A Weighted Combining Approach to Multiuser Detection in Macrodiversity

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Abstract—A new linear complexity algorithm using weighted combining is proposed for Multiuser Detection (MUD) in Macrodiversity for synchronous DS-CDMA. Error probability expression is derived for weighted combining based receivers and a framework for finding optimal weights is proposed. Simulation results show that the performance of the proposed algorithm compares with the optimal maximum-likelihood (ML) MUD under various situations.

I. INTRODUCTION

Macrodiversity refers to the situation where antennas, physically separated by a large distance, receive signals from different users and decode it jointly. Since users are at different distances from the antennas, each users signal arrives at various antennas with different powers. For detection purpose, a user with a very weak signal power at a particular antenna can be considered to be not at all present at that antenna. Each antenna can be considered to receive signals from different, but often overlapping, subsets of users. A recently proposed Conditional Metric Merge (CMM) algorithm reduces the computational complexity of the maximum likelihood multiuser detection (ML-MUD) for macrodiversity by a significant factor by exploiting the locality in space and time [4]-[5]. However, complexity is still exponential in number of users and depends on the distribution of the user set at different antennas [5].

A family of linear receivers is described in [1] and [2] for single antenna case and a more comprehensive treatment on the same is given in [3]. However, in a macrodiversity situation, users will often appear at more than one antennas. If detection is carried out locally at each of the antennas then there is a possibility of conflict in the sign of bits of a particular user detected at different antennas.

We present an efficient scheme for combining the decision variables obtained after employing a linear receiver at each of the antennas using a weighted combining approach. We also address the question of optimality of weights and compare the performance of the proposed scheme with the optimal ML-MUD.

A combined multiuser detection and antenna array processing has been addressed in [6] and [7] where optimum multiuser detection by antenna arrays is described for single path fading and multipath fading respectively. Computational complexity of both of these schemes grow exponentially with number of A. K. Chaturvedi Department of Electrical Engineering Indian Institute of Technology Kanpur Kanpur, India Email: akc@iitk.ac.in

users. [8] analyzes an iterative macrodiversity detection and decoding for the TDMA Cellular Uplink. All these schemes assume that all the users are present at all the antennas.

This paper is organized as follows. In section II, the system model is presented and linear receivers are introduced. In section III, weighted combining based recievers are described and performance bounds are derived. Simulation results are presented in section IV and finally section V contains the conclusions.

II. SYSTEM MODEL

Consider K users and M antennas. We use synchronous DS-CMDA channel model as described in [1] with BPSK modulation. The received signal is also assumed to be corrupted by additive white Gaussian noise (AWGN). Then the matched filter output at any antenna m can be written as

$$\mathbf{Y}^{\mathbf{m}} = \mathbf{R}\mathbf{A}^{\mathbf{m}}\mathbf{B} + \mathbf{N}^{\mathbf{m}},\tag{1}$$

where $\mathbf{Y}^{\mathbf{m}} = (y_1^m, y_2^m, \dots, y_K^m)^T$ with y_k^m denoting the matched filter output at antenna m when the received signal at antenna m is correlated with the signature of user k, \mathbf{R} is the $K \times K$ correlation matrix with $(i, j)^{th}$ entry being ρ_{ij} , $\mathbf{A}^{\mathbf{m}}$ is a $K \times K$ diagonal matrix with $(i, j)^{th}$ entry being ρ_{ij} , $\mathbf{A}^{\mathbf{m}}$ is a $K \times K$ diagonal matrix with its k^{th} diagonal element a_k^m denoting the received amplitude of the user k at antenna m, $\mathbf{B} = (b_1, b_2, \dots, b_K)^T$ is the transmitted data vector with $b_i \in \{+1, -1\}$ and $\mathbf{N}^{\mathbf{m}}$ follows Gaussian distribution with zero mean and variance $= E[\mathbf{N}^{\mathbf{m}} (\mathbf{N}^{\mathbf{m}})^{\mathbf{T}}] = \sigma_m^2 \mathbf{R}$. Here σ_m^2 is the noise power at antenna m and noise at different antennas is assumed to be independent. Also, if user k is not present at antenna m then $a_k^m = 0$.

Using the derivation in [1], log likelihood function for **B** at antenna m is $\Omega^m(\mathbf{B})$ where

$$\Omega^m(\mathbf{B}) = \mathbf{2}\mathbf{B}^{\mathbf{T}}\mathbf{A}^{\mathbf{m}}\mathbf{Y}^{\mathbf{m}} - \mathbf{B}^{\mathbf{T}}\mathbf{A}^{\mathbf{m}}\mathbf{R}\mathbf{A}^{\mathbf{m}}\mathbf{B}.$$
 (2)

The optimal detection at the local level involves maximizing the metric given by (2). Since noise is independent at different antennas, the overall optimal ML-MUD condition for macrodiversity for synchronous DS-CDMA is given by

$$\widehat{\mathbf{B}} = \max_{\mathbf{B} \in \{+1,-1\}^{K}} \sum_{m=1}^{M} \left(\mathbf{2B^{T}A^{m}Y^{m}} - \mathbf{B^{T}A^{m}RA^{m}B} \right).$$
(3)

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However, a receiver based on this scheme will have exponential complexity per bit and is not feasible for implementation in a real time system.

A wide range of sub optimal multi-user receivers, such as linear receivers [1], have been proposed as a trade-off between complexity and optimality for single antenna case. A linear receiver multiplies the matched filter output at antenna m, as given by (1), by a $K \times K$ matrix $\mathbf{L}^{\mathbf{m}}$ resulting in

$$\mathbf{L}^{\mathbf{m}}\mathbf{Y}^{\mathbf{m}} = \mathbf{L}^{\mathbf{m}}\mathbf{R}\mathbf{A}^{\mathbf{m}}\mathbf{B} + \mathbf{L}^{\mathbf{m}}\mathbf{N}^{\mathbf{m}}.$$
 (4)

The bit estimate can be written as

$$\widehat{\mathbf{B}} = sgn(\mathbf{L}^{\mathbf{m}}\mathbf{Y}^{\mathbf{m}}). \tag{5}$$

Using substitutions $\mathbf{F}^{\mathbf{m}} = \mathbf{L}^{\mathbf{m}}\mathbf{R}\mathbf{A}^{\mathbf{m}}, \mathbf{X}^{\mathbf{m}} = \mathbf{L}^{\mathbf{m}}\mathbf{N}^{\mathbf{m}},$ $\mathbf{Z}^{\mathbf{m}} = \mathbf{L}^{\mathbf{m}}\mathbf{Y}^{\mathbf{m}}$ and $\boldsymbol{\eta}^{\mathbf{m}} = \mathbf{L}^{\mathbf{m}}\mathbf{R}(\mathbf{L}^{\mathbf{m}})^{\mathbf{T}}$, we get

$$\mathbf{Z}^{\mathbf{m}} = \mathbf{F}^{\mathbf{m}}\mathbf{B} + \mathbf{X}^{\mathbf{m}},\tag{6}$$

where $\mathbf{X}^{\mathbf{m}}$ is Gaussian with zero mean and variance given by

$$E[\mathbf{X}^{\mathbf{m}}(\mathbf{X}^{\mathbf{m}})^{T}] = E[(\mathbf{L}^{\mathbf{m}}\mathbf{N}^{\mathbf{m}})(\mathbf{L}^{\mathbf{m}}\mathbf{N}^{\mathbf{m}})^{T}]$$
$$= \sigma_{m}^{2}\mathbf{L}^{\mathbf{m}}\mathbf{R}(\mathbf{L}^{\mathbf{m}})^{\mathbf{T}} = \sigma_{m}^{2}\boldsymbol{\eta}^{\mathbf{m}}.$$
 (7)

We denote the $(i, j)^{th}$ element of $\mathbf{F}^{\mathbf{m}}$ and $\boldsymbol{\eta}^{\mathbf{m}}$ are denoted by f_{ij}^{m} and η_{ij}^{m} respectively.

III. THE WEIGHTED COMBINING APPROACH

If local decisions are made at each of the antennas then an intuitively obvious solution to the problem of conflict in signs of bits is to use majority voting rule. However, this scheme is not efficient as equal weights are assigned to local decisions made at each of the antennas irrespective of error probabilities associated with such decisions. Below we describe a new approach to solve this problem efficiently.

Let S_k denote the set of antennas at which user k is present, S_k^i be the i^{th} antenna of the set S_k and $|S_k|$ be its cardinality.

Using (6), the output of a linear receiver at antenna S_k^i for user k is given by

$$z_k^{S_k^i} = \sum_{j=1}^K f_{kj}^{S_k^i} b_j + x_k^{S_k^i}.$$
(8)

We can combine the decision variables of a user, obtained at different antennas, as given by (8), using a weighted combining approach as following: Consider any arbitrary choice of weights $w_{S_{L}^{i}}$. We define

$$\gamma_k = \sum_{i=1}^{|S_k|} w_{S_k^i} z_k^{S_k^i}, \tag{9}$$

and take the detection rule to be $\hat{b}_k = sgn(\gamma_k)$. Using (8), (9) can be rewritten as

$$\gamma_k = \sum_{j=1}^K b_j \sum_{i=1}^{|S_k|} w_{S_k^i} f_{kj}^{S_k^i} + \sum_{i=1}^{|S_k|} w_{S_k^i} x_k^{S_k^i}.$$
 (10)

Since $\mathbf{X}^{\mathbf{m}}$ is Gaussian with zero mean and variance $\sigma_m^2 \boldsymbol{\eta}^{\mathbf{m}}$ and noise at different antennas is independent, we have

$$E[\sum_{i=1}^{|S_k|} w_{S_k^i} x_k^{S_k^i}] = 0;$$
(11)

$$E[(\sum_{i=1}^{|S_k|} w_{S_k^i} x_k^{S_k^i})^2] = \sum_{i=1}^{|S_k|} w_{S_k^i}^2 \sigma_{S_k^i}^2 \eta_{kk}^{S_k^i}.$$
 (12)

In Appendix I, we derive the error probability for user k if weighted combining of linear receivers output is carried out. The final expression is given by (24).

We can minimize the error probability given by (24) by a suitable choice of weights. However, no closed form expression can be found for optimal weights since (24) involves summation of Q function terms.

As an example, a simple weighted combining (SWC) can be done as following. If $P^{S_k^i}$ is the error probability associated with the local decision made at antenna S_k^i about the k^{th} user bit then one possible choice of weights is $w_{S_k^i} = \frac{1-2P^{S_k^i}}{\sum_{i=1}^{|S_k|}(1-2P^{S_k^i})}$. This choice ensures that smaller weights are assigned when error probability is high with weights becoming zero when $P^{S_k^i} = \frac{1}{2}$, and $w_{S_k^i} \ge 0$ since $0 < P^{S_k^i} < \frac{1}{2}$. Although this particular choice of weights may not be optimal, this scheme takes into account the confidence associated with the local decisions made at each antennas and hence would be better than the majority voting rule.

A. Lower Bound on Error Probability

Using (24) and the inequality (29) derived in Appendix II, we have

$$P_{k} \geq 2^{1-K} \sum_{e_{1} \in \{+1,-1\}} \dots \sum_{e_{j \neq k} \in \{+1,-1\}} \dots \sum_{e_{K} \in \{+1,-1\}} Q\left(\sqrt{\sum_{i=1}^{|S_{k}|} \frac{\left(f_{kk}^{S_{k}^{i}} + \sum_{j \neq k}^{K} e_{j} f_{kj}^{S_{k}^{i}}\right)^{2}}{\sigma_{S_{k}^{i}}^{2} \eta_{kk}^{S_{k}^{i}}}} \right), \quad (13)$$

where equality holds when weights are given by (28), as shown in Appendix II.

However, it is not possible to reach the lower bound on error probability derived above for any arbitrary choice of $\mathbf{L}^{\mathbf{m}}$ in (4). Each of the Q-function term in (24) attains its minimum value for a particular choice of weights, as given by (28). Weights might be in conflict since they depend on values of $e_j, j \neq k$. However, if we can make the terms $f_{kj}^{S_k^i}, j \neq k$ to be zero, choice of weights will no longer depend on $e_j, j \neq k$. Each of the Q-function term will then attain its minimum for the same set of weights and the lower bound given by (13) can be achieved. This prompts us to employ a decorrelator locally at each of the antennas.

B. Decorrelated Weighted Combining (DWC)

For a decorrelator, we take $\mathbf{L}^{\mathbf{m}} = (\mathbf{R}\mathbf{A}^{\mathbf{m}})^{-1}$ in (4). However, since in a macrodiversity situation, if user *i* is not present at antenna *m* then the *i*th entry of the diagonal matrix A^m will be zero and hence matrix $\mathbf{R}\mathbf{A}^{\mathbf{m}}$ will not be invertible. However, notice that for such situation, $f_{ki}^m = 0$. Hence, such terms do not contribute in the analysis and hence can be ignored. We redefine our notations by deleting the terms corresponding to users that are not present at a particular antenna *m*.

Let U_m denote the set of users at the antenna m and U_m^j be the j^{th} user at antenna m. We write

$$\tilde{\mathbf{Y}}^{\mathbf{m}} = \tilde{\mathbf{R}}^{\mathbf{m}} \tilde{\mathbf{A}}^{\mathbf{m}} \tilde{\mathbf{B}}^{\mathbf{m}} + \tilde{\mathbf{N}}^{\mathbf{m}}, \qquad (14)$$

where $\tilde{R}_{ij}^m = R_{U_m^i, U_m^j}$, $\tilde{A}_{ij}^m = A_{U_m^i, U_m^j}^m$, $\tilde{b}_i^m = b_{U_m^i}$. $\tilde{\mathbf{N}}^{\mathbf{m}}$ is Gaussian with zero mean and variance $\sigma_m^2 \tilde{\mathbf{R}}^{\mathbf{m}}$. We choose $\tilde{\mathbf{L}}^{\mathbf{m}} = (\tilde{\mathbf{R}}^{\mathbf{m}} \tilde{\mathbf{A}}^{\mathbf{m}})^{-1}$. This can be done as $\tilde{\mathbf{A}}^{\mathbf{m}}$ is a diagonal matrix with diagonal elements being non zero. Therefore,

$$\tilde{\mathbf{L}}^{\mathbf{m}}\tilde{\mathbf{Y}}^{\mathbf{m}} = \tilde{\mathbf{L}}^{\mathbf{m}}\tilde{\mathbf{R}}^{\mathbf{m}}\tilde{\mathbf{A}}^{\mathbf{m}}\tilde{\mathbf{b}}^{\mathbf{m}} + \tilde{\mathbf{L}}^{\mathbf{m}}\tilde{\mathbf{N}}^{\mathbf{m}} = \tilde{\mathbf{b}}^{\mathbf{m}} + \tilde{\mathbf{L}}^{\mathbf{m}}\tilde{\mathbf{N}}^{\mathbf{m}}.$$
 (15)

We again use substitutions $\tilde{\mathbf{F}^{m}} = \tilde{\mathbf{L}}^{m} \tilde{\mathbf{R}}^{m} \tilde{\mathbf{A}}^{m}$, $\tilde{\mathbf{X}}^{m} = \tilde{\mathbf{L}}^{m} \tilde{\mathbf{N}}^{m}$, $\tilde{\mathbf{Z}}^{m} = \tilde{\mathbf{L}}^{m} \tilde{\mathbf{Y}}^{m}$ and $\tilde{\eta}^{m} = \tilde{\mathbf{L}}^{m} \tilde{\mathbf{R}}^{m} (\tilde{\mathbf{L}}^{m})^{T}$. Hence, we have

$$\tilde{\mathbf{Z}}^{\mathbf{m}} = \tilde{\mathbf{F}}^{\mathbf{m}} \tilde{\mathbf{b}}^{\mathbf{m}} + \tilde{\mathbf{X}}^{\mathbf{m}}, \qquad (16)$$

where \tilde{F}^m is an identity matrix and \tilde{X}^m is Gaussian with zero mean and variance given by

$$E[\tilde{\mathbf{X}}^{\mathbf{m}}(\tilde{\mathbf{X}}^{\mathbf{m}})^{\mathbf{T}}] = E[(\tilde{\mathbf{L}}^{\mathbf{m}}\tilde{\mathbf{N}}^{\mathbf{m}})(\tilde{\mathbf{L}}^{\mathbf{m}}\tilde{\mathbf{N}}^{\mathbf{m}})^{T}]$$
$$= \sigma_{m}^{2}\tilde{\mathbf{L}}^{\mathbf{m}}\tilde{\mathbf{R}}^{\mathbf{m}}(\tilde{\mathbf{L}}^{\mathbf{m}})^{\mathbf{T}} = \sigma_{m}^{2}\tilde{\boldsymbol{\eta}}^{\mathbf{m}}.$$
(17)

The $(i, j)^{th}$ element of $\tilde{\mathbf{F}}^{\mathbf{m}}$ and $\tilde{\eta}_{\mathbf{m}}$ are denoted by \tilde{f}^m_{ij} and $\tilde{\eta}^m_{ij}$ respectively. Let $\lambda^m(k)$ be that element of set U_m for which $U_m^{\lambda^m(k)} = k$.

Since $\tilde{f}_{ij}^m = 0$, $i \neq j$, and $\tilde{f}_{ii}^m = 1$, $w_{S_k^i}$, as given by (28) will be independent of e_j . Hence individual Q-function terms in (24) are minimized for the same set of weights. Thus for decorrelated weighted combining, minimum error probability as described by (24), is achieved. Expression for optimal weights and corresponding error probability for such a receiver is given by

$$w_{S_{k}^{i}} = \left(\frac{1}{\sigma_{S_{k}^{i}}^{2}\tilde{\eta}_{\lambda^{S_{k}^{i}}(k),\lambda^{S_{k}^{i}}(k)}^{2}}\right) \left(\frac{1}{\sum_{i=1}^{|S_{k}|} \frac{1}{\sigma_{S_{k}^{i}}^{2}\tilde{\eta}_{\lambda^{S_{k}^{i}}(k),\lambda^{S_{k}^{i}}(k)}^{S_{k}^{i}}}\right);$$

$$P_{k,min} = Q\left(\sqrt{\sum_{i=1}^{|S_{k}|} \frac{1}{\sigma_{S_{k}^{i}}^{2}\tilde{\eta}_{\lambda^{S_{k}^{i}}(k)\lambda^{S_{k}^{i}}(k)}^{S_{k}^{i}}}\right).$$
(19)

The complexity of this scheme is linear in number of users since weight computations and matrix inversion can be done offline. Moreover, since a decorrelator is employed locally at each of the antennas, performance is expected to be invariant to the power of interferers.

IV. SIMULATION RESULTS

Simulations are carried out to test the performance of DWC algorithm. A set of 10 users and 4 antennas are considered with the following distribution of the user set: User set at antenna $1 = \{1 \ 2 \ 3 \ 4 \ 5 \}$, at antenna $2 = \{2 \ 3 \ 4 \ 5 \ 6 \ 7 \}$, at antenna $3 = \{4 \ 5 \ 6 \ 7 \ 8\}$ and at antenna $4 = \{2 \ 7 \ 8 \ 9 \ 10\}$. A synchronous CDMA channel with AWGN is considered. Power of all the users present at a particular antenna are taken to be equal for Figs. 1, 2 and 3 while in Fig. 4 power of user 4 is varied.

First, a performance comparison is done between the Majority Voting Algorithm, SWC algorithm, DWC algorithm and ML-MUD. Results are shown in Figs. 1 and 2 for low and high correlation respectively. In Fig. 1, Gold sequences of length 31 are used as signatures while in Fig. 2 correlation between any pair of users is 0.6. Average of bit error rate (BER) of all users is taken as the performance measure parameter. While the performance of DWC algorithm is found to be very close to that of ML-MUD for low correlation, it is comparable to ML-MUD even for high correlation, unlike majority voting and simple weighted combining.

In Fig. 3, the simulation results for the DWC algorithm are compared with the theoretical expression as given in (19). Result is shown only for user 6. Simulation results are found to be in close agreement with the theoretical plot. Similar thing has been observed for all the other users.

Finally, to study the effect of unequal power on DWC algorithm, power of user 4 is varied from 0 db to 20 db while keeping the power of all other users fixed at 0 db and SNR fixed at 5 db. Gold sequences of length 31 are used as signatures. Average of BER of users is taken as the performance measure parameter. Since users $\{9,10\}$ are appearing only at antenna 4 while user 4 is not present at antenna 4, only users $\{1, 2, 3, 5, 6, 7, 8\}$ are considered for computing the average BER. Results are shown in Fig. 4. Average BER is found to be almost invariant to the power variation of user 4 and is comparable with the performance of optimal ML-MUD.

V. CONCLUSION

A weighted combining based approach to MUD in macrodiversity has been proposed here. The complexity of the scheme is linear in the number of users. The performance of decorrelated weighted combining is found to be close to that of optimal ML-MUD for identical signal power. Average BER is also observed to be invariant to the power of a dominant interferer. BER obtained through simulations is found to be in agreement with the theory.

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APPENDIX I Error probability for weighthed combining Approach

Error probability for user k is given by

$$P_k = \frac{1}{2} \left(P[\gamma_k > 0 | b_k = -1] + P[\gamma_k < 0 | b_k = 1] \right).$$
 (20)

Since all symbols are equally likely,

$$P[\gamma_k > 0|b_k = -1] = P[\gamma_k < 0|b_k = 1], \qquad (21)$$

and

$$P[\gamma_k < 0|b_k = 1] = 2^{1-K} \sum_{e_1 \in \{1, -1\}} \dots \sum_{e_{j \neq k} \in \{1, -1\}} \dots \sum_{e_{j \neq k} \in \{1, -1\}} P[\gamma_k < 0|b_k = 1, b_j = e_j, j \neq k].$$
(22)

From (10) we get

$$P[\gamma_{k} < 0|b_{k} = 1, b_{j} = e_{j}, j \neq k]$$

$$= P\left[\sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}} f_{kk}^{S_{k}^{i}} + \sum_{j\neq k}^{K} e_{j} \sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}} f_{kj}^{S_{k}^{i}} < -\sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}} x_{k}^{S_{k}^{i}}\right]$$

$$= Q\left(\frac{\sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}} f_{kk}^{S_{k}^{i}} + \sum_{j\neq k}^{K} e_{j} \sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}} f_{kj}^{S_{k}^{i}}}{\sqrt{\sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}}^{2} \sigma_{S_{k}^{i}}^{2} \eta_{kk}^{S_{k}^{i}}}}\right), \quad (23)$$

where we have used the fact that the quantity $\sum_{i=1}^{|S_k|} w_{S_k^i} x^{S_k^i,k}$ is Gaussian with mean and variance given by (11) and (12). From (20) - (23), we can write

$$P_{k} = 2^{1-K} \sum_{e_{1} \in \{+1,-1\}} \cdots \sum_{e_{j \neq k} \in \{+1,-1\}} \cdots \sum_{e_{K} \in \{+1,-1\}} \left(\frac{\sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}} f_{kk}^{S_{k}^{i}} + \sum_{j \neq k}^{K} e_{j} \sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}} f_{kj}^{S_{k}^{i}}}{\sqrt{\sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}}^{2} \sigma_{S_{k}^{i}}^{2} \eta_{kk}^{S_{k}^{i}}}} \right).$$
(24)

APPENDIX II

Consider the argument of any Q-function term in (24). Value of Q-function is minimum when its argument is maximum. The argument can be rewritten as:

$$\frac{\sum_{i=1}^{|S_k|} w_{S_k^i} \left(f_{kk}^{S_k^i} + \sum_{j \neq k}^{K} e_j f_{kj}^{S_k^i} \right)}{\sqrt{\sum_{i=1}^{|S_k|} w_{S_k^i}^2 \sigma_{S_k^i}^2 \eta_{kk}^{S_k^i}}} = \frac{\sum_{i=1}^{|S_k|} w_{S_k^i} \beta_{ki} \left(\alpha_{ki} / \beta_{ki} \right)}{\sqrt{\sum_{i=1}^{|S_k|} w_{S_k^i}^2 \beta_{ki}^2}} = \frac{\sum_{i=1}^{|S_k|} w_{S_k^i} \beta_{ki} \left(\alpha_{ki} / \beta_{ki} \right)}{\sqrt{\sum_{i=1}^{|S_k|} w_{S_k^i}^2 \beta_{ki}^2}} \\ \leq \frac{\left(\sum_{i=1}^{|S_k|} w_{S_k^i}^2 \beta_{ki}^2 \right)^{\frac{1}{2}} \left(\sum_{i=1}^{|S_k|} \alpha_{ki}^2 / \beta_{ki}^2 \right)^{\frac{1}{2}}}{\sqrt{\sum_{i=1}^{|S_k|} w_{S_k^i}^2 \beta_{ki}^2}} = \left(\sum_{i=1}^{|S_k|} \frac{\alpha_{ki}^2}{\beta_{ki}^2} \right)^{\frac{1}{2}}, \tag{25}$$

where for convenience of notation, we have defined $\alpha_{ki} = f_{kk}^{S_k^i} + \sum_{j \neq k}^{K} e_j f_{kj}^{S_k^i}$ and $\beta_{ki}^2 = \sigma_{S_k^i}^2 \eta_{kk}^{S_k^i}$. Inequality (25) is obtained by applying Cauchy-Schwarz inequality. Equality holds when

$$\frac{w_{S_k^i}\beta_{ki}}{(\alpha_{ki}/\beta_{ki})} = L, \quad i = 1, 2\dots, |S_k|,$$
(26)

for some constant L. Taking $\sum_{i=1}^{|S_k|} w_{S_k^i} = 1$, we get

$$L = \left(\sum_{i=1}^{|S_k|} \alpha_{ki} / \beta_{ki}^2\right)^{-1}, \qquad (27)$$

and choice of $w_{S_{l.}^{i}}$ for which equality holds to be

$$w_{S_{k}^{i}} = \frac{\alpha_{ki}}{\beta_{ki}^{2} \left(\sum_{i=1}^{|S_{k}|} \alpha_{ki} / \beta_{ki}^{2}\right)} = \frac{f_{kk}^{S_{k}^{i}} + \sum_{j \neq k}^{K} e_{j} f_{kj}^{S_{k}^{i}}}{\sigma_{S_{k}^{i}}^{2} \eta_{kk}^{S_{k}^{i}} \left(\sum_{i=1}^{|S_{k}|} \frac{f_{kk}^{S_{k}^{i}} + \sum_{j \neq k}^{K} e_{j} f_{kj}^{S_{k}^{i}}}{\sigma_{S_{k}^{i}}^{2} \eta_{kk}^{S_{k}^{i}}}\right)}.$$
 (28)

Since Q function decreases monotonically with its argument, from (25) we get

$$Q\left(\frac{\sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}} f_{kk}^{S_{k}(i)} + \sum_{j \neq k}^{K} e_{j} \sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}} f_{kj}^{S_{k}(i)}}{\sqrt{\sum_{i=1}^{|S_{k}|} w_{S_{k}^{i}}^{2} \sigma_{S_{k}^{i}}^{2} \eta_{kk}^{S_{k}^{i}}}}\right)$$
$$\geq Q\left(\sqrt{\sum_{i=1}^{|S_{k}|} \frac{\left(f_{kk}^{S_{k}^{i}} + \sum_{j \neq k}^{K} e_{j} f_{kj}^{S_{k}^{i}}\right)^{2}}{\sigma_{S_{k}^{i}}^{2} \eta_{kk}^{S_{k}^{i}}}}\right).$$
(29)

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Fig. 1. Comparison of average BER of Majority Voting, SWC, DWC and Optimal ML-MUD based detectors with equal signal power and low correlation



Fig. 2. Comparison of average BER of Majority Voting, SWC, DWC and Optimal ML-MUD based detectors with equal signal power and high correlation of 0.6 between any two users

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Fig. 3. Theoretical plot vs Simulation Result for DWC algorithm with equal signal power and low correlation for a typical user



Fig. 4. Performance in the case of varying power of interferers with a fixed SNR of 5db