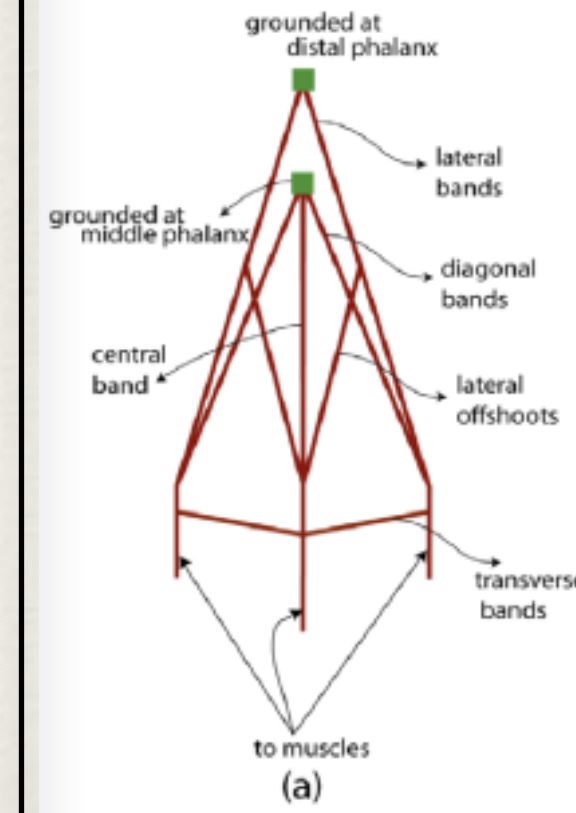
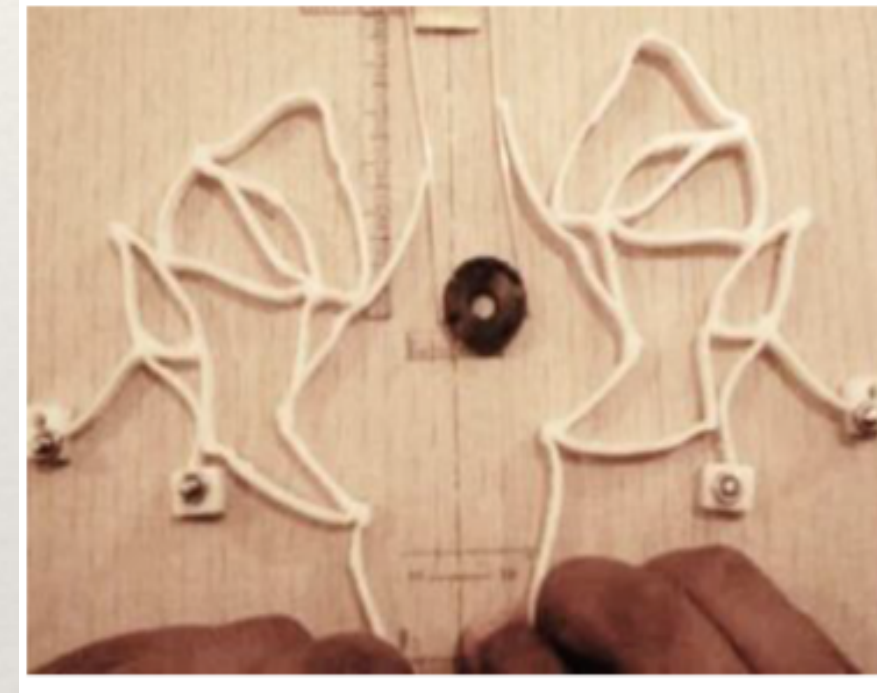
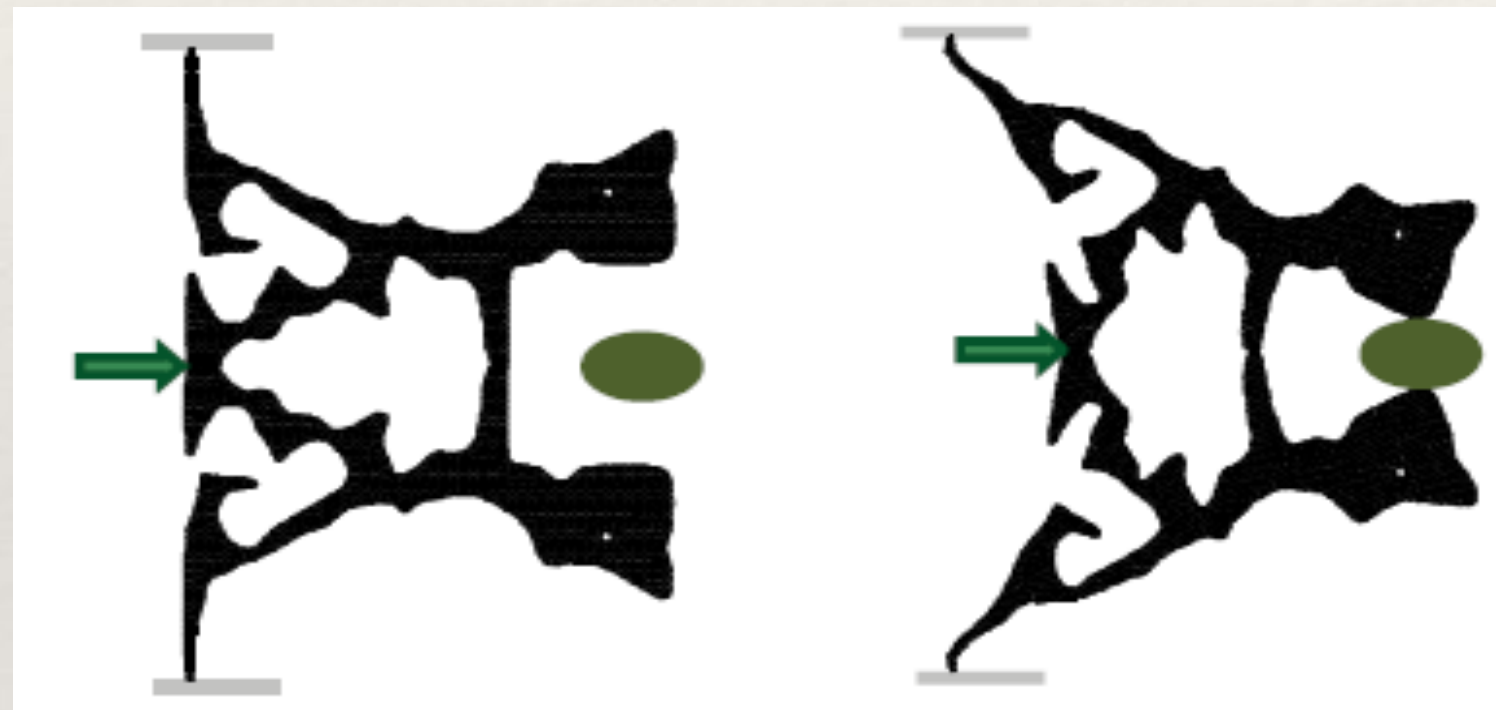
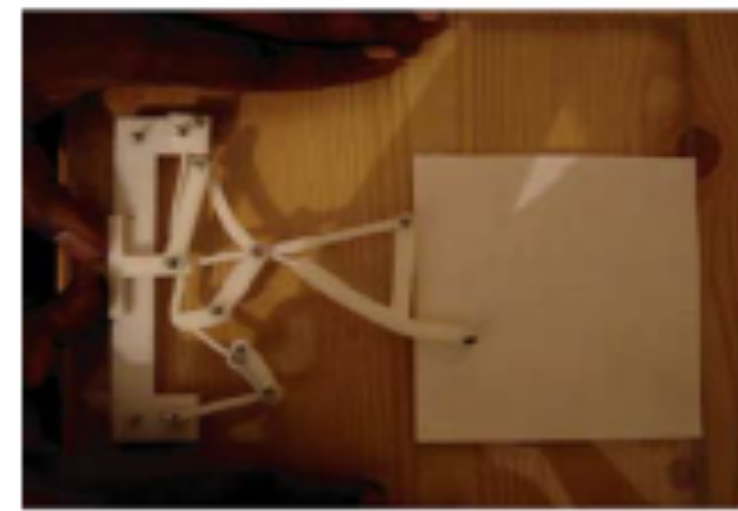
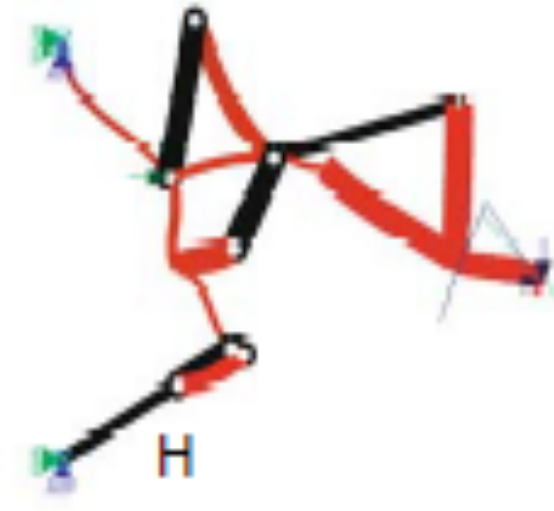
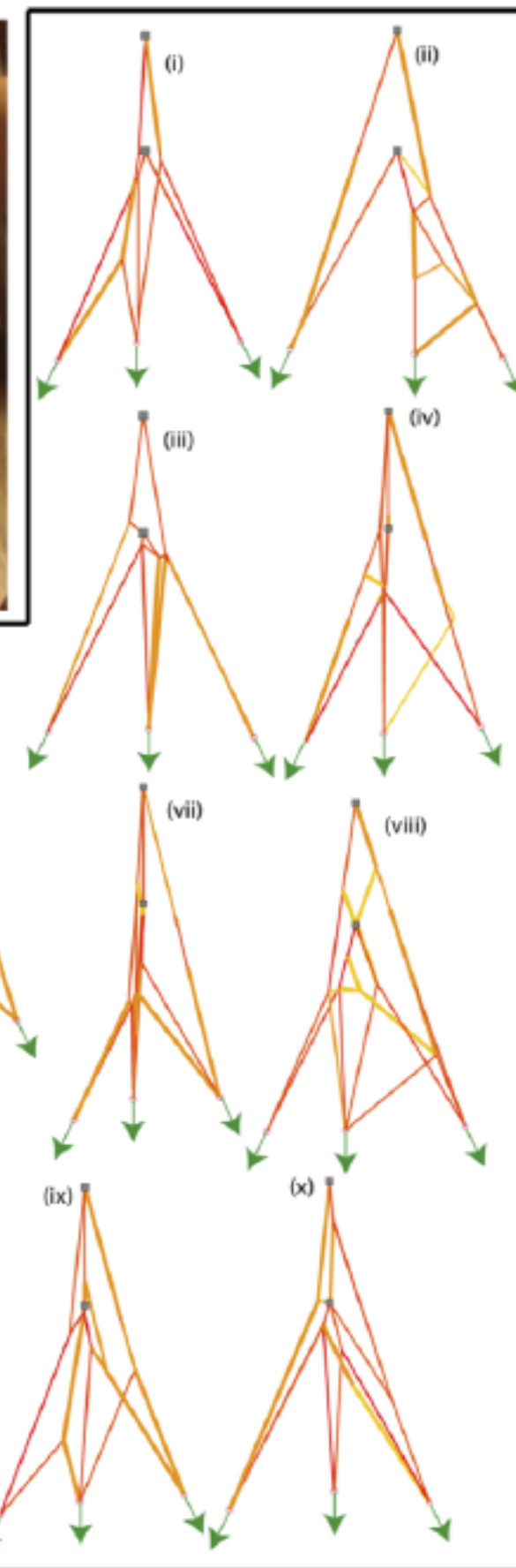




(a)



(b)



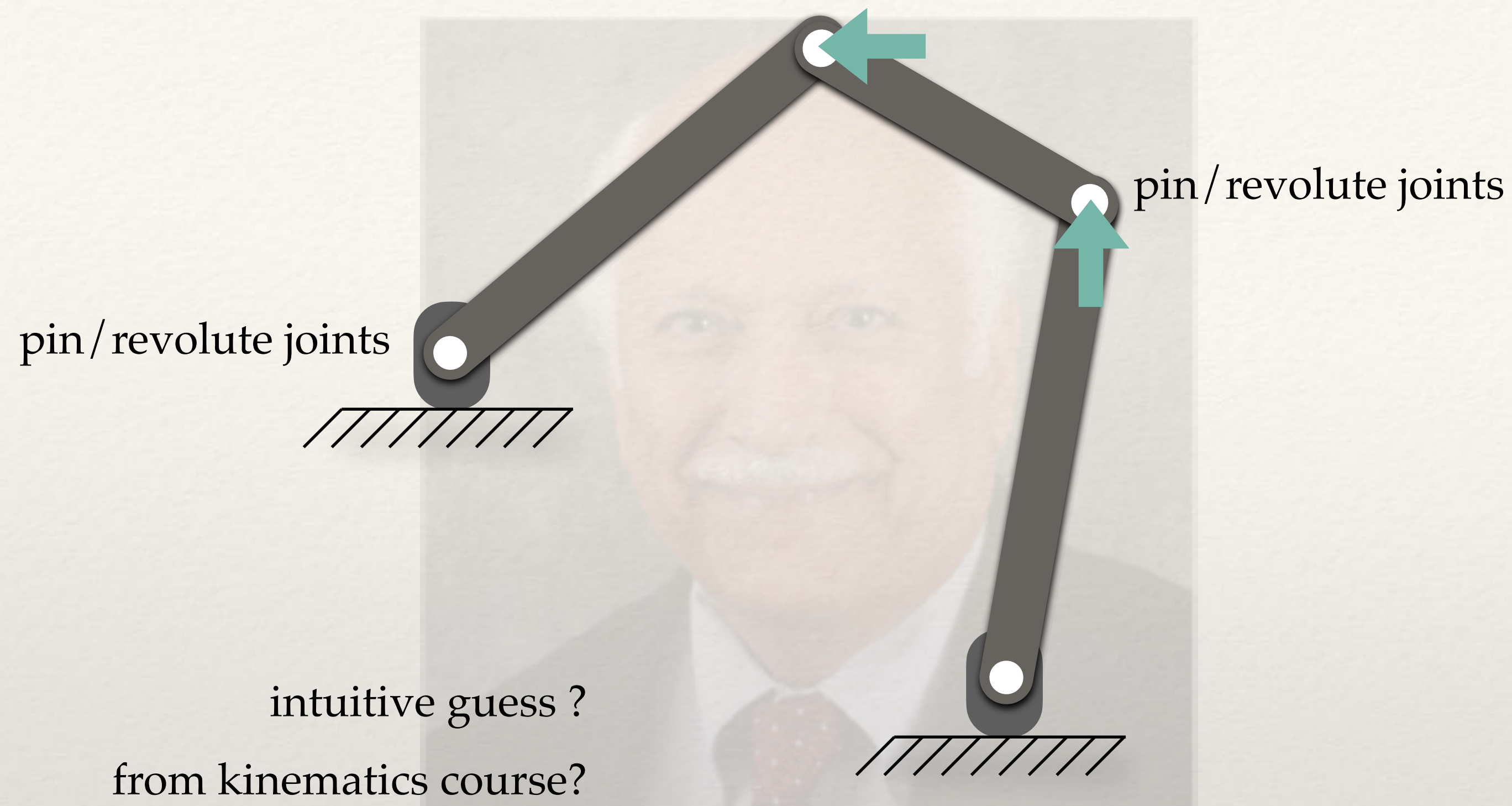
(c)



(d)

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor
Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

MECHANISMS

MANIPULATORS

they (must) move
to transmit force, motion, energy

MOBILITY ANALYSIS

is

STEP I

in quantification of motion

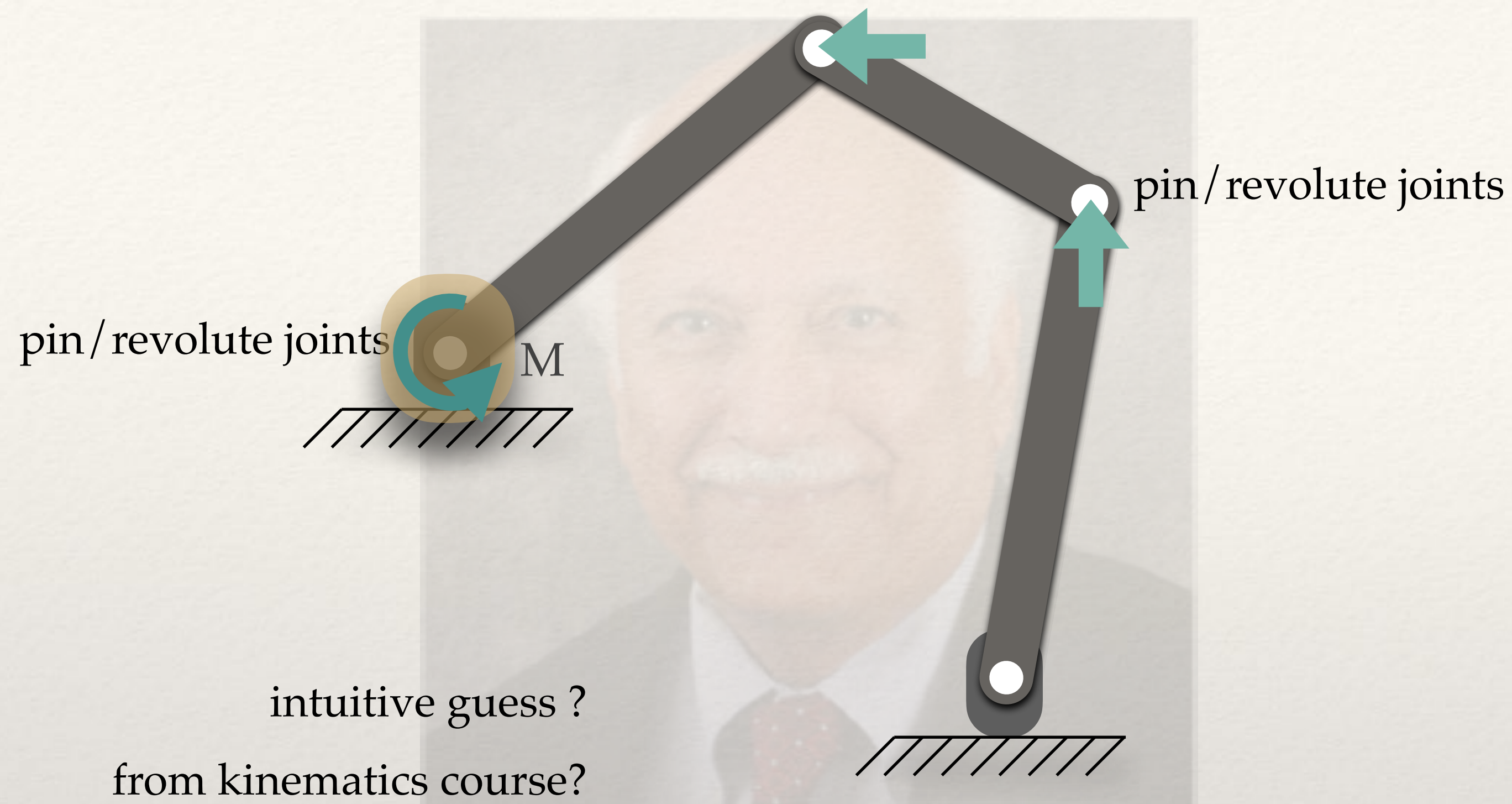
Q: How does this 4-bar linkage move?

Q: How many (minimum number of)
actuators required ?

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

MECHANISMS

MANIPULATORS

they (must) move
to transmit force, motion, energy

MOBILITY ANALYSIS

is

STEP I

in quantification of motion

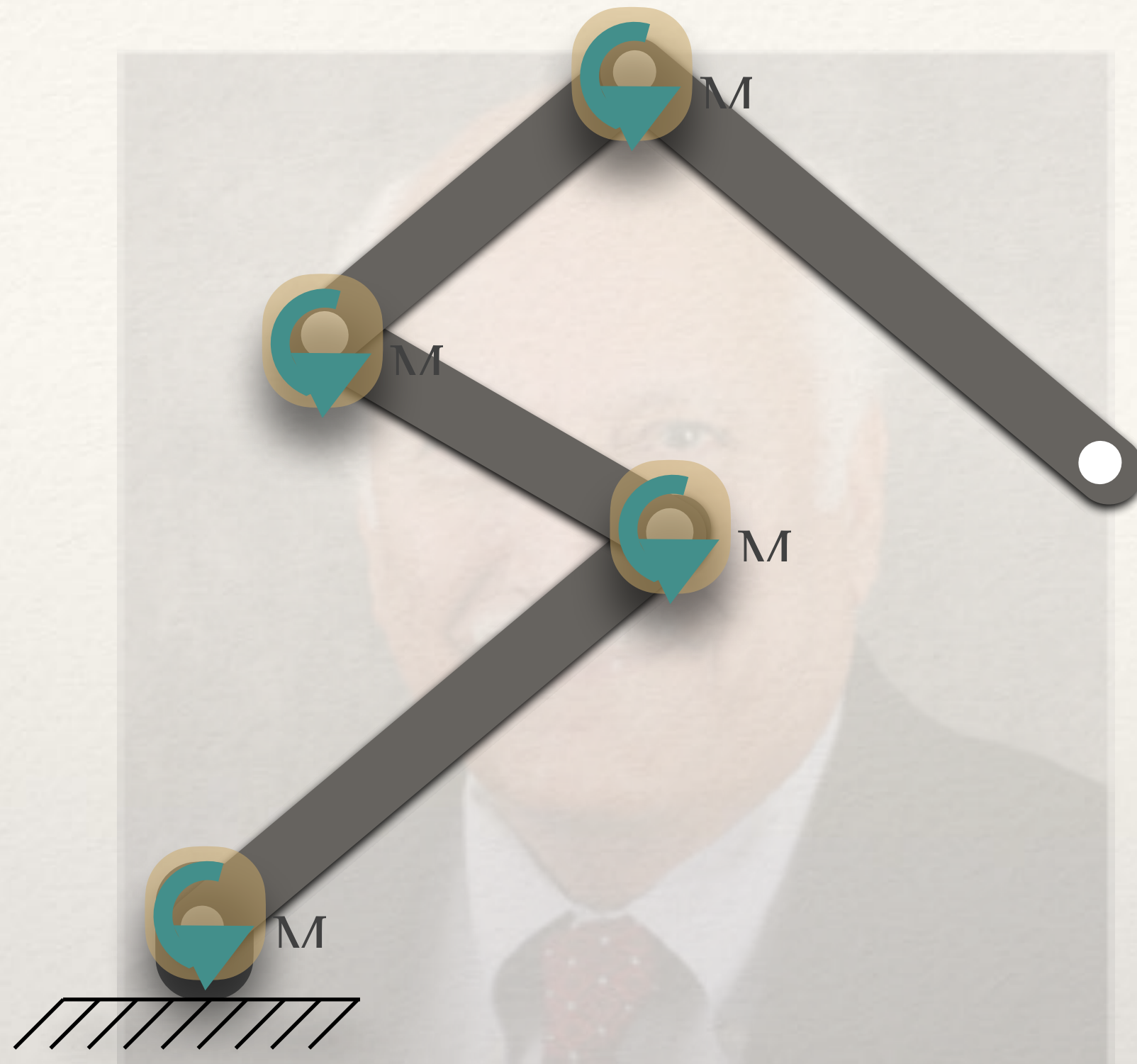
Q: How does this 4-bar linkage move?

Q: How many (minimum number of)
actuators required ?

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

MECHANISMS

MANIPULATORS

Open Chain manipulator

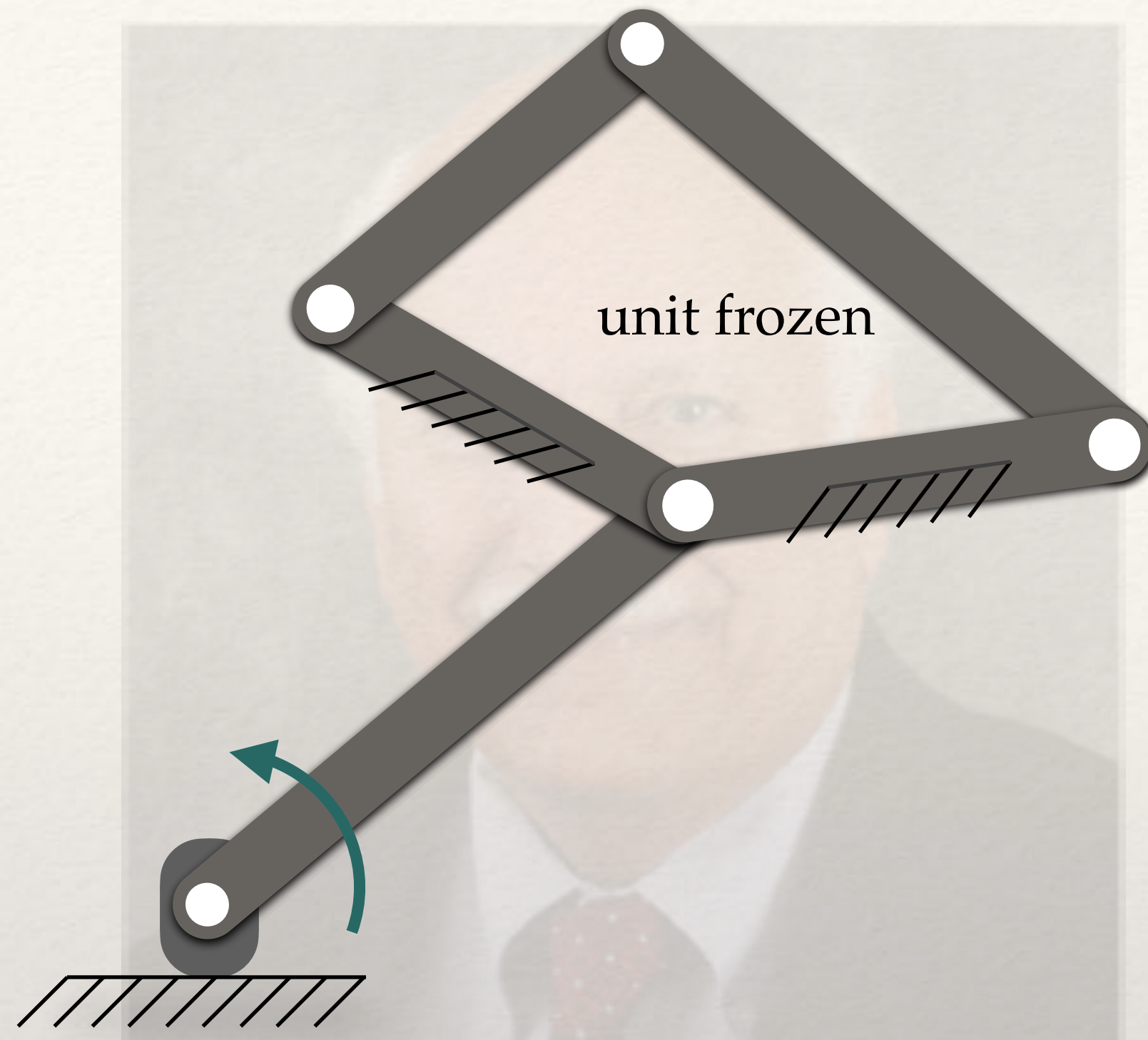
MOBILITY ANALYSIS

how many ways could this move?

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

MECHANISMS

MANIPULATORS

Open Chain manipulator

MOBILITY ANALYSIS

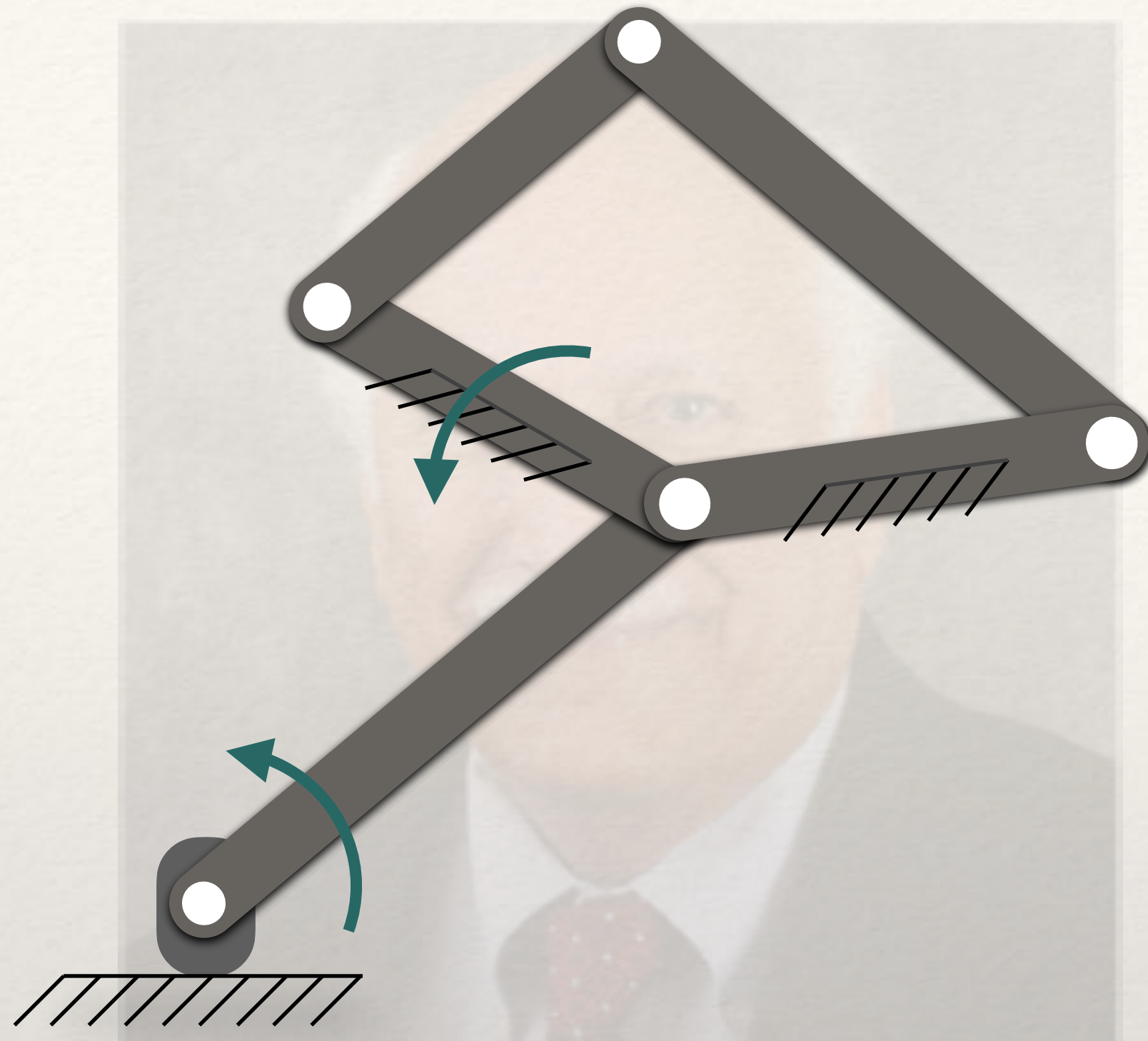
how many ways could this move?

how many ways, now, could this move?

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

MECHANISMS

MANIPULATORS

Open Chain manipulator

MOBILITY ANALYSIS

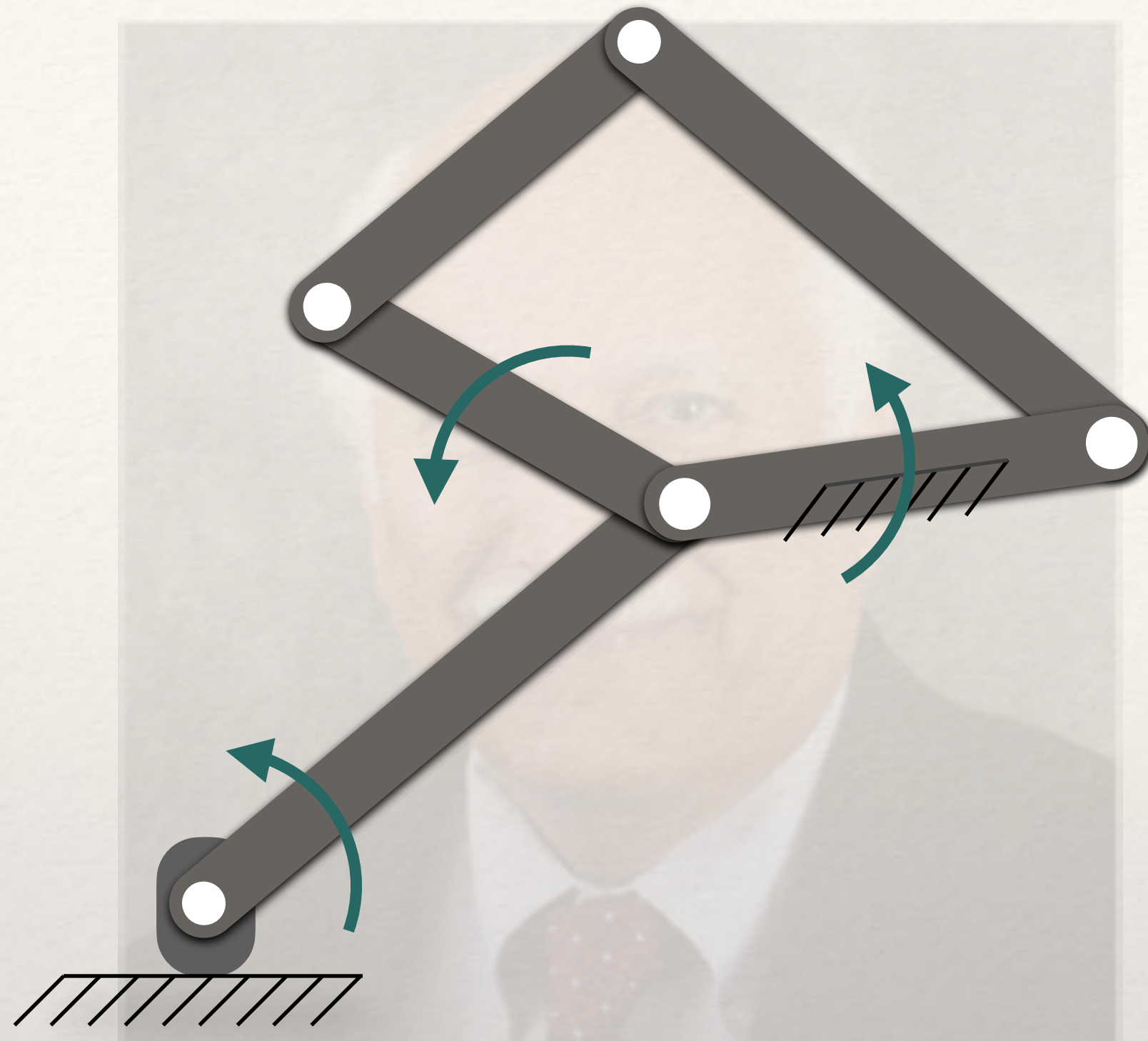
how many ways could this move?

how many ways, now, could this move?

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

MECHANISMS

MANIPULATORS

Open Chain manipulator

MOBILITY ANALYSIS

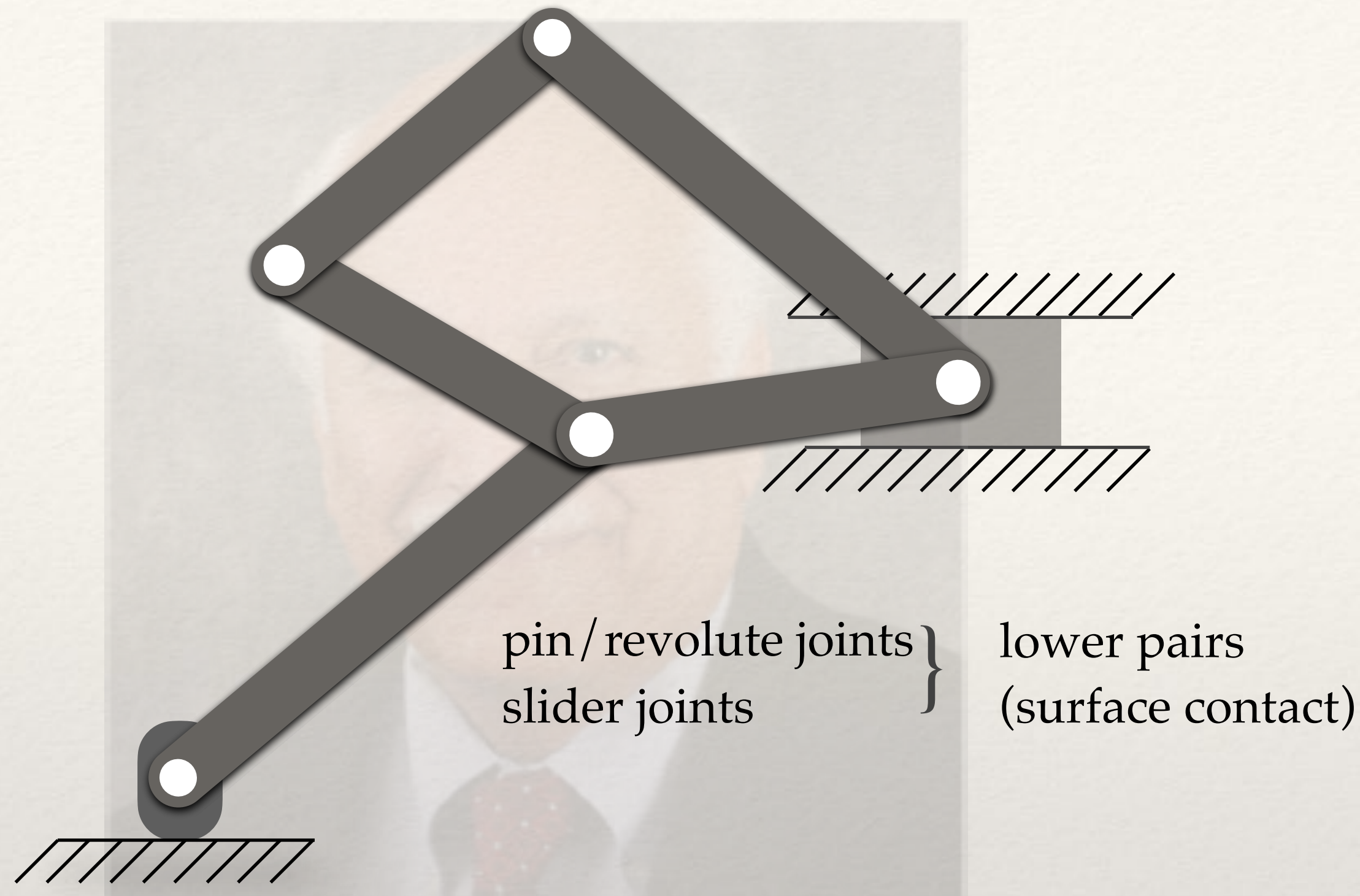
how many ways could this move?

how many ways, now, could this move?

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



pin / revolute joints } lower pairs
slider joints } (surface contact)

<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

MECHANISMS

MANIPULATORS

Open Chain manipulator

MOBILITY ANALYSIS

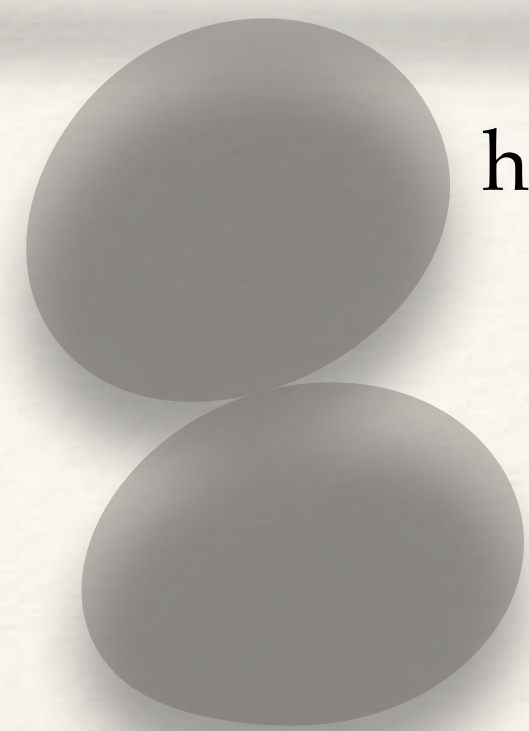
how many ways could this move?

how many ways, now, would this system move?

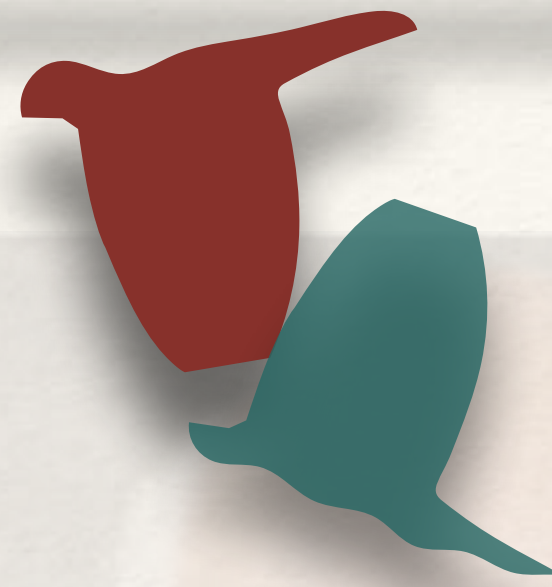
Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

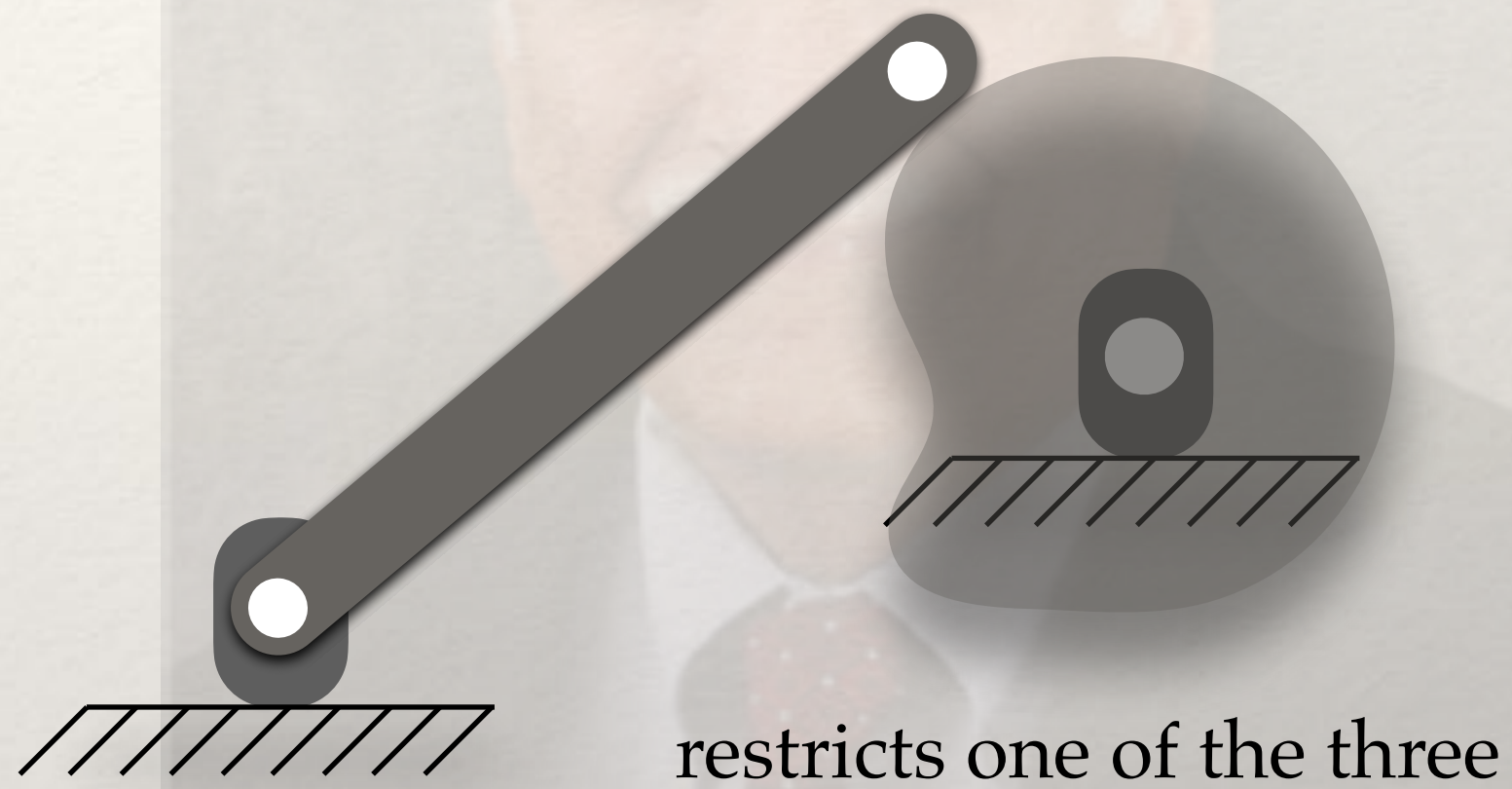
Indian Institute of Technology Kanpur



higher pairs



cam-follower joint } higher pair
(line / point contact)



restricts one of the three relative motions

<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

MECHANISMS

MANIPULATORS

Open Chain manipulator

MOBILITY ANALYSIS

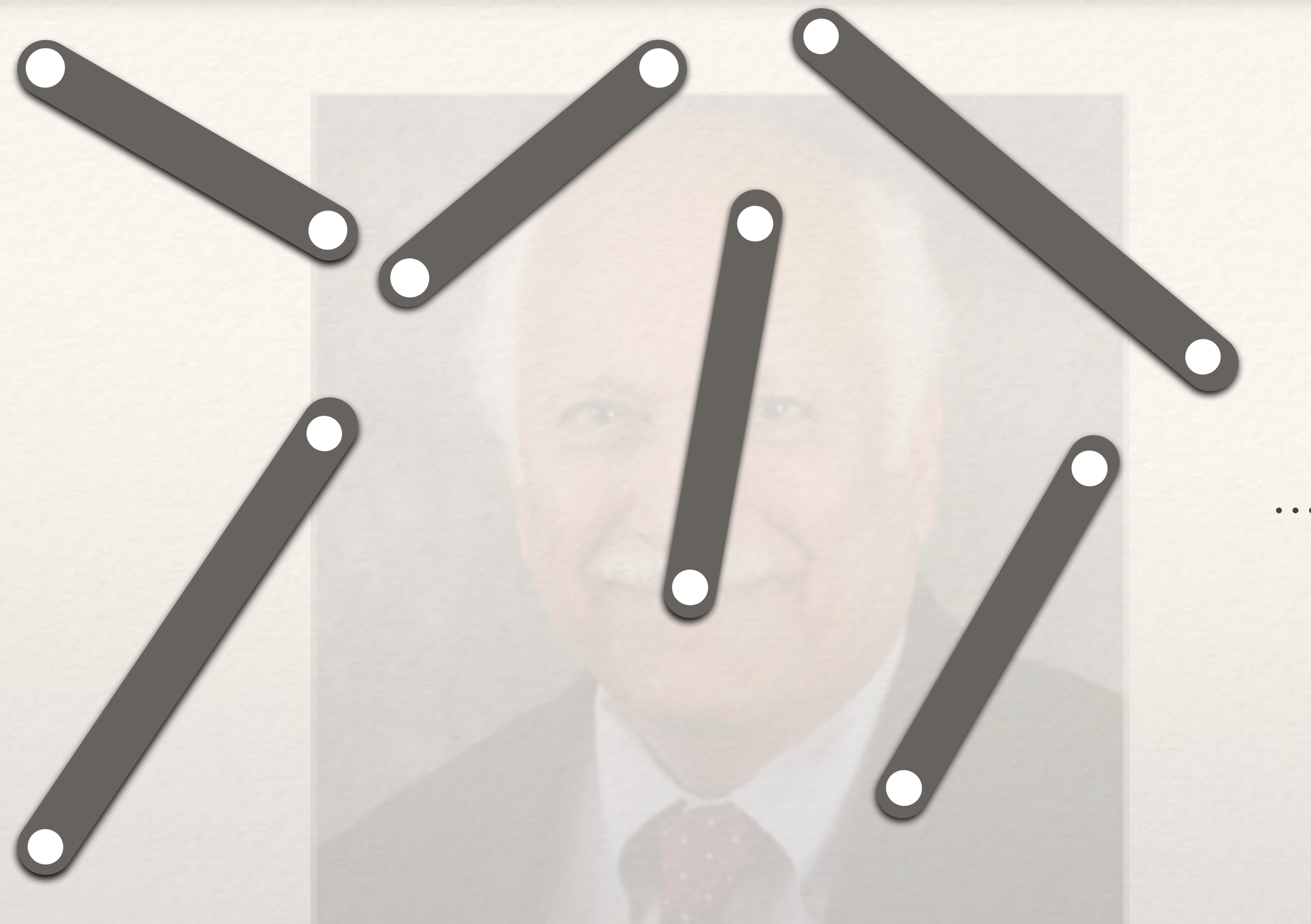
how many ways could this move?

how many ways, now, would this system move?

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

Grübler's (Mobility) Criterion Planar Motion

n rigid bodies

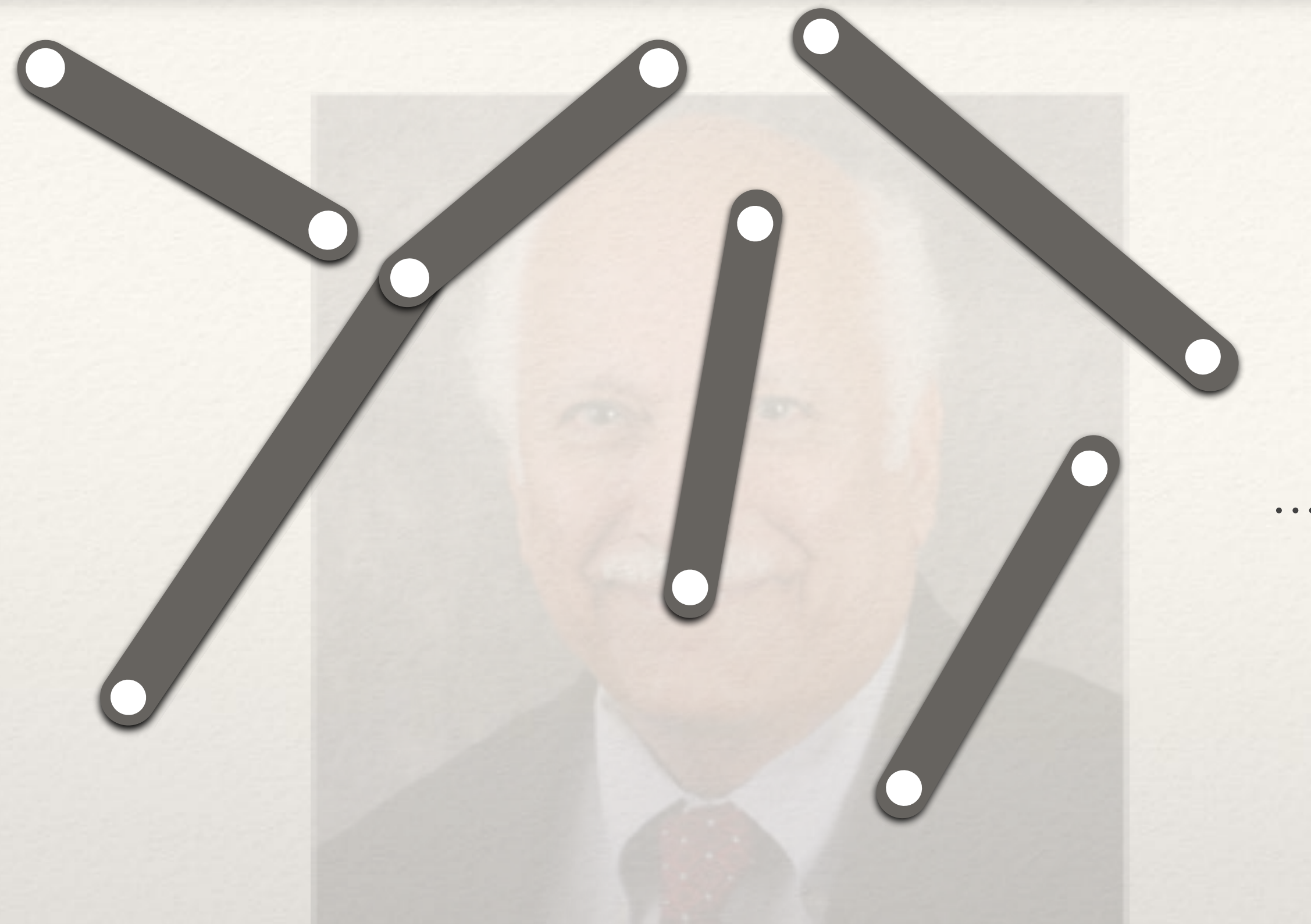
each body can translate and rotate

Net freedom in motion: $3n$

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

Grübler's (Mobility) Criterion Planar Motion

n rigid bodies

each body can translate and rotate

Net freedom in motion: $3n$

two motions **arrested** relative to a
rigid body

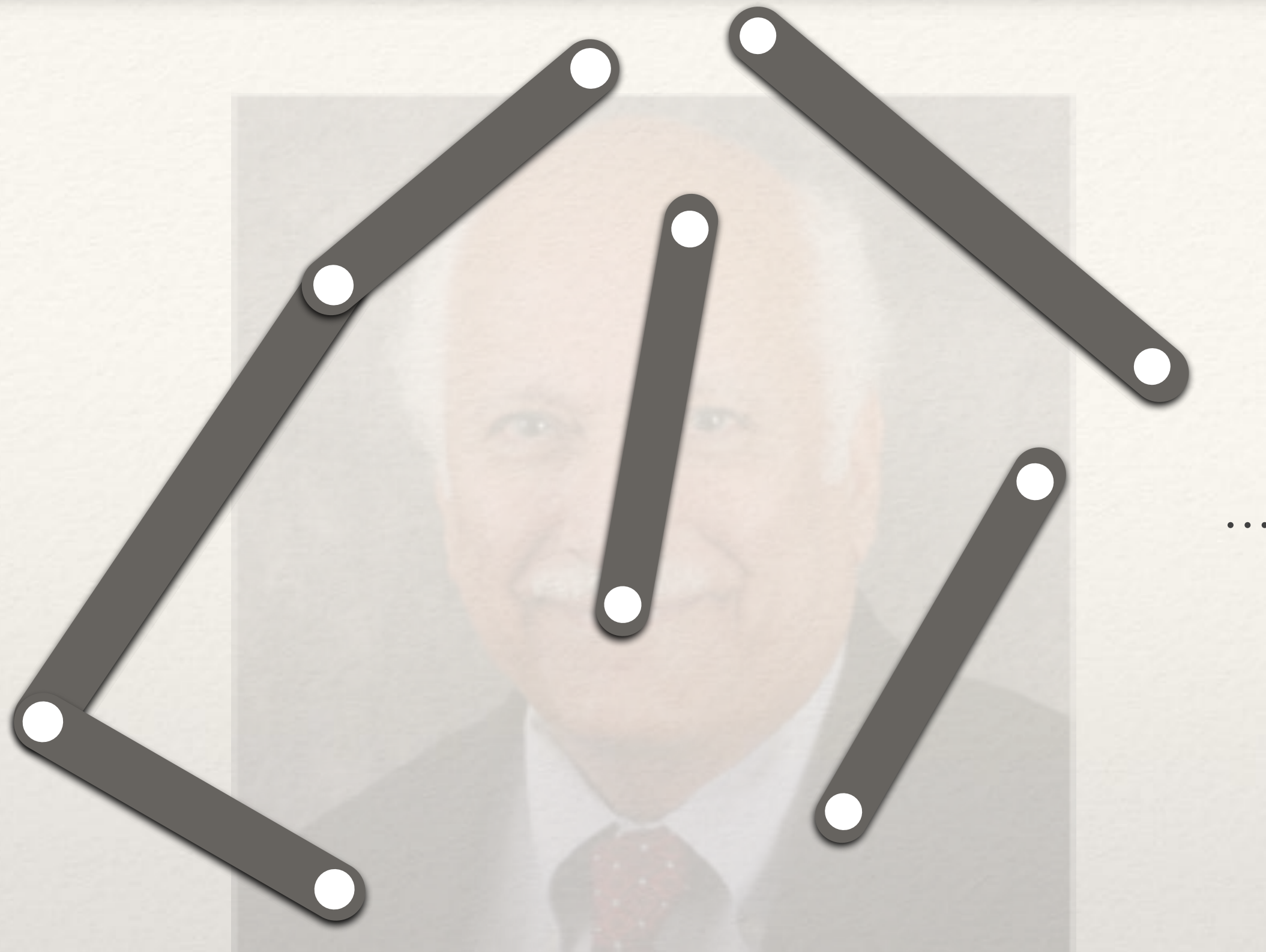
a revolute joint (kinematic-pair) is created

Net freedom in motion: $3n - 2$

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

Grübler's (Mobility) Criterion Planar Motion

n rigid bodies

each body can translate and rotate

Net freedom in motion: $3n$

two **more** motions **arrested** relative to a
rigid body

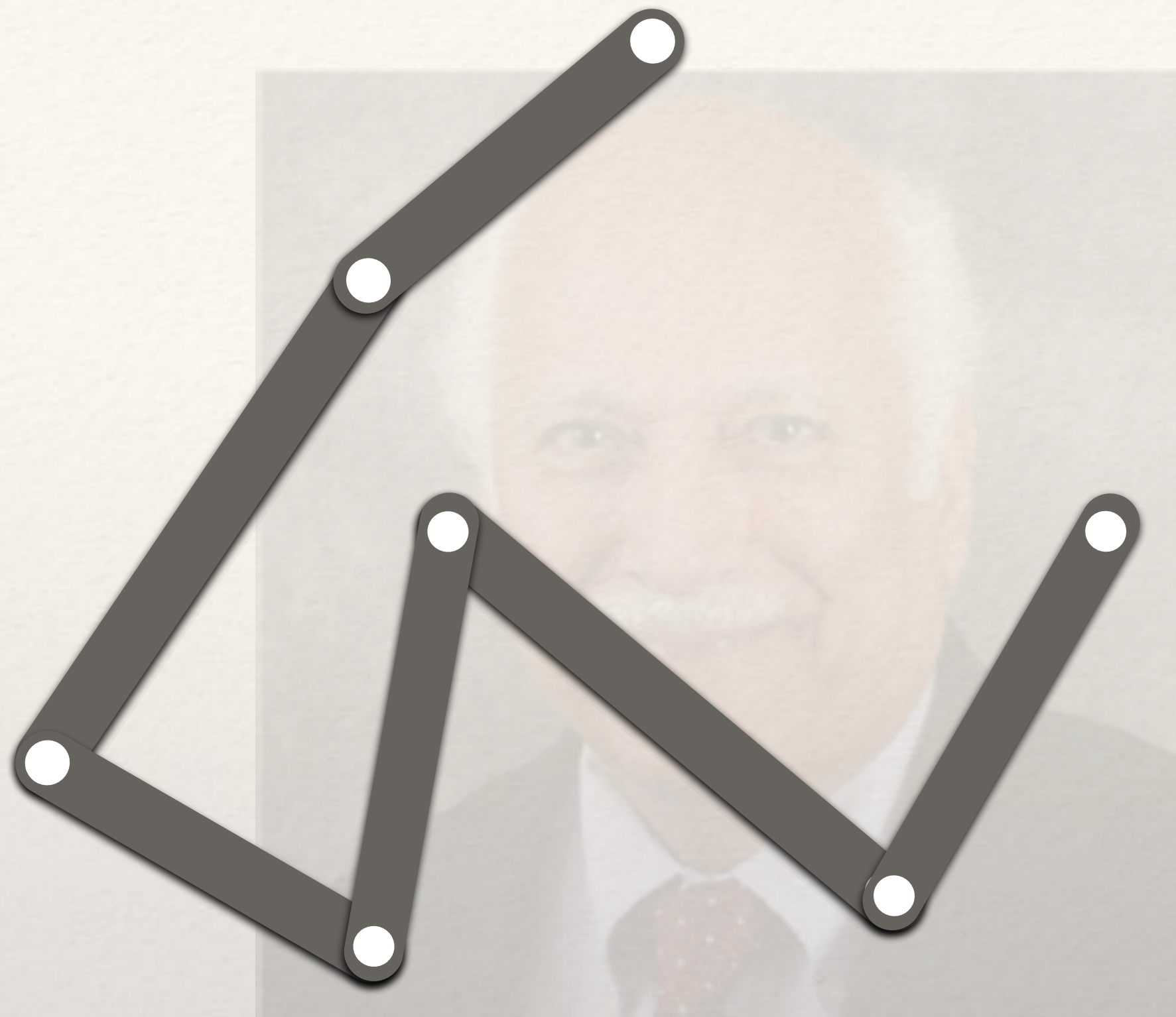
two revolute joints (kinematic-pairs) created

Net freedom in motion: $3n - 4$

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



...

<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

Grübler's (Mobility) Criterion Planar Motion

n rigid bodies

each body can translate and rotate

Net freedom in motion: $3n$

three **more** motions **arrested** relative to a rigid body

three additional revolute joints created

Net freedom in motion: $3n - 10$

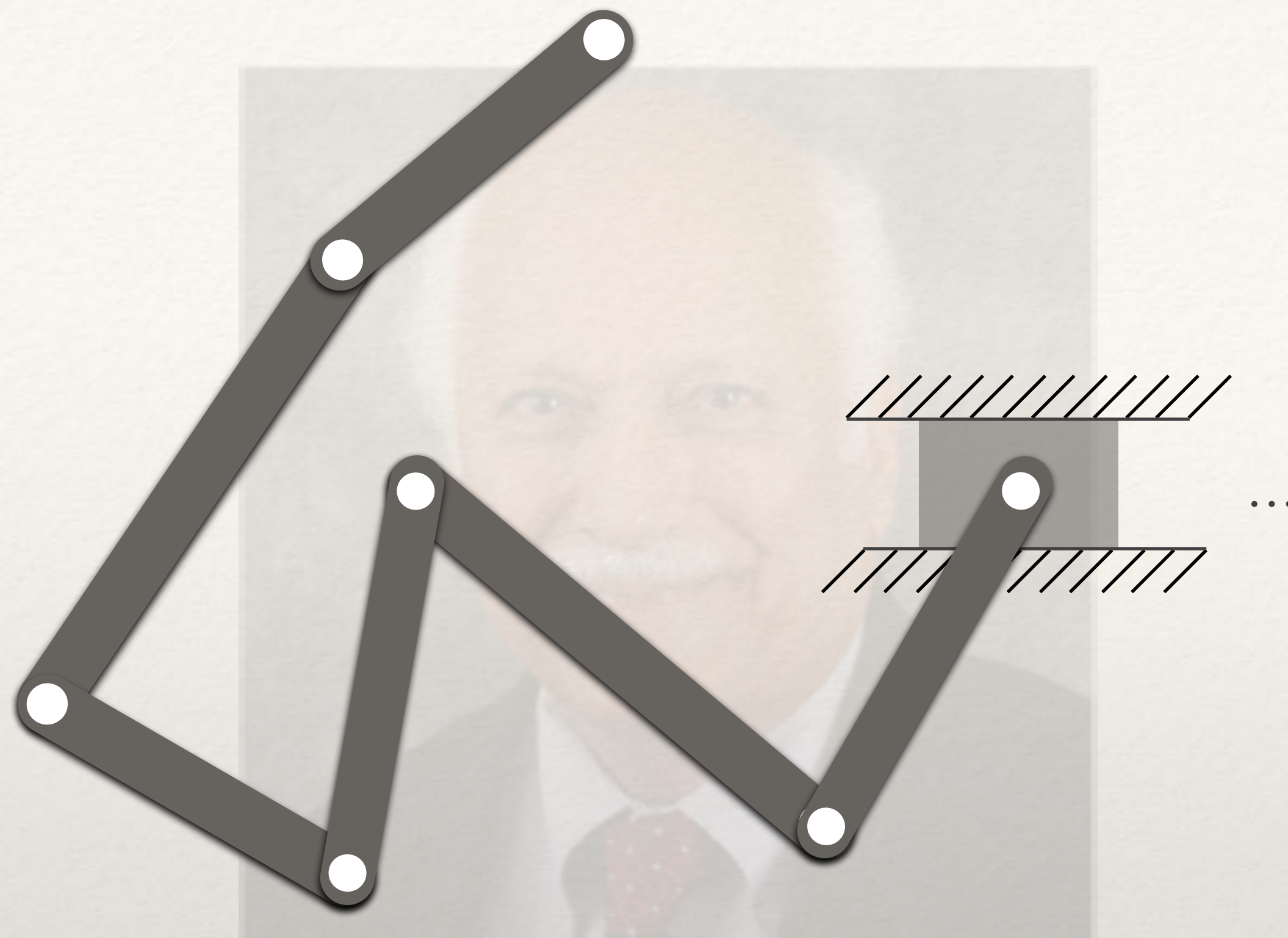
In general, net freedom in motion: $3n - 2rp$

rp : revolute pairs

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

Grübler's (Mobility) Criterion Planar Motion

n rigid bodies

each body can translate and rotate

Net freedom in motion: $3n$

In general, net freedom in motion: $3n - 2rp$

A link gets generated, which can only translate

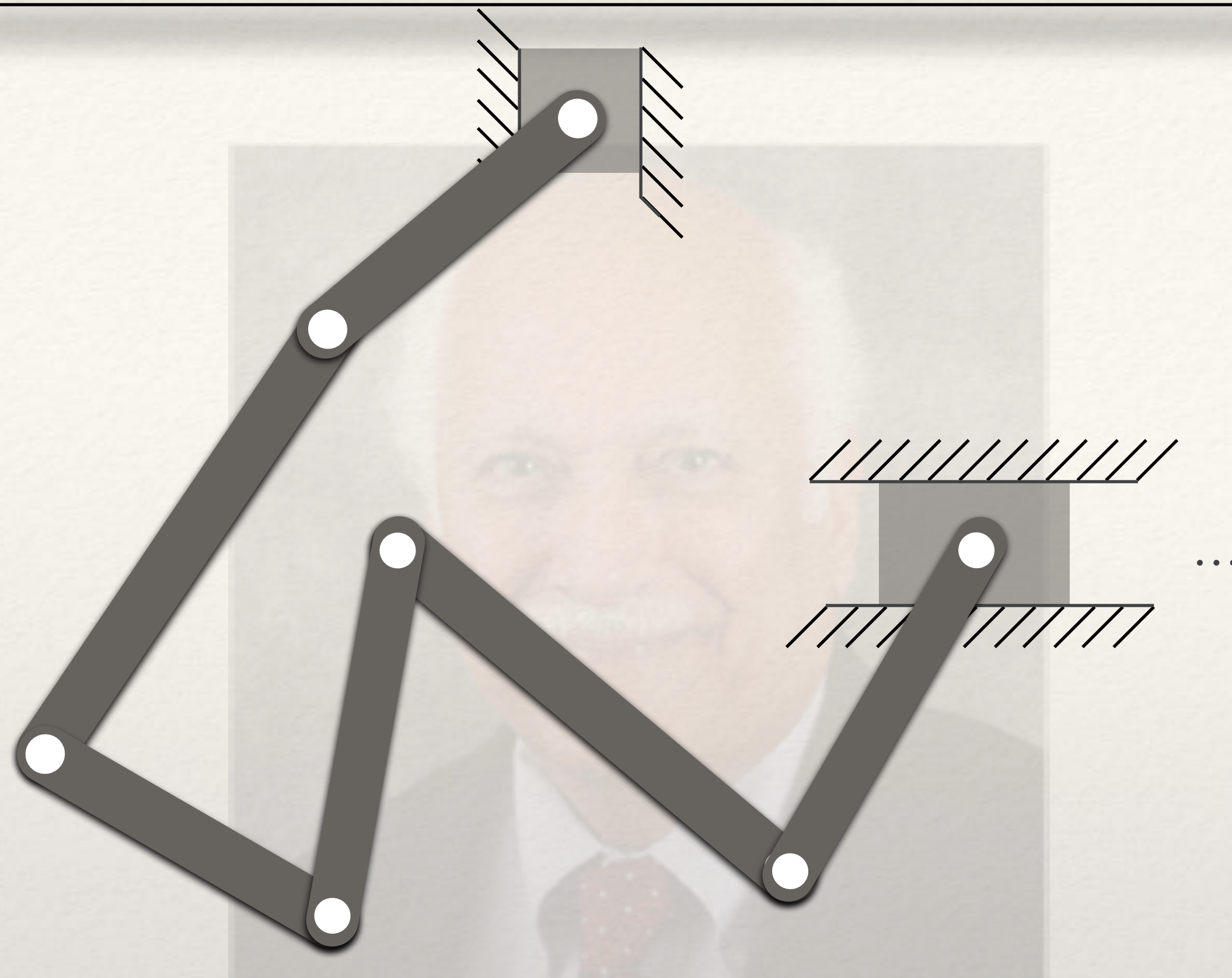
net freedom in motion: $3(n + 1) - 2rp - 2$
 $3n - 2rp - 2$

n : TOTAL number of rigid bodies

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

Grübler's (Mobility) Criterion Planar Motion

n rigid bodies

each body can translate and rotate

Net freedom in motion: $3n$

In general, net freedom in motion:
 $3n - 2rp - 2$

One more link gets generated, which can only translate

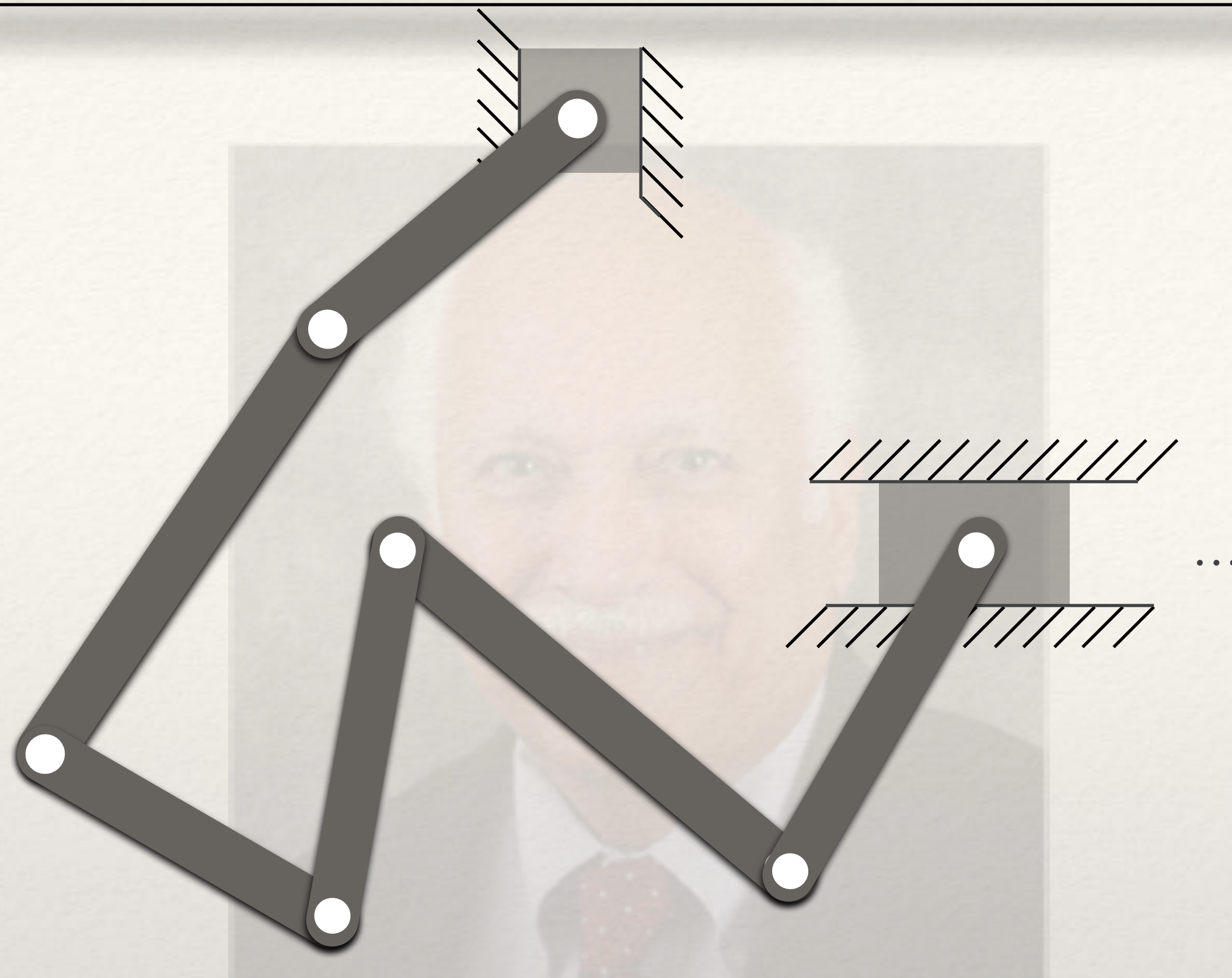
net freedom in motion: $3(n + 1) - 2rp - 4$
 $3n - 2rp - 4$

n : TOTAL number of rigid bodies

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

Grübler's (Mobility) Criterion Planar Motion

n rigid bodies

each body can translate and rotate

Net freedom in motion: $3n$

In general, net freedom in motion:
 $3n - 2rp - 2sp$

sp : slider pairs

net freedom in motion: $3n - 2j_1$

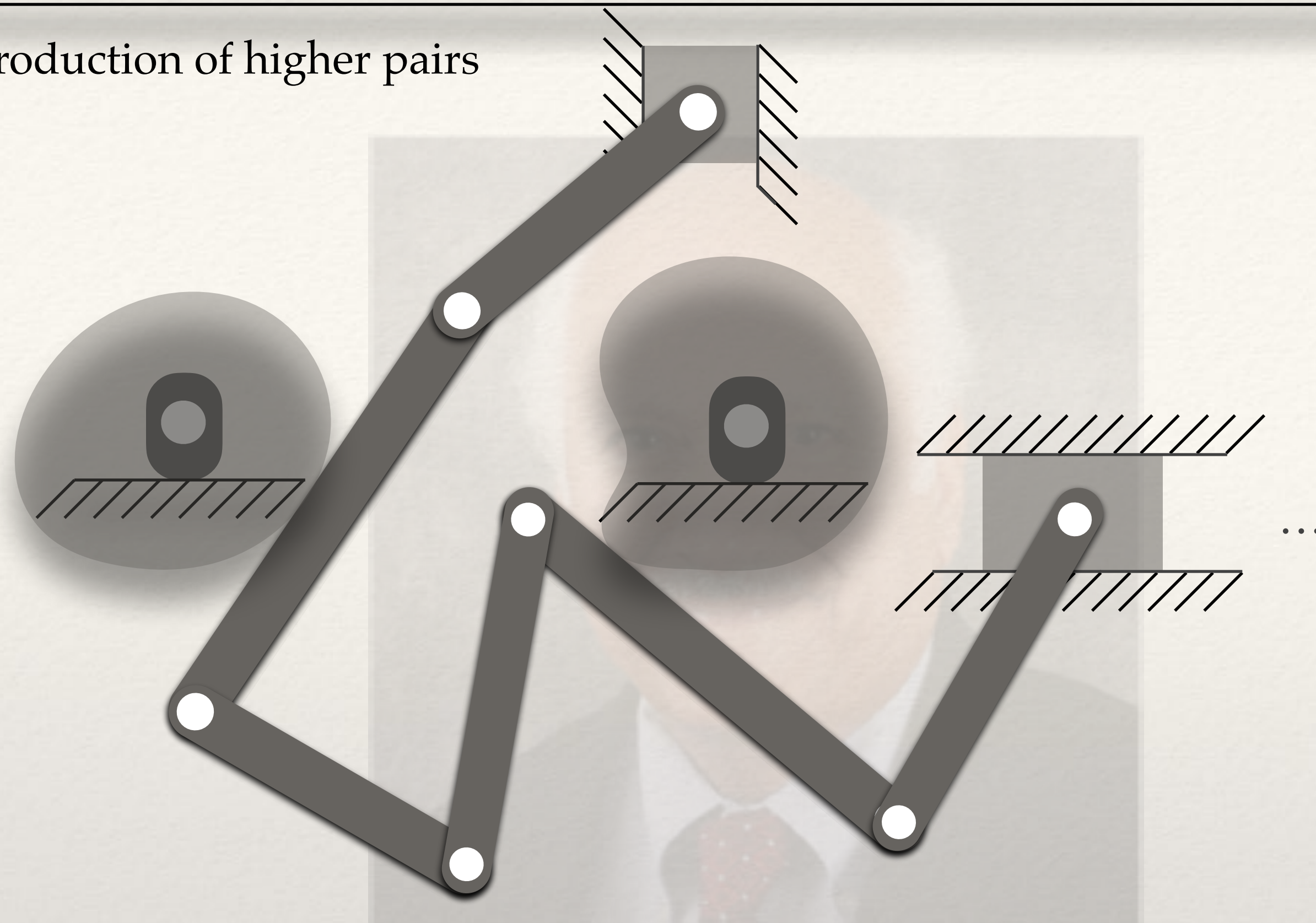
$j_1 = rp + sp$ number of lower order pairs

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur

Introduction of higher pairs



<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

Grübler's (Mobility) Criterion Planar Motion

n rigid bodies

each body can translate and rotate

net freedom in motion: $3n - 2j_1$

With each higher pair, one extra link or rigid-body introduced

Those many, i.e., j_2 motions reduced

net freedom in motion:

$$DOF : 3n - 2j_1 - j_2 - 3$$

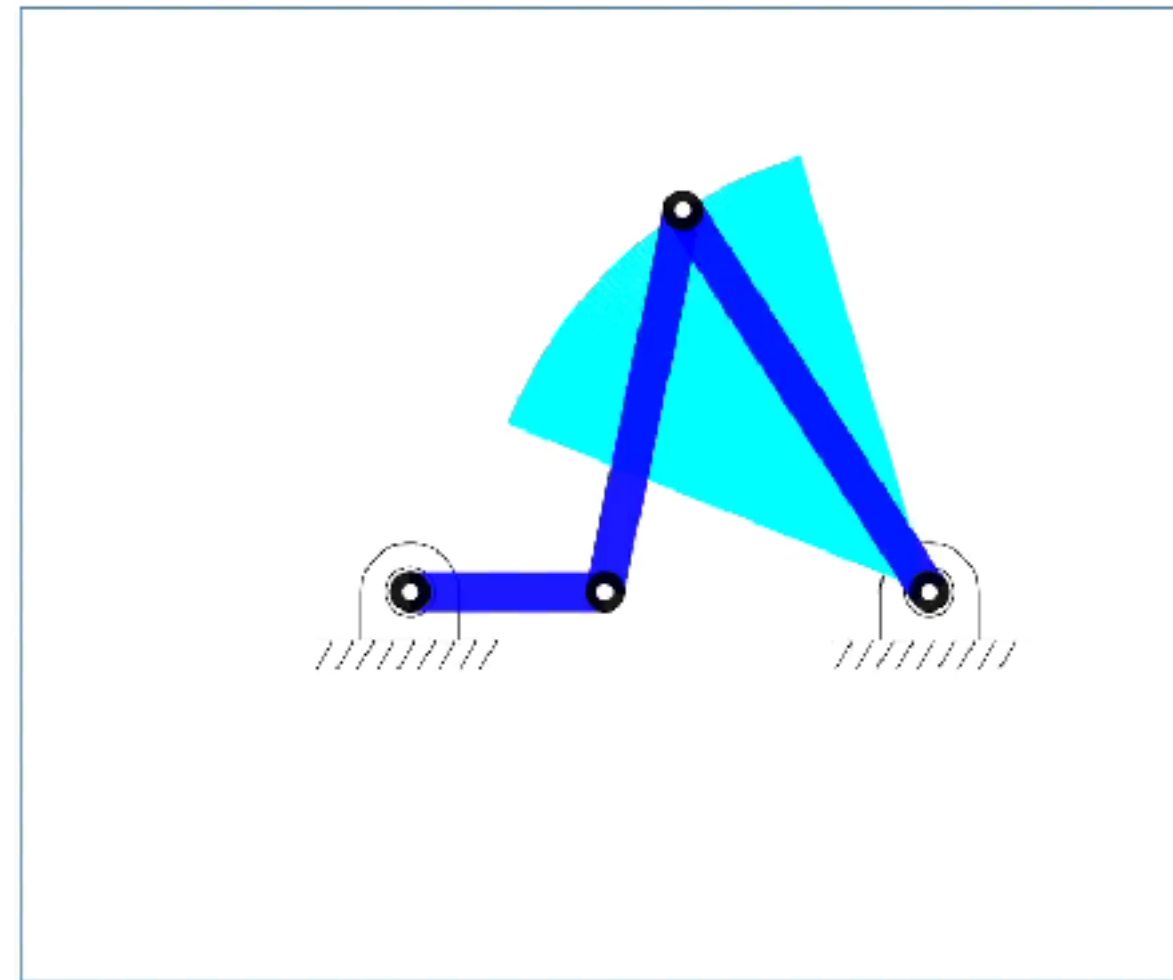
to arrest 2T and 1R of the entire system

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur

Some Examples



No. links: $n = 4$

No. joints: $j_1 = 4$

<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

Grübler's (Mobility) Criterion

Planar Motion

n rigid bodies

each body can translate and rotate

net freedom in motion: $3n - 2j_1$

With each higher pair, one extra link or rigid-body introduced

Those many, i.e., j_2 motions reduced

net freedom in motion:

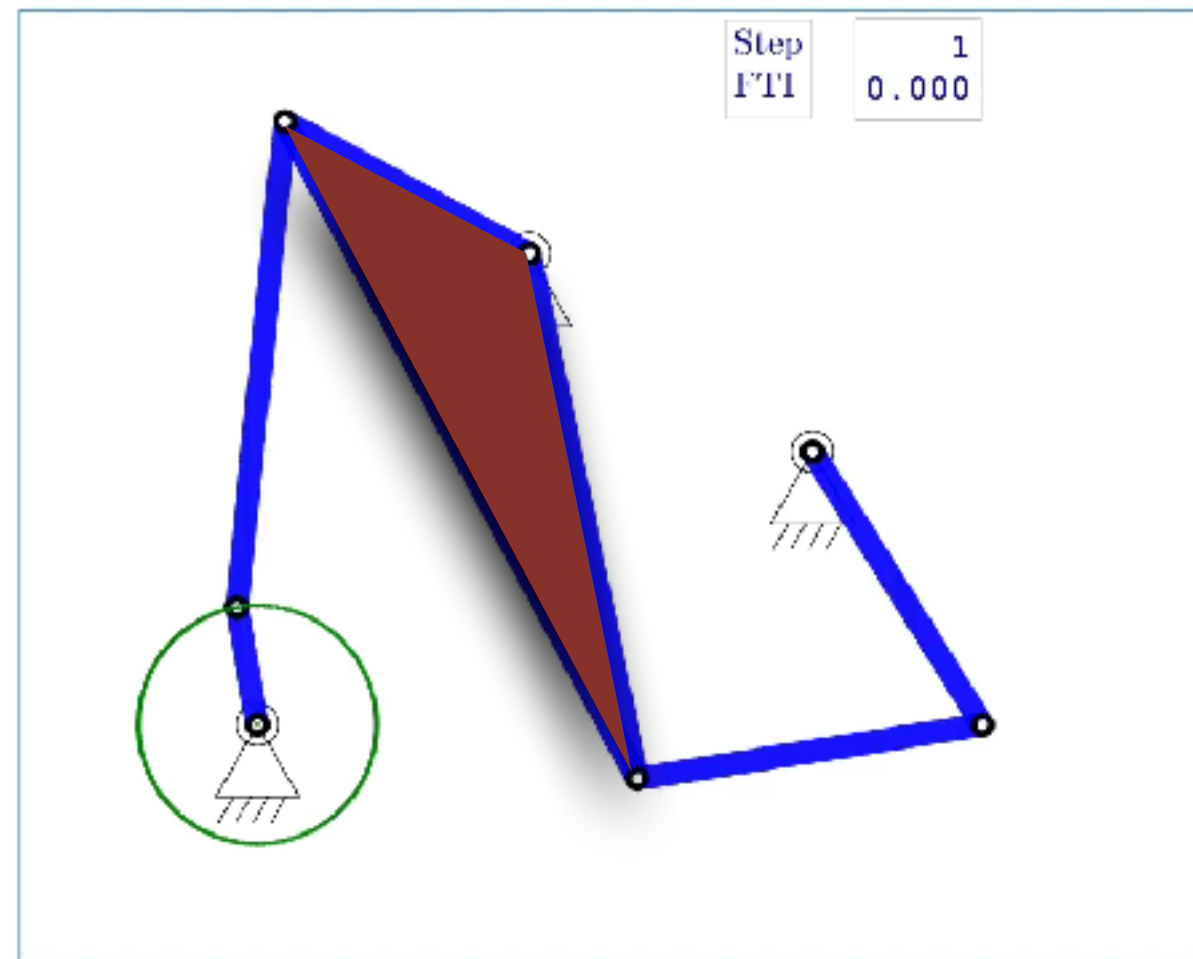
$$DOF : 3(n - 1) - 2j_1 - j_2$$

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

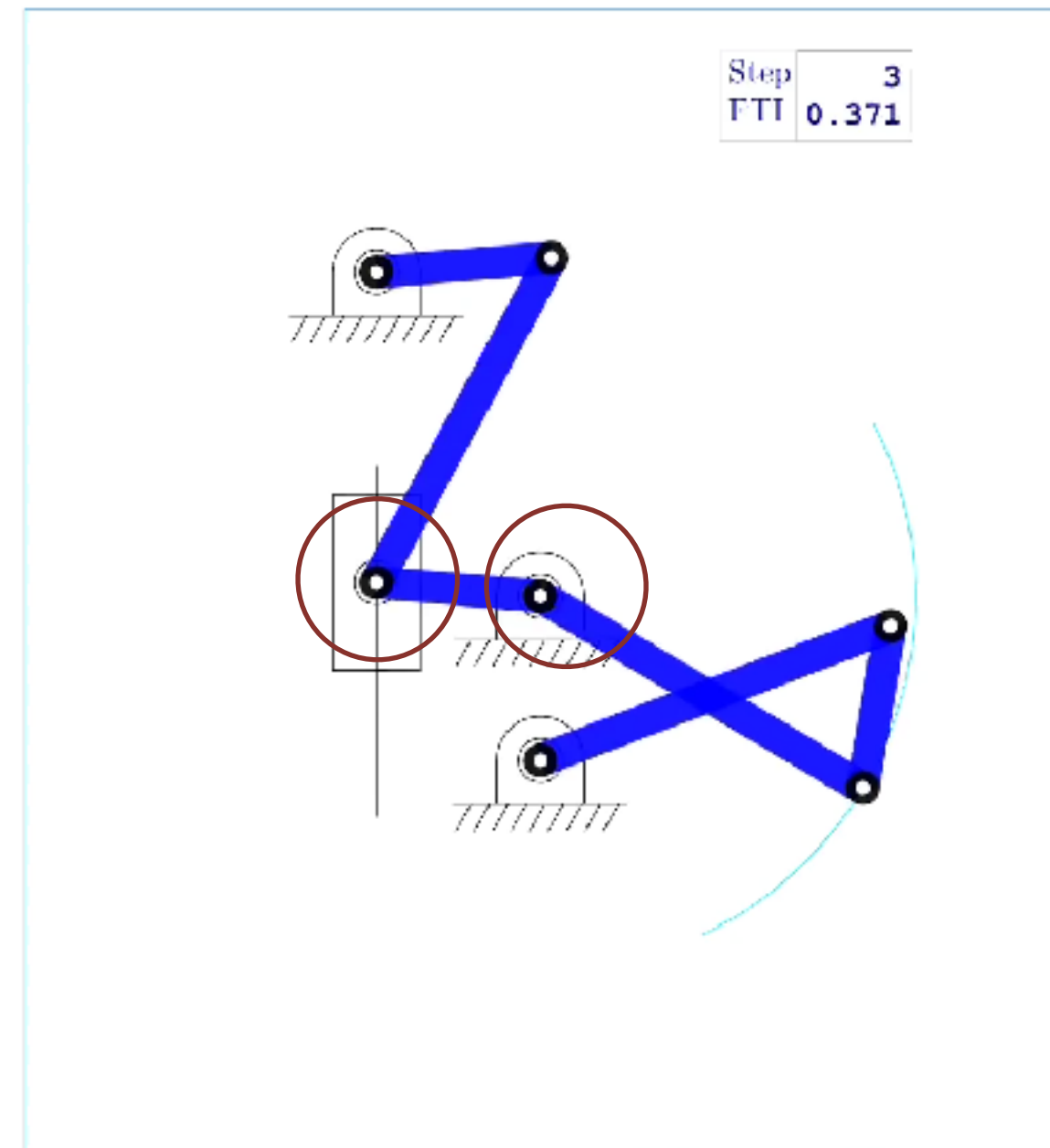
Indian Institute of Technology Kanpur

Some Examples



No. links: $n = 6$

No. joints: $j_1 = 7$



No. links: $n = 8$ No. joints: $j_1 = 10$

<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Prof. Ashok Midha

Grübler's (Mobility) Criterion Planar Motion

n rigid bodies

each body can translate and rotate

net freedom in motion: $3n - 2j_1$

With each higher pair, one extra link or rigid-body introduced

Those many, i.e., j_2 motions reduced

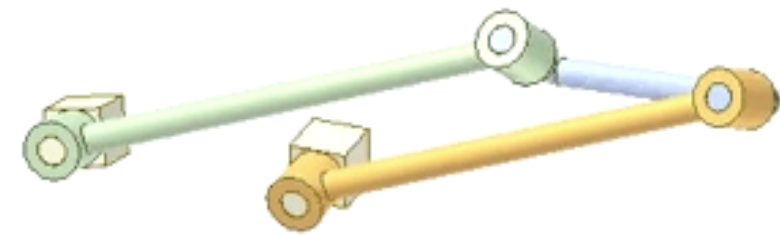
net freedom in motion:

$$DOF : 3(n - 1) - 2j_1 - j_2$$

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur



Bennett 4R(revolute) mechanism

<http://youtu.be/q433oAXwHuU>

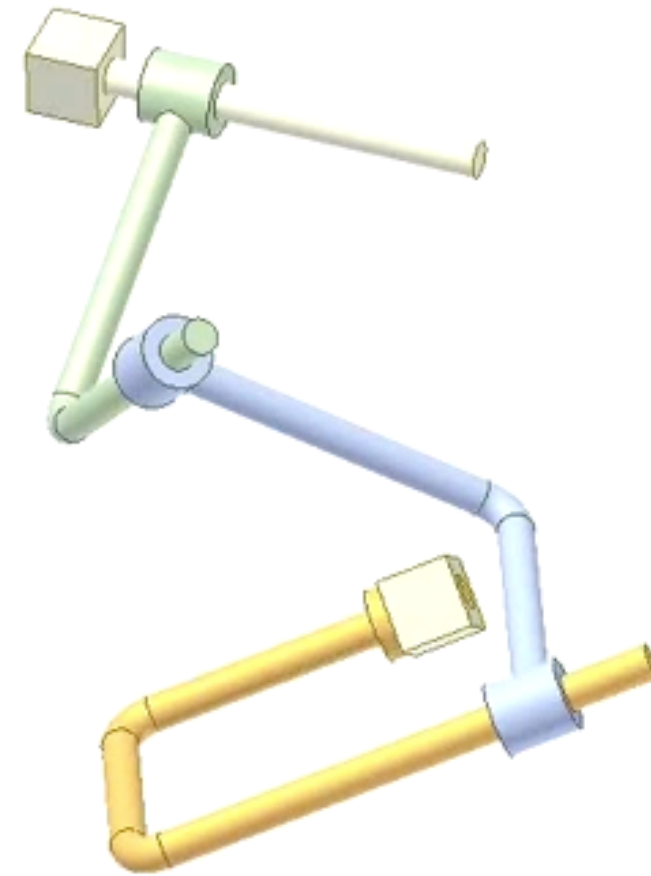
$$4L, 4j_1: DOF = 6(3) - 5(4) = -2$$

<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>

Opposite links: a, b, equal length

Opposite twist angles: A, B, equal but opposite signs

$$\sin A/a = \sin B/b$$



RCCC mechanism

<http://youtu.be/9mcEF2s8QZU>

$$4L, 1j_1, 3j_2: DOF = 6(3) - 5(1) - 4(3) = 1$$

Prof. Ashok Midha

Kutzbach (Mobility) Criterion Spatial Motion

straightforward extension

$$DOF : 3(n - 1) - 2j_1 - j_2$$

n rigid bodies

j_1 joints restricting 5 relative motions

j_2 joints restricting 4 relative motions

j_3 joints restricting 3 relative motions

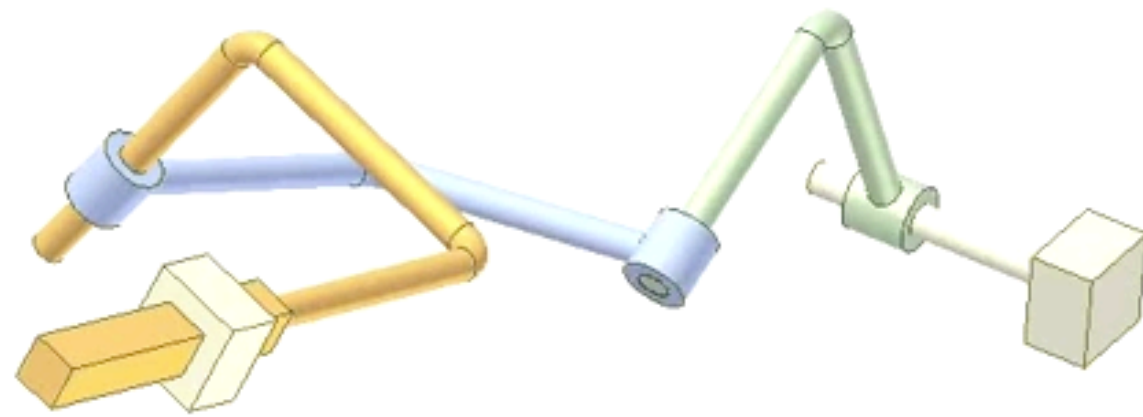
j_5 joints restricting 1 relative motions

$$DOF : 6(n - 1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5$$

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

Indian Institute of Technology Kanpur

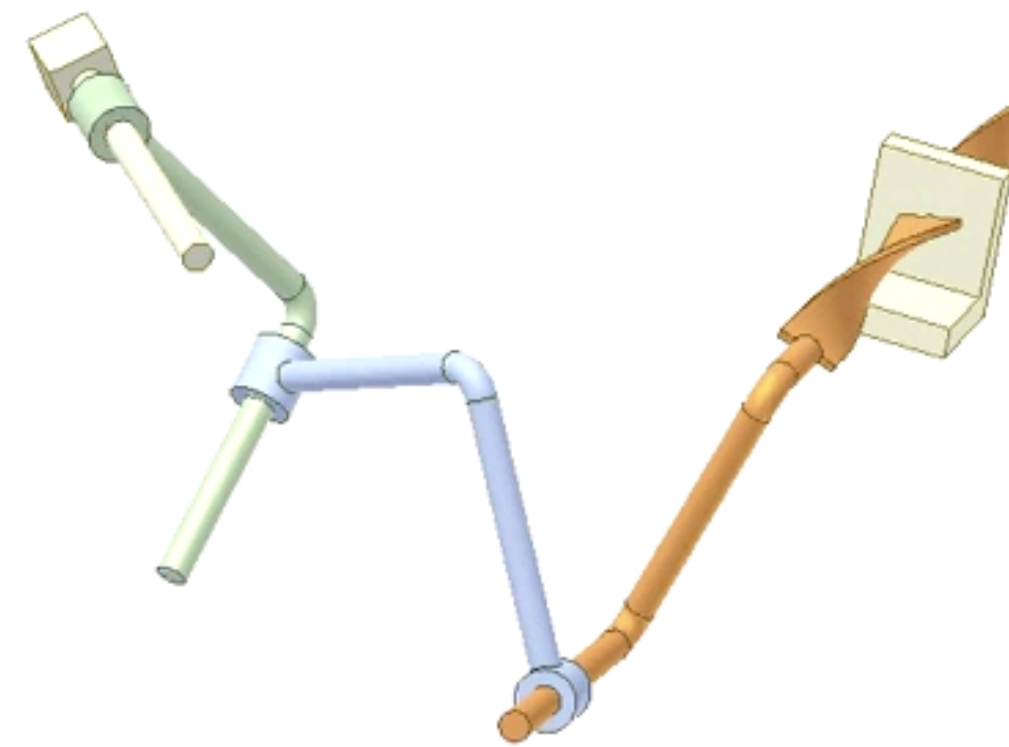


PCCC mechanism

<http://youtu.be/nK66lwNJG78>

$$4L, 1j_1, 3j_2: DOF = 6(3) - 5(1) - 4(3) = 1$$

<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>



HCCC mechanism

<http://youtu.be/aUllcT74mXM>

$$4L, 1j_1, 3j_2: DOF = 6(3) - 5(1) - 4(3) = 1$$

Prof. Ashok Midha

Kutzbach (Mobility) Criterion Spatial Motion

straightforward extension

$$DOF : 3(n - 1) - 2j_1 - j_2$$

n rigid bodies

j_1 joints restricting 5 relative motions

j_2 joints restricting 4 relative motions

j_3 joints restricting 3 relative motions

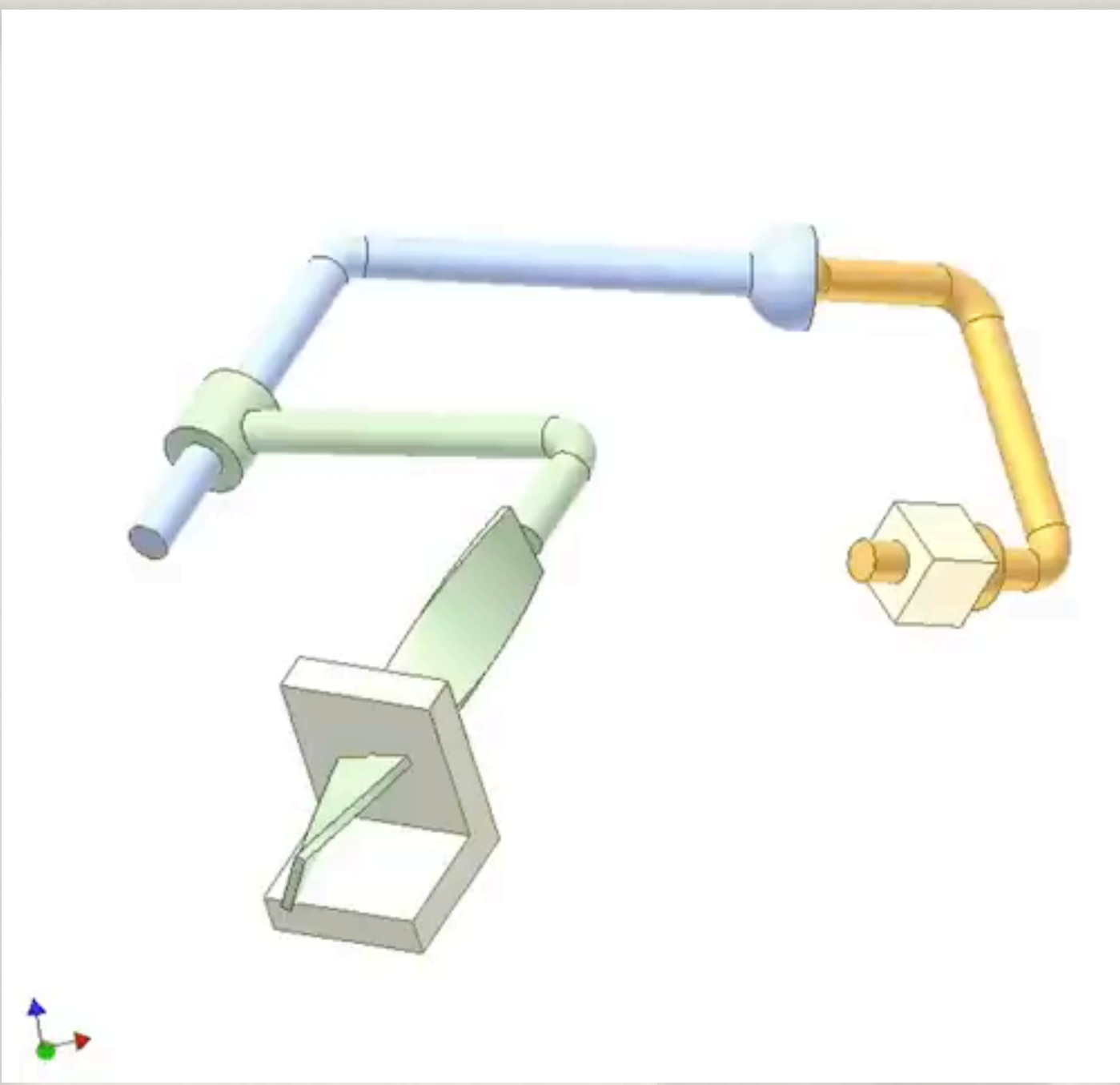
j_5 joints restricting 1 relative motions

$$DOF : 6(n - 1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5$$

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor

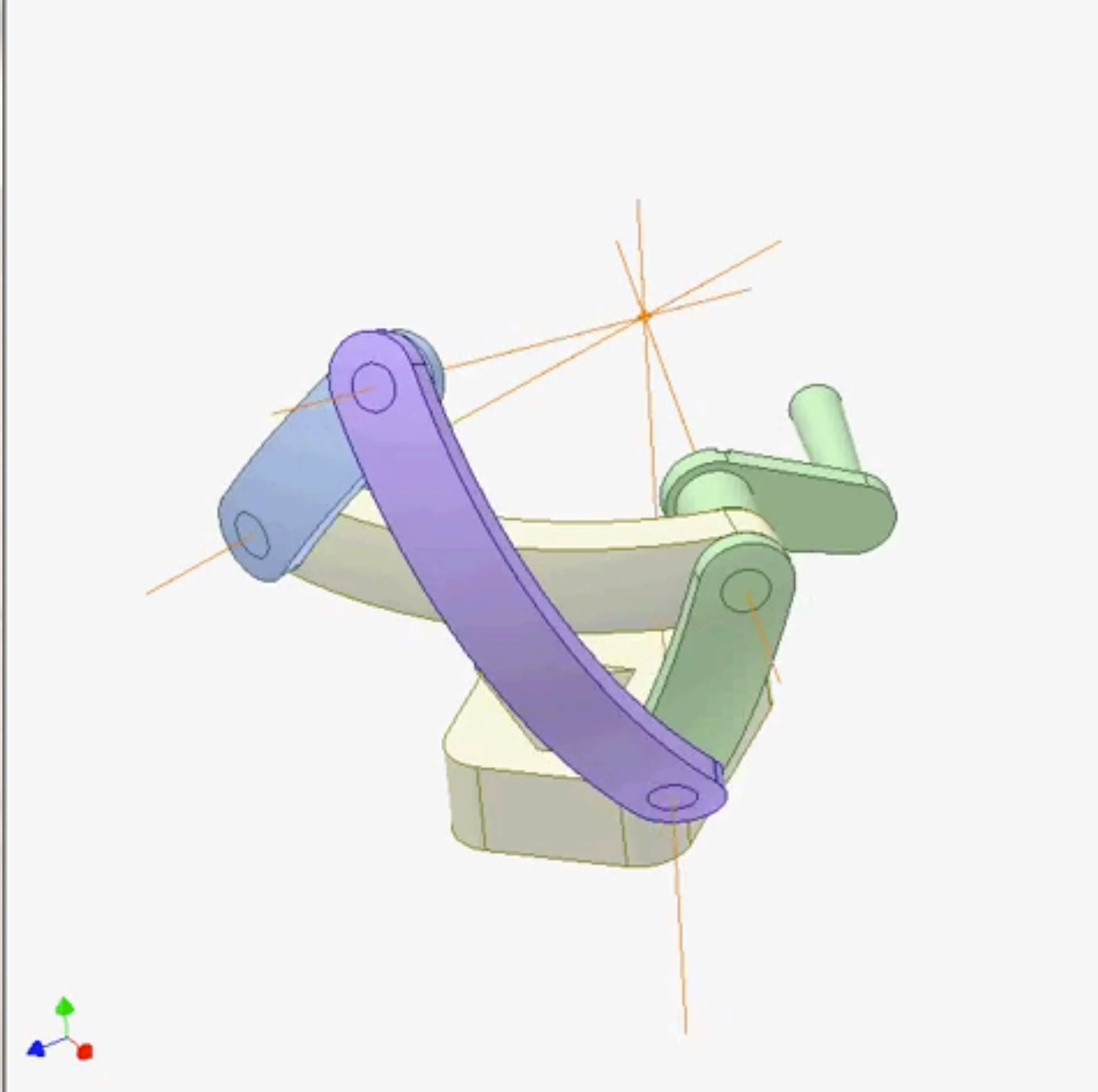
Indian Institute of Technology Kanpur



RSCH mechanism
<http://youtu.be/Gg8Q6nUZc1c>

$$4L, 2j_1, 1j_2, 1j_3: DOF = 6(3) - 5(2) - 4(1) - 3(1) = 1$$

<https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG>



Spherical 4R mechanism
<http://www.youtube.com/watch?v=q0erDDuPO7w>

$$4L, 4j_1: DOF = 6(3) - 5(4) = -2$$

Prof. Ashok Midha

Axes of R joints cocentric

Kutzbach (Mobility) Criterion Spatial Motion

straightforward extension

$$DOF : 3(n - 1) - 2j_1 - j_2$$

n rigid bodies

j_1 joints restricting 5 relative motions

j_2 joints restricting 4 relative motions

j_3 joints restricting 3 relative motions

j_5 joints restricting 1 relative motions

$$DOF : 6(n - 1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5$$

Compliant Mechanisms (ME 851)

Anupam Saxena
 Professor

Indian Institute of Technology Kanpur