Impact of Noise Imbalance on the Performance of Predetection Dual-EGC Receivers over Rayleigh Fading Channels

Prabhat Patel
Department of Electrical Engineering
Indian Institute of Technology
Kanpur, India – 208016.
Email: ppatel@iitk.ac.in

A. K. Chaturvedi
Department of Electrical Engineering
Indian Institute of Technology
Kanpur, India – 208016.
Email: akc@iitk.ac.in

***Abstract*—A new expression for the probability density function of the signal-to-noise ratio (SNR) at the output of a predetection dual-equal-gain-combining (dual-EGC) receiver over independent but non-identical Rayleigh fading paths with different noise levels is derived. A closed-form expression for the average bit error rate (ABER) for coherent binary modulation schemes is also derived. The effect of the degree of noise imbalance on ABER is studied. From the results we determine the ranges of noise imbalance for a given average SNR per path over which the performance of predetection dual-EGC is better or worse than SC and single path receivers.**

I. Introduction

Much attention has been devoted in the literature towards performance analysis of various diversity techniques under the assumption that noise powers in different paths are equal. However, in practice, as has been recently pointed out in [1], there may be significant noise power imbalance in different diversity paths. In [1], authors have derived closed-form expressions and analyzed the effect of noise imbalance on imperfect maximal-ratio combining (MRC) over Rayleigh fading path and found that for a 5 dB imbalance the performance of imperfect MRC is worse than even selection combining (SC). In view of this it would be of interest to analyze the effect of noise imbalance on the performance of a predetection dual EGC receiver.

We begin by listing some available closed-form results regarding the performance of EGC. The probability distribution function (pdf) of a dual diversity EGC output in Rayleigh fading [2]. Cumulative distribution function (cdf) of the output envelope of dual-diversity EGC over independent non-identical Rayleigh paths [3]. Cdf and pdf of the output SNR of dual-diversity EGC over independent identical Rayleigh paths as well as the corresponding ABER for coherent binary signals [4]. ABER for binary signals with dual and triple-diversity EGC over independent non-identical Rayleigh paths [5]. Pdf of combined output SNR and average symbol error rate (ASER) of M-ary phase-shift keying signals [6]. Study of performance of dual predetection EGC in correlated Rayleigh fading with unequal path SNRs [7].

In this paper we have considered a new scenario where average signal power as well as average noise power of the two paths are allowed to be non-identical. We analyze the performance of dual-EGC in terms of ABER for coherent binary modulation schemes. The expressions are derived in closed-form and compared with the performance of SC and single path receivers.

The remainder of this paper is organized as follows. Section II presents the system and channel models under consideration. In Section III we derive the pdf of the output SNR. A closed-form expression for ABER for coherent binary modulation schemes is derived in section IV. Section V presents simulation and numerical results illustrating the impact of noise imbalance on the performance of dual-EGC and compare it with SC and single path receivers. Finally, the conclusion is stated in section VI.

II. System and Channel Models

We assume the signal is received at the receiver over two independent, slowly varying, flat Rayleigh fading paths with fading amplitudes α_1 and α_2 whose pdf is given by

$$p_{\alpha_l}(\alpha_l, \Omega_l) = \frac{2\alpha_l}{\Omega_l} e^{-\frac{\alpha_l^2}{\Omega_l}},\ \alpha_l \geq 0, \tag{1}$$

where $\Omega_l = E[\alpha_l^2]$. After passing through the fading path, the l^{th} replica of the signal is perturbed by additive white Gaussian noise (AWGN) with power spectral density denoted by σ_l^2 (W/Hz). We assume here nonidentical noise power for each path, i.e., $\sigma_1^2 \neq \sigma_2^2$. Further, the noise is assumed to be statistically independent for each path and also independent of the fading amplitudes α_1 and α_2. For equally likely transmitted symbols, the output SNR per symbol, γ_{egc} , at the output of the EGC combiner is given by [10]

$$\gamma_{egc} = \frac{(\alpha_1 + \alpha_2)^2 E_s}{\sigma_1^2 + \sigma_2^2}, \tag{2}$$

where E_s (J) is the average energy per transmitted symbol.

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III. PDF OF OUTPUT SNR

The need for finding the pdf of γ arises form the fact that the existing expressions for the same in literature are not suitable to study the effect of noise imbalance on the performance of predetection dual-EGC as all the previous works have assumed equal noise variances in both paths. Next we derive the expression for pdf of γ.

Let $\beta = \alpha_1 + \alpha_2$, then the pdf of β is given by [8]

$$p_\beta(\beta) = \int_0^\beta p_{\alpha_1}(\alpha_1) p_{\alpha_2}(\beta - \alpha_1) d\alpha_1. \quad (3)$$

Substituting (1) in (3) and solving the integral, algebraic manipulations leads to (see Appendix A)

$$\begin{aligned} p_\beta(\beta) = & \frac{4e^{-\beta^2/(\Omega_1+\Omega_2)}}{\Omega_1\Omega_2} \\ & \{B(2,1)a^2b\ {}_2F_2(1/2,1;3/2,2;-k_1a^2) \\ & + B(2,2)a^3\ {}_2F_2(1,3/2;2,5/2;-k_1a^2) \\ & + B(2,1)ab^2\ {}_2F_2(1/2,1;3/2,2;-k_1b^2) \\ & + B(2,2)b^3\ {}_2F_2(1,3/2;2,5/2;-k_1b^2)\}, \end{aligned} \quad (4)$$

where k_1, a, and b have their meanings as defined in the Appendix A. $B(c,d)$ is Beta function given in (8.380.1) of [9], and ${}_2F_2$ is generalized hypergeometric function as defined in (9.14) of [9].

Substituting $B(2,1) = 1/2$ and $B(2,2) = 1/6$ and substituting the value of k_1, a, b, we get

$$\begin{aligned} p_\beta(\beta) = & \frac{2\beta^3 e^{-\beta^2/(\Omega_1+\Omega_2)}}{\Omega_1\Omega_2(\Omega_1+\Omega_2)^3} \\ & \Big\{\Omega_1^2\Omega_2\ {}_2F_2(1/2,1;3/2,2;-\frac{\beta^2\Omega_1}{\Omega_2(\Omega_1+\Omega_2)}) \\ & + \frac{1}{3}\Omega_1^3\ {}_2F_2(1,3/2;2,5/2;-\frac{\beta^2\Omega_1}{\Omega_2(\Omega_1+\Omega_2)}) \\ & + \Omega_1\Omega_2^2\ {}_2F_2(1/2,1;3/2,2;-\frac{\beta^2\Omega_2}{\Omega_1(\Omega_1+\Omega_2)}) \\ & + \frac{1}{3}\Omega_2^3\ {}_2F_2(1,3/2;2,5/2;-\frac{\beta^2\Omega_2}{\Omega_1(\Omega_1+\Omega_2)})\Big\}. \end{aligned} \quad (5)$$

Let $\gamma_x = \beta^2$, then from [8], $p_{\gamma_x}(\gamma_x) = \frac{1}{2\sqrt{\gamma_x}} p_\beta(\sqrt{\gamma_x})$.

Finally, since $\gamma_{egc} = \frac{\gamma_x E_s}{\sigma_1^2+\sigma_2^2}$, hence

$$p_{\gamma_{egc}}(\gamma_{egc}) = \frac{\sigma_1^2+\sigma_2^2}{E_s} p_{\gamma_x}(\frac{\gamma_{egc}(\sigma_1^2+\sigma_2^2)}{E_s}). \quad (6)$$

Denoting noise imbalance by $\eta = \frac{\sigma_2^2}{\sigma_1^2}$, average SNR of first path by $\bar{\gamma_1} = \frac{\Omega_1 E_s}{\sigma_1^2}$, and average SNR of second path by $\bar{\gamma_2} = \frac{\Omega_2 E_s}{\sigma_2^2}$, the pdf of γ_{egc}, henceforth denoted by γ, can be found to be

$$\begin{aligned} p_\gamma(\gamma) = & \frac{(1+\eta)^2\gamma e^{\frac{-(1+\eta)\gamma}{\bar{\gamma_1}+\eta\bar{\gamma_2}}}}{(\bar{\gamma_1}+\eta\bar{\gamma_2})^3} \times \\ & \Big\{\bar{\gamma_1}\ {}_2F_2(1/2,1;3/2,2;-\frac{(1+\eta)\bar{\gamma_1}\gamma}{\eta\bar{\gamma_2}(\bar{\gamma_1}+\eta\bar{\gamma_2})}) \\ & + \frac{1}{3}\frac{\bar{\gamma_1}^2}{\eta\bar{\gamma_2}}\ {}_2F_2(1,3/2;2,5/2;-\frac{(1+\eta)\bar{\gamma_1}\gamma}{\eta\bar{\gamma_2}(\bar{\gamma_1}+\eta\bar{\gamma_2})}) \\ & + \eta\bar{\gamma_2}\ {}_2F_2(1/2,1;3/2,2;-\frac{\eta(1+\eta)\bar{\gamma_2}\gamma}{\bar{\gamma_1}(\bar{\gamma_1}+\eta\bar{\gamma_2})}) \\ & + \frac{1}{3}\frac{(\eta\bar{\gamma_2})^2}{\bar{\gamma_1}}\ {}_2F_2(1,3/2;2,5/2;-\frac{\eta(1+\eta)\bar{\gamma_2}\gamma}{\bar{\gamma_1}(\bar{\gamma_1}+\eta\bar{\gamma_2})})\Big\}, \end{aligned} \quad (7)$$

which reduces to the expression provided in (4) of [6] for the balanced noise case (i.e., $\eta = 1$) as shown in Appendix A.

IV. AVERAGE BIT ERROR RATE

The conditional BER of an EGC receiver is given by [10]

$$P_b(E|\gamma) = Q(\sqrt{2g\gamma}), \quad (8)$$

where $g = 1$ for BPSK and $g = 1/2$ for orthogonal BFSK. $Q(.)$ denotes the Gaussian Q-function defined in (4.1) of [10]. Hence the ABER is given by

$$P_b(E) = \int_0^\infty Q(\sqrt{2g\gamma}) p_\gamma(\gamma) d\gamma. \quad (9)$$

Substituting for $Q(\sqrt{2g\gamma})$ and changing the order of integration, we get

$$\begin{aligned} P_b(E) &= \frac{1}{\sqrt{2\pi}} \int_0^\infty \int_0^{x^2/2g} p_\gamma(\gamma) d\gamma e^{-x^2/2} dx \\ &= \frac{1}{\sqrt{2\pi}} \int_0^\infty I(x) e^{-x^2/2} dx, \end{aligned} \quad (10)$$

where $I(x) = \int_0^{x^2/2g} p_\gamma(\gamma) d\gamma$. The integration in $I(x)$ can be solved using steps shown in Appendix B and is given in (19).

Substituting (19) in (10), after some algebraic manipulations (see Appendix B) $P_b(E)$ can be given by

$$\begin{aligned} P_b(E) = & \frac{1}{(\bar{\gamma_1}+\eta\bar{\gamma_2})} \sum_{n=0}^\infty \frac{(-1)^n}{2^{2(n+2)}} \binom{2n+3}{n+2} \times \\ & \Big\{\frac{1}{(2n+1)}\left(\frac{\bar{\gamma_1}^{n+1}}{(\eta\bar{\gamma_2})^n} + \frac{(\eta\bar{\gamma_2})^{n+1}}{\bar{\gamma_1}^n}\right) \\ & + \frac{1}{(2n+3)}\left(\frac{\bar{\gamma_1}^{n+2}}{(\eta\bar{\gamma_2})^{n+1}} + \frac{(\eta\bar{\gamma_2})^{n+2}}{\bar{\gamma_1}^{n+1}}\right)\Big\} \times \\ & \left(\frac{(1+\eta)}{g(\bar{\gamma_1}+\eta\bar{\gamma_2})}\right)^{n+2} \times \\ & {}_2F_1(n+5/2, n+2; n+3; -\frac{1+\eta}{g(\bar{\gamma_1}+\eta\bar{\gamma_2})}). \end{aligned} \quad (11)$$

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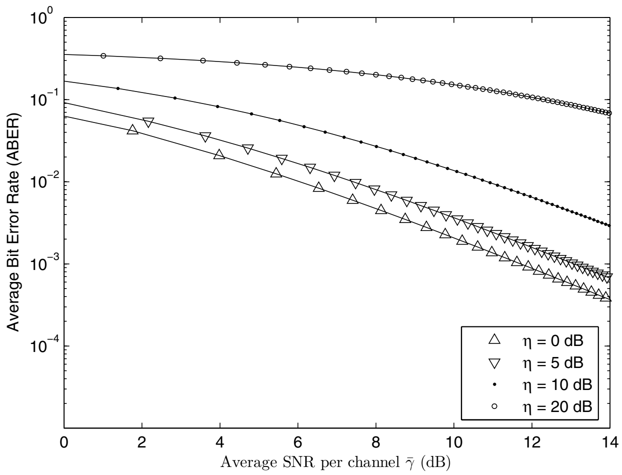

Fig. 1. ABER versus average SNR per path $\bar{\gamma}$ for several values of η for dual-EGC.

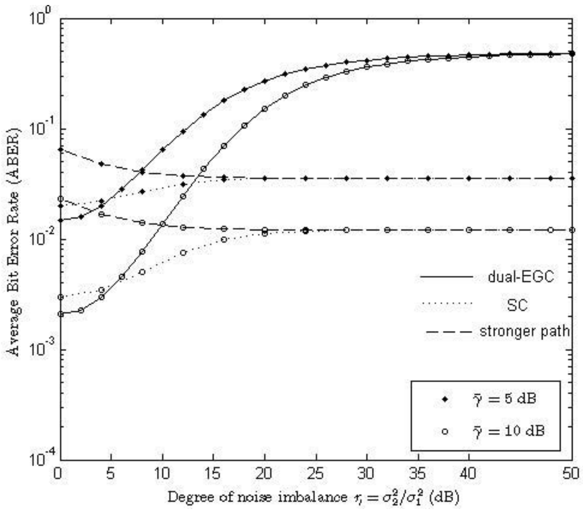

Fig. 2. ABER versus η for dual-EGC, SC and single path for various values of average SNR per path $\bar{\gamma}$.

V. Simulation and Numerical results

In this section we present the ABER performance of a dual-EGC receiver for various degrees of noise imbalance keeping the average signal power same in both the paths i.e. $\Omega_1 = \Omega_2$. Hence, in this case the noise imbalance η turns out to be same as $\bar{\gamma_1}/\bar{\gamma_2}$. We compare the ABER performance with the SC and the single path receivers. By a single path receiver we mean a receiver that takes only one path as the input and that path is the one with the higher average SNR. Hence its performance cannot be better than the SC receiver. We have used (11) to plot the ABER curves for dual-EGC receiver and the standard expressions for the SC and single path receivers [10]. Fig. 1 presents the ABER for BPSK for dual-EGC as a function of $\bar{\gamma} = (\bar{\gamma_1} + \bar{\gamma_2})/2$ for different values of η. The markers in the figure represent the simulation results whereas solid lines represent the curves obtained by numerically evaluating the derived expression. From the figure it can be clearly observed that, as expected, the performance of the dual-EGC receiver degrades in the presence of noise imbalance. In fact, larger the imbalance worse is the performance. For example, for an ABER of 10^{-2}, the SNR penalty of the dual-EGC receiver is about 1 dB for $\eta = 5$ dB and 5 dB for $\eta = 10$ dB where the comparison is with respect to the case when $\eta = 0$ dB.

Fig. 2 presents the ABER for BPSK for dual-EGC, SC and single path receivers with respect to η for different values of $\bar{\gamma}$. From the figure, it can be seen that the performance of dual-EGC degrades as η increases and as expected for large η it approaches the worst possible BER i.e. 0.5, for both the values of $\bar{\gamma}$. It can also be seen that for a given $\bar{\gamma}$, beyond some value of η the performance of dual-EGC is worse than that of SC. Thus for $\bar{\gamma} = 10$ dB the performance of dual-EGC is worse than that of SC when η is greater than 5.2 dB where the two curves intersect. Interestingly, for the same $\bar{\gamma}$, the dual-EGC and single path curves intersect at $\eta = 10$ dB. Thus for $\eta > 10$ dB dual-EGC is worse than even the single path receiver. It can also be observed that for a given $\bar{\gamma}$, as η increases the performances of SC and single path approach each other and become identical for large η. The explanation of this lies in the fact that for higher values of η the probability of weaker path to be chosen by SC at any instant becomes very low which results in the average performance of SC being same as the single path receiver.

Table I lists for various values of $\bar{\gamma}$, the values of $\bar{\gamma_1}$, $\bar{\gamma_2}$ and η at which the ABER curves of dual-EGC and SC intersect.

TABLE I
INTERSECTION POINTS OF THE BER FOR DUAL-EGC AND SC FOR VARIOUS VALUES OF AVERAGE SNR PER PATH ($\bar{\gamma}$)

$\bar{\gamma}$	$\bar{\gamma_1}$	$\bar{\gamma_2}$	η
0	1.59	-2.51	4.1
2	3.67	-0.73	4.4
4	5.71	1.11	4.6
6	7.77	2.97	4.8
8	9.81	4.81	5.0
10	11.86	6.66	5.2
12	13.91	8.51	5.4
14	15.93	10.43	5.5
16	17.93	12.43	5.5
18	19.95	14.35	5.6
20	21.95	16.35	5.6

TABLE II
INTERSECTION POINTS OF THE BER FOR DUAL-EGC AND SINGLE PATH FOR VARIOUS VALUES OF AVERAGE SNR PER PATH ($\bar{\gamma}$)

$\bar{\gamma}$	$\bar{\gamma_1}$	$\bar{\gamma_2}$	η
0	2.09	-4.21	6.3
2	4.2	-2.7	6.9
4	6.29	-1.11	7.4
6	8.4	0.2	8.2
8	10.5	1.5	9.0
10	12.59	2.59	10.0
12	14.66	3.86	10.8
14	16.72	5.02	11.7
16	18.78	6.08	12.7
18	20.82	7.22	13.6
20	22.86	8.26	14.6

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All the entries are in dB. The intersection values of η have been obtained from the curves plotted using (11) and standard expressions of SC for ABER versus η for different values of $\bar{\gamma}$. The values of $\bar{\gamma_1}$ and $\bar{\gamma_2}$ listed in the table have been evaluated using the relations $\bar{\gamma_1} = 2\bar{\gamma}\eta/(1+\eta)$ and $\bar{\gamma_2} = 2\bar{\gamma}/(1+\eta)$.

Similarly, Table II is for the case when dual-EGC is compared with the single path receiver. It can be seen that, as expected, the value of the intersection points η in Table I are always less than the corresponding intersection points in Table II.

Based on the entries of the tables it can be ascertained which of the three systems provides a better performance for a given value of $\bar{\gamma}$ and η. For example, for $\bar{\gamma} = 10$ dB and $\eta \leq 5.2$ dual-EGC performs better than the other two systems, while for $5.2 < \eta \leq 10$ dual-EGC performs worse than SC but better than single path, and for $\eta > 10$ dual-EGC performs worse than both SC and single path receivers.

VI. Conclusion

We derived a new expression for the probability density function of the output SNR of the dual-EGC receiver when the noise variances in two paths are different. Using this expression, a closed-form expression for ABER for binary modulation schemes has been derived. We conclude that the imbalance in noise variances has a severe negative impact on the performance of EGC receiver. The performance of the EGC receiver is also compared with the performance of SC and single path receivers for various degrees of noise imbalance. From these comparisons it is concluded that depending on the average SNR and noise imbalance, a dual-EGC receiver may give an ABER performance which is worse than SC and for even higher values of noise imbalance it becomes worse than even a single path receiver.

Appendix

A: PDF of Output SNR

Substituting (1) in (3) yields

$$p_\beta(\beta) = \frac{4e^{-\beta^2/(\Omega_1+\Omega_2)}}{\Omega_1\Omega_2} \int_0^\beta (\beta\alpha_1 - \alpha_1^2) e^{-k_1(\alpha_1 - k_2/k_1)^2} d\alpha_1, \quad (12)$$

where $k_1 = 1/\Omega_1 + 1/\Omega_2$ and $k_2 = \beta/\Omega_2$.

Now put $\alpha_1 - k_2/k_1 = x$, such that $d\alpha_1 = dx$, when $\alpha_1 = 0$, $x = -k_2/k_1$, and when $\alpha_1 = \beta$, $x = \beta - k_2/k_1$. Equation (12) after simplification becomes

$$p_\beta(\beta) = \frac{4e^{-\beta^2/(\Omega_1+\Omega_2)}}{\Omega_1\Omega_2} \int_{-\beta\Omega_1/(\Omega_1+\Omega_2)}^{\beta\Omega_2/(\Omega_1+\Omega_2)} (\frac{\beta\Omega_2}{\Omega_1+\Omega_2} - x) (\frac{\beta\Omega_1}{\Omega_1+\Omega_2} + x) e^{-k_1x^2} dx. \quad (13)$$

Denoting $a = \frac{\beta\Omega_1}{\Omega_1+\Omega_2}$ and $b = \frac{\beta\Omega_2}{\Omega_1+\Omega_2}$, above equation can be Simplified using (3.478.3) of [9] to get (4).

The equivalence of (4) of [6] and (7) can be established as follows:

For $\eta = 1$, (7) reduces to

$$\begin{aligned} p_\gamma(\gamma) = &\frac{4\gamma e^{\frac{-2\gamma}{\bar{\gamma_1}+\bar{\gamma_2}}}}{(\bar{\gamma_1}+\bar{\gamma_2})^3} \Big\{ \bar{\gamma_1}\ {}_2F_2(1/2,1;3/2,2;-\frac{2\bar{\gamma_1}\gamma}{\bar{\gamma_2}(\bar{\gamma_1}+\bar{\gamma_2})}) \\ &+ \frac{1}{3}\frac{\bar{\gamma_1}^2}{\bar{\gamma_2}}\ {}_2F_2(1,3/2;2,5/2;-\frac{2\bar{\gamma_1}\gamma}{\bar{\gamma_2}(\bar{\gamma_1}+\bar{\gamma_2})}) \\ &+ \bar{\gamma_2}\ {}_2F_2(1/2,1;3/2,2;-\frac{2\bar{\gamma_2}\gamma}{\bar{\gamma_1}(\bar{\gamma_1}+\bar{\gamma_2})}) \\ &+ \frac{1}{3}\frac{(\bar{\gamma_2})^2}{\bar{\gamma_1}}\ {}_2F_2(1,3/2;2,5/2;-\frac{2\bar{\gamma_2}\gamma}{\bar{\gamma_1}(\bar{\gamma_1}+\bar{\gamma_2})}) \Big\}. \end{aligned} \quad (14)$$

Using Table 7.12.1 of [11] the generalized hypergeometric functions in the above equation can be written in terms of confluent hypergeometric functions. Applying Kummer transformation using (9.212.1) of [9] to these functions and using Table 7.11.2 of [11] simple algebraic manipulations lead to

$$p_\gamma(\gamma) = \frac{4\gamma e^{\frac{-2\gamma}{\bar{\gamma_1}+\bar{\gamma_2}}}}{(\bar{\gamma_1}+\bar{\gamma_2})^3}(A+B), \quad (15)$$

where

$$\begin{aligned} A = &\frac{\bar{\gamma_2}(\bar{\gamma_1}+\bar{\gamma_2})}{2\gamma} e^{\frac{-2\bar{\gamma_1}\gamma}{\bar{\gamma_2}(\bar{\gamma_1}+\bar{\gamma_2})}} \\ &+ \sqrt{\frac{\pi\bar{\gamma_1}\bar{\gamma_2}(\bar{\gamma_1}+\bar{\gamma_2})^3}{32\gamma^3}} \left(\frac{4\gamma}{\bar{\gamma_1}+\bar{\gamma_2}} - 1\right) \\ &\left(1 - 2Q(2\sqrt{\frac{\bar{\gamma_1}\gamma}{\bar{\gamma_2}(\bar{\gamma_1}+\bar{\gamma_2})}})\right) \end{aligned}$$

and

$$\begin{aligned} B = &\frac{\bar{\gamma_1}(\bar{\gamma_1}+\bar{\gamma_2})}{2\gamma} e^{\frac{-2\bar{\gamma_2}\gamma}{\bar{\gamma_1}(\bar{\gamma_1}+\bar{\gamma_2})}} \\ &+ \sqrt{\frac{\pi\bar{\gamma_1}\bar{\gamma_2}(\bar{\gamma_1}+\bar{\gamma_2})^3}{32\gamma^3}} \left(\frac{4\gamma}{\bar{\gamma_1}+\bar{\gamma_2}} - 1\right) \\ &\left(1 - 2Q(2\sqrt{\frac{\bar{\gamma_2}\gamma}{\bar{\gamma_1}(\bar{\gamma_1}+\bar{\gamma_2})}})\right). \end{aligned}$$

Further simplifying (15) we get (4) of [6].

B: Average Bit Error Rate

Integration of $I(x)$:

Substituting for $p_\gamma(\gamma)$ from (7) in the integral and changing the variable $y = \frac{(1+\eta)\gamma}{\bar{\gamma_1}+\eta\bar{\gamma_2}}$ we get

$$\begin{aligned} I(x) = &\frac{1}{(\bar{\gamma_1}+\eta\bar{\gamma_2})} \int_0^{\frac{(1+\eta)x^2}{2g(\bar{\gamma_1}+\eta\bar{\gamma_2})}} ye^{-y} \\ &\Big\{ \bar{\gamma_1}\ {}_2F_2(1/2,1;3/2,2;-\frac{\bar{\gamma_1}y}{\eta\bar{\gamma_2}}) \\ &+ \frac{1}{3}\frac{\bar{\gamma_1}^2}{\eta\bar{\gamma_2}}\ {}_2F_2(1,3/2;2,5/2;-\frac{\bar{\gamma_1}y}{\eta\bar{\gamma_2}}) \end{aligned}$$

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$$+\eta\bar{\gamma_2}\ {}_2F_2(1/2,1;3/2,2;-\frac{\eta\bar{\gamma_2}y}{\bar{\gamma_1}}) + \frac{1}{3}\frac{(\eta\bar{\gamma_2})^2}{\bar{\gamma_1}}\ {}_2F_2(1,3/2;2,5/2;-\frac{\eta\bar{\gamma_2}y}{\bar{\gamma_1}})\Big\}dy. \quad (16)$$

Now, since [9]

$${}_2F_2(1/2,1;3/2,2;z) = \sum_{n=0}^{\infty}\frac{\alpha_n z^n}{n!},\ where\ \alpha_0 = 1,$$
$$\frac{\alpha_{n+1}}{\alpha_n} = \frac{(n+1/2)(n+1)}{(n+3/2)(n+2)},\ and$$
$$\alpha_n = \frac{1}{(n+1)(2n+1)}, \quad (17)$$

also

$${}_2F_2(1,3/2;2,5/2;z) = \sum_{n=0}^{\infty}\frac{\alpha_n z^n}{n!},\ where\ \alpha_0 = 1,$$
$$\frac{\alpha_{n+1}}{\alpha_n} = \frac{(n+1)(n+3/2)}{(n+2)(n+5/2)},\ and$$
$$\alpha_n = \frac{3}{(n+1)(2n+3)}. \quad (18)$$

Substituting these in (16), simplifying and using (3.381.1) of [9], yields

$$I(x) = \frac{1}{(\bar{\gamma_1}+\eta\bar{\gamma_2})}\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)!}\times \Bigg\{\frac{1}{(2n+1)}\left(\frac{\bar{\gamma_1}^{n+1}}{(\eta\bar{\gamma_2})^n}+\frac{(\eta\bar{\gamma_2})^{n+1}}{\bar{\gamma_1}^n}\right) + \frac{1}{(2n+3)}\left(\frac{\bar{\gamma_1}^{n+2}}{(\eta\bar{\gamma_2})^{n+1}}+\frac{(\eta\bar{\gamma_2})^{n+2}}{\bar{\gamma_1}^{n+1}}\right)\Bigg\}\times \Upsilon\left(n+2,\frac{(1+\eta)x^2}{2g(\bar{\gamma_1}+\eta\bar{\gamma_2})}\right), \quad (19)$$

where $\Upsilon(\cdot,\cdot)$ is the incomplete gamma function.

Evaluation of ABER:

Substituting (19) in (10), and changing the order of integration and summation we get

$$P_b(E) = \frac{1}{\sqrt{2\pi}(\bar{\gamma_1}+\eta\bar{\gamma_2})}\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)!}\times \Bigg\{\frac{1}{(2n+1)}\left(\frac{\bar{\gamma_1}^{n+1}}{(\eta\bar{\gamma_2})^n}+\frac{(\eta\bar{\gamma_2})^{n+1}}{\bar{\gamma_1}^n}\right) + \frac{1}{(2n+3)}\left(\frac{\bar{\gamma_1}^{n+2}}{(\eta\bar{\gamma_2})^{n+1}}+\frac{(\eta\bar{\gamma_2})^{n+2}}{\bar{\gamma_1}^{n+1}}\right)\Bigg\}\times \int_0^{\infty} e^{-x^2/2}\,\Upsilon\left(n+2,\frac{(1+\eta)x^2}{2g(\bar{\gamma_1}+\eta\bar{\gamma_2})}\right)dx. \quad (20)$$

Put $x^2/2 = u$, so that $xdx = du\ or\ dx = du/\sqrt{2u}$, when $x = 0, u = 0$ and $x = \infty, u = \infty$, above equation becomes

$$P_b(E) = \frac{1}{2\sqrt{\pi}(\bar{\gamma_1}+\eta\bar{\gamma_2})}\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)!}\times \Bigg\{\frac{1}{(2n+1)}\left(\frac{\bar{\gamma_1}^{n+1}}{(\eta\bar{\gamma_2})^n}+\frac{(\eta\bar{\gamma_2})^{n+1}}{\bar{\gamma_1}^n}\right) + \frac{1}{(2n+3)}\left(\frac{\bar{\gamma_1}^{n+2}}{(\eta\bar{\gamma_2})^{n+1}}+\frac{(\eta\bar{\gamma_2})^{n+2}}{\bar{\gamma_1}^{n+1}}\right)\Bigg\}\times \int_0^{\infty} u^{-1/2}e^{-u}\,\Upsilon\left(n+2,\frac{(1+\eta)u}{g(\bar{\gamma_1}+\eta\bar{\gamma_2})}\right)du. \quad (21)$$

Solving the above integral using (6.455.2) of [9], yields

$$P_b(E) = \frac{1}{2\sqrt{\pi}(\bar{\gamma_1}+\eta\bar{\gamma_2})}\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)!}\times \Bigg\{\frac{1}{(2n+1)}\left(\frac{\bar{\gamma_1}^{n+1}}{(\eta\bar{\gamma_2})^n}+\frac{(\eta\bar{\gamma_2})^{n+1}}{\bar{\gamma_1}^n}\right) + \frac{1}{(2n+3)}\left(\frac{\bar{\gamma_1}^{n+2}}{(\eta\bar{\gamma_2})^{n+1}}+\frac{(\eta\bar{\gamma_2})^{n+2}}{\bar{\gamma_1}^{n+1}}\right)\Bigg\}\times \frac{\left(\frac{(1+\eta)}{g(\bar{\gamma_1}+\eta\bar{\gamma_2})}\right)^{(n+2)}\Gamma(n+5/2)}{(n+2)\left(\frac{(1+\eta)}{g(\bar{\gamma_1}+\eta\bar{\gamma_2})}+1\right)^{n+5/2}}\times {}_2F_1\left(1,n+5/2;n+3;\frac{1}{1+\frac{g(\bar{\gamma_1}+\eta\bar{\gamma_2})}{(1+\eta)}}\right). \quad (22)$$

Simplifying this equation using (9.131.1) and (8.339.2) of [9], we get (11).

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