Application of Steiner Design for Capacity Enhancement of Frequency Hopping Spread Spectrum Wireless systems

S. Chakrabarti A. K. Chaturvedi
Department of Electrical Engineering, Indian Institute of Technology, Kanpur, India
Email: akc@iitk.ac.in

ABSTRACT

A slow frequency hopping code diversity system with a bandwidth efficient modulation scheme based on balanced incomplete block (BIB) design — the Steiner design has been proposed. The proposed system uses more than one frequency bins simultaneously to transmit a symbol. The system performance is evaluated and compared with the MFSK slow frequency hopping code diversity system for a multi-user environment considering non-fading and frequency non-selective slowly time varying Rayleigh fading channel. The performance has also been studied with matched frequency hopping (MFH), an efficient addressing technique for slow frequency selective dispersive channels.

INTRODUCTION

In a conventional fast frequency hopping M-ary frequency shift keying (FFH-MFSK) system [1,2,3] the signaling set is formed by M orthogonal waveforms. An extension to this modulation scheme is a multiple tone FSK system [4], which can further be extended to a modulation scheme [5] where the MFSK modulator divides equally its energy among more than one waveforms and results in a bandwidth efficiency improvement. A code diversity slow frequency hopping (SFH) multiple access system has been investigated in [6]. Here we have studied the performance of a SFH code diversity system with bandwidth efficient modulation scheme based on combinatorial theory of balanced incomplete block (BIB) design — Steiner design.

SYSTEM MODEL

In Steiner system based on BIB design, the M-ary FSK modulator divides equally the energy of an FSK block among $w > 1$ waveforms. The signaling frames are then represented by arrangements of v elements into b blocks such that each block contains w distinct elements, each element occurs in exactly r different blocks and every pair of distinct elements occurs together in λ blocks [7], so that the minimum Hamming distance, which determines effective diversity of the system [8], between the frames is increased. For $\lambda = 1$, the system is called Steiner system with parameters (v, b, r, w, λ) for the admissible values of v and w [7]. The transmitter selects, according to the input code word (of k bits), a block of w elements from Steiner set. Then a v-ary FSK modulator divides its energy among w orthogonal waveforms. Each of these waveforms is now frequency hopped by a random hop code generator in L distinct frequency bins out of q available. Now, these $L \times w$ waveforms are transmitted simultaneously. The receiver (synchronized to the transmitter and knows the hopping pattern used by a given user) consists of a group of v matched filters followed by noncoherent detectors.

PERFORMANCE ANALYSIS

(A) Basic Assumptions

1) The data symbols for each transmitter are statistically independent and equally probable and each of $\binom{q}{L}$ possible hopping patterns is chosen with equal probability.

2) The set of N interferers (for a transmitter T) be partitioned into v symbol groups $G_1, \ldots, G_v$. Each interferer in group G_i transmits symbol i, $i = 1, \ldots, v$ and we denote the number of such interferers by N_i. In the case of complete hit, the receiver takes random decision [6].

Let $Q(i / n_s)$ be the probability of having exactly i of T's L frequency bins hit by the n_s interferers transmitting symbol 's'. The probability $Q(i / n_s)$ can be calculated using the following recursive equation [6].

$$Q(i/n_s) = \sum_{j=0}^{i} Q(i-j/n_s) \frac{\binom{L-i+j}{j}\binom{q-L+i-j}{L-j}}{\binom{q}{L}} \qquad (1)$$

With initial condition

0-7803-5893-7/00/$10.00 © 2000 IEEE

$$Q(i/n_s) = \begin{cases} 1 & for \quad i=0 \\ 0 & otherwise \end{cases} \tag{2}$$

The probability of symbol error due to user interference is given by [6]

$$P_e(N,q,L/N_2=n_2,N_3=n_3,......N_v) = \frac{1}{2}\sum_{i=2}^{v} P_i\prod_{j\neq i}(1-P_j)+\frac{2}{3}\sum_{2\leq i<j\leq v} P_iP_j\prod_{k\neq i,j}(1-P_k)+... + \frac{(v-1)}{v}\prod_{i=2}^{v}P_i \tag{3}$$

Where $P_i = Q(1/n_i)$, i ∈ {2, 3, … , v} and n_2 is the number of users transmitting the symbol "2" and so on. The probability associated with a particular distribution of the interferers is given by the multinomial distribution [9].

$$\Pr\{N_2=n_2,N_3=n_3,......N_v\}=\frac{M!}{n_2!n_3!....n_v!}\left(\frac{1}{v}\right)^{N} \tag{4}$$

Further

$$P_e(N,q,L)=\sum_{n_2+n_3.....+n_v=N}\Pr\{N_2=n_2,N_3=n_3,......N_v=n_v\} \times P_e(N,q,L/N_2=n_2,N_3=n_3,.....N_v=n_v), \tag{5}$$

Therefore, the probability of symbol error due to user interference is obtained using equations (1) –(5).

(B) Non-fading Channel

Let $B_j = (b_{j1}, b_{j2}, \dots, b_{jw})$, for $j = 1, 2, \dots, b$, be a block in the Steiner set. The receiver is based on the square law combining of the matched filter outputs R_{jm}. Therefore; each decision variable is obtained as [5]

$$U_j = \sum_{m=1}^{w}\left|R_{jm}\right|^2 \quad \text{for } j=1,\dots,b \tag{6}$$

The noncoherent detected outputs R_{jm}'s are square-law combined to obtain a decision variable.

Let B_i be the transmitted block, with U_i its decision variable and consider a block B_j ($j \neq i$), with a decision variable U_j. Two cases are then observed [5]:

- $B_j \in X_0(B_i)$, (B_j has no element in common to B_i)
- $B_j \in X_1(B_i)$, (B_j has only one element in common to B_i)

The number of blocks in above cases is given by x_0 and x_1 respectively. Now, a decision error is made when the block B_j ($j \neq i$) is detected. Let U_j^0 and U_j^1 be the decision variables corresponding to $B_j^0 \in X_0(B_i)$ and $B_j^1 \in X_1(B_i)$ respectively. The probability of symbol error can be upper bounded by [5]

$$P_d(\gamma,w) \leq x_0P(U_j^0 > U_i) + x_1P(U_j^1 > U_i) \tag{7}$$

where

$$P(U_i - U_j^0 < 0) = P(E_0 < 0) \tag{8a}$$

$$E_0 = \sum_{m=1}^{w}(\left|R_{im}\right|^2 - \left|R_{jm}\right|^2) \tag{8b}$$

$$P(U_i - U_j^1 < 0) = P(E_1 < 0) \tag{9a}$$

$$E_1 = \sum_{m=1}^{w-1}(\left|R_{im}\right|^2 - \left|R_{jm}\right|^2) \tag{9b}$$

The decision error probability for two blocks separated by a Hamming distance of l is bounded by [10]

$$P(\gamma,l) \leq \frac{e^{-k\gamma/2}}{2^{2l-1}}\sum_{i=0}^{l-1}\frac{1}{i!}\left(\frac{k\gamma}{2}\right)^{i}\sum_{s=0}^{l-1-n}\binom{2l-1}{s} \tag{10}$$

where γ is the signal-to-noise ratio per bit. Therefore, equation (8a) and (9a) can be obtained as [5]

$$P(E_0 < 0) = P(\gamma, l = w) \tag{11a}$$

$$P(E_1 < 0) = P(\gamma, l = w-1) \tag{11b}$$

Let P_u = Pr {user interference on $i \leq w$ Steiner elements} $= \binom{w}{i}P_e^{\,i}(1-P_e)^{w-i}$ (12)

where P_e is the probability of user interference in a symbol given by (5). If $P_d(\gamma, w|j)$ is the probability that j slots (among w frequency slots) in a Steiner block are hit by other interfering users, the probability of Steiner symbol error when (w-1) slots are interfered is bounded by [5]

$$P_d(\gamma, w|w-1) \leq x_1P(E_1 < 0) \tag{13}$$

When all w transmitted slots are hit, the Steiner symbol error probability is bounded by

$$P_d(\gamma, w|w) \leq x_0P(E_0 < 0) + x_1P(E_1 < 0) \tag{14}$$

Therefore, the Steiner coded symbol error probability is given by

$$P_{St} = \sum_{i=d_{min}/2}^{w} P_i P_d(\gamma, w|i) \quad (15)$$

The bit error probability is obtained as [10]

$$P_b = \frac{2^{k-1}}{2^k - 1} \sum_{j=(L-1)/2}^{L} \binom{L}{j} P_{St}^{\,j} (1 - P_{St})^{L-j} \quad (16)$$

Fig.1 shows simulation plot of bit error rate (BER) vs. diversity (L) at Eb/No=25 dB for non-fading channel (considering AWGN) for N = 20, 25 & 30 users. BER attains a minimum value near L = 3 for Steiner system & near L = 5 for MFSK system.

Fig.2 shows the BER performance with optimum code diversity L=3, q=100, v=13 for Steiner system & with L = 5 (other parameters remaining same) for MFSK system. The frequency band is from 1.25 to 651.25 kHz for Steiner system and from 1.25 to 801.25 kHz for MFSK system. Input bit rate is 1 kbps. The BER performance for MFSK system is better for lower values of Eb/No and becomes comparable to Steiner system at higher SNR. The advantage we get for the System is the better bandwidth efficiency.

Figure 3 shows simulation and upper bounds (equation 16) on the same plot for N = 20 & 30 users.

(C) Frequency non selective slowly Fading Channel

The bit error rate performance is derived when the signals are transmitted over a frequency non-selective, slowly fading channel considering AWGN. The received equivalent low pass signal in one singling interval is represented by

$$r_n(t) = \alpha_n e^{-j\phi_n} u_m^n(t) + z_n(t), \quad 0 \le t \le T \quad (17)$$

where $n = \{1, 2, \ldots, w\}$, $m = \{1, 2, \ldots, v\}$, $\{u_m^n(t)\}$ are the equivalent low pass transmitted signals and $z_n(t)$ are Gaussian noise random processes. The probability of error is given by [10]

$$p_r = \left(\frac{1}{1+\gamma_c}\right)^w \sum_{i=0}^{w-1} \binom{w-1+i}{i} \times \left(\frac{1+\gamma_c}{2+\gamma_c}\right)^i \quad (18)$$

$$\gamma_c = \frac{k\xi}{wN_0} \sum_{i=1}^{w} E(a_i^2) = \frac{k}{w}\gamma_b \quad (19)$$

where ξ is the signal energy per bit, γ_c is the average SNR per branch and γ_b is the average SNR per bit. Using the union bound, the detection error probability can be upper bounded by [10]

$$P_d(\gamma_c, w) \le (M-1) p_r \quad (20)$$

This result is used in (11a) with w and, in (11b) with $w = w - 1$ to obtain the average decision error probabilities. These equations are put in (14) to bound Steiner coded symbol error probability. Finally, the expression for bit error rate is obtained by using the equations (15) – (16).

From BER vs. code diversity (L) plot for fading channel, the optimum value of diversity obtained (at Eb/No = 25 dB) for Steiner system was 3 & for MFSK system it was 5.

The BER performance in fading channel for 15, 20 and 25 users for the same parameters as in Fig.2 is shown in Fig 4. The BER performance for MFSK system is better for lower values of Eb/No and becomes comparable to Steiner system at higher SNR as before.

Figure 5 shows simulation and upper bounds (equations 11a, 11b, 14-16, 20) for frequency non-selective Rayleigh fading channel on the same plot for N = 20 & 25 users.

CHANNEL MATCHED FREQUENCY HOPPING (MFH) PATTERNS

Matched frequency hopping (MFH) is an efficient signaling technique in frequency selective slowly fading dispersive channel [11]. In the MFH scheme, a suitable channel measurement technique is assumed to provide the information about the channel transfer function. Using this information, the hopping frequencies are selected to be in those regions of the channel in which signal attenuation is minimum. For such channels, Rummler [12] has developed a three-path model. The model uses a channel transfer function given by [12]

$$H(f) = a[1 - \beta e^{-j2\pi(f-f_0)\tau}] \quad (21)$$

where a is the overall attenuation parameter, β is called a shape parameter, f_0 is the frequency of the fade minimum and τ is the relative time delay between the direct and the multipath components.

Figure 6 shows BER performance of the Steiner system with MFH pattern for 20 number of users.

CONCLUSION

The system incorporates frequency diversity (simultaneous transmission of L frequency bins for each symbol) and error correcting capability due to the use of the Steiner design. For lower values of code diversity (L) the error rate is high due to channel noise and user interference, but with the increase of L the system rapidly recovers most of the loss due to interference from other users. However, beyond the optimum diversity, where the BER minima is achieved, any further increase in diversity degrades the performance due to interference among the users. It is seen that the performance of MFSK-SFH system (with diversity 5) is better than that of the Steiner system (with diversity 3) at lower SNR and these two systems are comparable at higher SNR. The advantage we have got from Steiner system is its better bandwidth efficiency. Further, a lot of improvement is obtained with MFH, an effective addressing scheme for frequency selective fading channel.

REFERENCES

[1] V. K. Prabhu, D. J. Goodman & P. S. Henry, "Frequency-Hopped Multilevel FSK for Mobile Radio," *Bell System Technical Journal,* vol. 59, No. 7, pp. 1257-1275, 1980.

[2] U. Timor, "Multistage Decoding of Frequency-Hopped FSK System," *Bell System Technical Journal*, vol. 60, pp. 471-483, April 1981.

[3] B. G. Haskell, "Computer Simulation Results on Frequency Hopped MFSK Mobile Radio-Noiseless Case," *IEEE Transactions on Communications,* vol. COM-29, pp. 125-132, 1982.

[4] G. E. Atkin & I. F. Blake, "Performance of Multitone FFH/MFSK Systems in the Presence of Jamming," *IEEE Transactions on Information Theory,* vol. 35, No. 2, pp. 428-435, 1989.

[5] G. E. Atkin & H. P. Corrales, "An Efficient Modulation/Coding Scheme for MFSK Systems on Bandwidth Constrained Channels," *IEEE Journal on Selected Areas in Communications,* vol. 7, No. 9, pp. 1396-1401, December 1989.

[6] C. T. Ong & C. Leung, "Code Diversity Transmission in a Slow-Frequency-Hopped Spread Spectrum Multiple-Access Communication System," *IEEE Transactions on Communications,* vol. 43, No. 12, pp. 2897-2899, December 1995.

[7] J. M. Hall, *Combinatorial Theory,* Second Edition, Wiley-Interscience Publication, 1986.

[8] J. F. Piper, J. G. Proakis, R. R. Reed and J. K. Wolf, "Design of efficient Coding and Modulation for a Rayleigh Fading Channel," *IEEE Transactions on Information Theory,* vol. IT-24, pp. 457-468, July 1978.

[9] William Feller, *An Introduction to Probability Theory and Its Applications*, vol. 1, Third Edition, New York: Wiley, 1968.

[10] J. G. Proakis, *Digital Communications*, Third Edition, McGraw-Hill Inc, 1995.

[11] E. El-Khamy, "Matched Frequency-Hopping (MFH) Signals for Slowly Fading Dispersive channels," *IEEE Transactions on Vehicular technology,* vol. 47, No. 1, February 1998.

[12] W. D. Rummler, "A New Selective Fading Model: Application to Propagation Data," *The Bell System Technical Journal,* vol. 58, No. 5, pp. 1037-1071, May-June 1979.

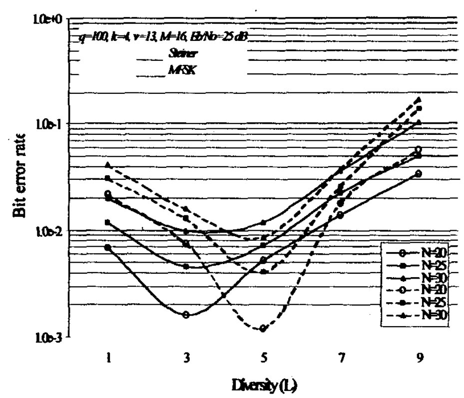

Figure 1: BER vs. diversity plot at Eb/No=25 dB for non-fading channel

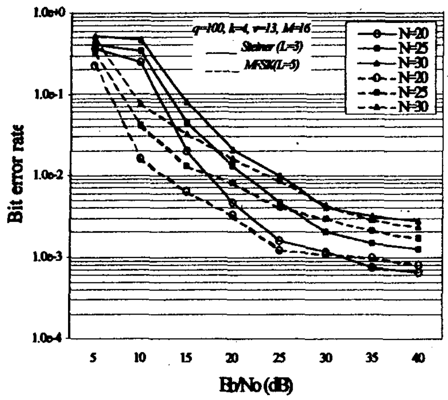

Figure 2: BER performance for non-fading channel

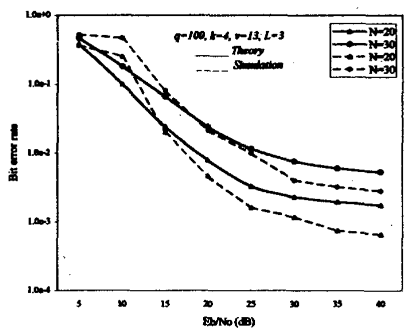

Figure3: Comparison between simulation and upper bounds for non-fading channel

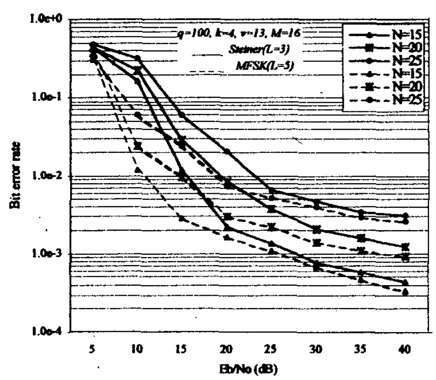

Figure 4: BER performance for Rayleigh fading channel.

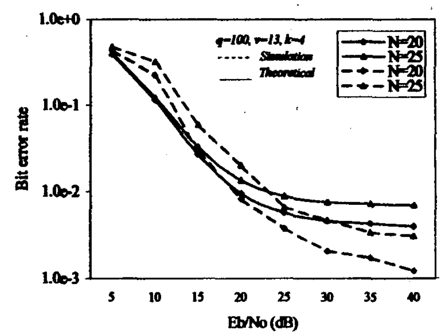

Figure 5: Comparison between simulation and upper bounds for fading channel

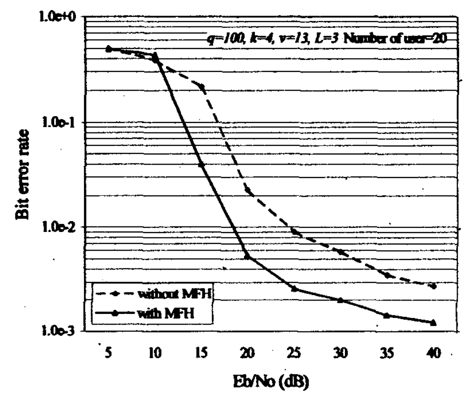

Figure 6: Performance of the Steiner system with MFH for frequency selective fading channel