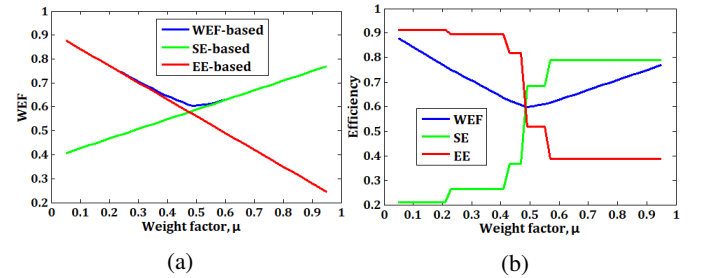


Fig. 1: Variation of the mode selection efficiency results with the weight factor,  $\mu$ . (a) WEF obtained using SE-based, EE-based and WEF-based mode selection. (b) WEF, SE and EE obtained using WEF-based mode selection.

Fig. 2 illustrates the WEF results of the 3 techniques as a function of  $P_{tr,max}$  and  $R_{min}$  at  $\mu = 0.5$ . Here, we examine the efficacy of the proposed WEF-based method with respect to the SE-based and EE-based approaches when the constraints are varied over a range of values. It is obvious from the figures that the WEF-based mode selection always yields greater or equal WEF as compared to SE-based and EE-based mode selections. This proves that the proposed approach outperforms the other two irrespective of the values chosen for the constraints. It is also worth noting that EE in Fig. 2(a) and SE in Fig. 2(b) do not vary at all. This is because EE has an inverse relation with power and SE has a linear relation with data rate and hence their maximums are independent of  $P_{tr,max}$  and  $R_{min}$  respectively.

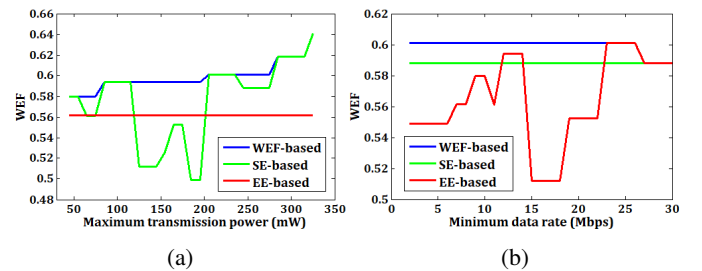


Fig. 2: Comparison of WEF obtained using SE-based, EE-based and WEF-based mode selection as a function of (a)  $P_{tr,max}$ , (b)  $R_{min}$  at  $\mu = 0.5$ .

## V. CONCLUSION

In this paper, we have designed a method for choosing the transmission configuration for an SM-based MIMO system. We adopted the approach of predefining a set of transmission modes and then selecting the best one on the basis of a function that can provide the desired balance between SE and EE. For this purpose, we proposed a function of SE and EE, called

WEF that employs a linear combination of the two metrics in the optimization problem. Next, we verified, with the help of simulation results, that the proposed methodology always yields greater or equal weighted sum of the 2 efficiencies as compared to the state of the art. The WEF-based mode selection also allows for a desired ratio of contribution of SE and EE in that sum, thereby fulfilling the purpose of this work.

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