



(c)

(d)

(e)

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Grübler's (Mobility) Criterion Planar Motion

n rigid bodies

 j_1 lower pairs j_2 higher pairs

net freedom in motion: $DOF: 3(n-1) - 2j_1 - j_2$





 \mathbf{x}

4L, $1j_1$, $3j_2$: DOF = 6(3) - 5(1) - 4(3) = 1

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Prof. Ashok Midha

Compliant Mechanisms (ME 851)

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Kutzbach (Mobility) Criterion Spatial Motion

straightforward extension $DOF: 3(n-1) - 2j_1 - j_2$

n rigid bodies j_1 joints restricting 5 relative motions j_2 joints restricting 4 relative motions j_3 joints restricting 3 relative motions j_5 joints restricting 1 relative motions $DOF: 6(n-1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5$





Spherical 4R mechanism

Axes of R joints cocentric

Kutzbach (Mobility) Criterion Spatial Motion

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Compliant Mechanisms (ME 851)

What happens when Compliance is introduced?

Understanding Compliance

Flexural pivot

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Distributed beams

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Four bar linkages

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Identical topologies ?

Not quite — connectivity at central bar changes

Example of TYPE SYNTHESIS

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Example of TYPE SYNTHESIS

DE-Vol. 71, Machine Elements and Machine Dynamics

ON THE MOBILITY OF COMPLIANT MECHANISMS

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Prof. Ashok

Compliant Mechanisms (ME 851)

Topological synthesis for a compliant mechanism... 1994

For mechanisms containing flexible members, response to inputs, is comprised of rigid body and elastic members.

The paper presents a technique for determination of Mobility Characteristics of **Compliant Mechanisms**

Compliant Mechanisms (ME 851)

Type Synthesis

Olsen et al. define Type synthesis

as

process of determining possible mechanism structures to perform a given task or their combination without regard to *component dimensions*

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Compliant Mechanisms (ME 851)

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Many researchers have made significant contributions to the systematic study of mechanism design. The focus of several efforts has been in the area of type synthesis. Olson et al. (1985) defined type synthesis as "the process of determining possible mechanism structures to perform a given task or combination of tasks without regard to the dimensions of a component." The formulation of design procedures for rigidbody mechanisms has benefited from the application of typesynthesis techniques. Therefore, type synthesis is seen as a useful tool in the development of design procedures for compliant mechanisms as well. Murphy (1993) and Murphy et al. (1993) provided a mathematically rigorous method for the topological synthesis of compliant mechanisms that included compliance content information. One drawback of this technique is that a large number of design options can result from the application of this process. In order to arrive at the evaluation criteria to select a final design, it is helpful to investigate the mobility characteristics of the chains enumerated in the topological synthesis process.

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Segment compliance of CA: $s_c = 3$ $d\theta_A dx_A$ dy_A Joint variables, dx_A , dy_A , $d\theta_A$

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MATHEMATICAL COMPLIANT MODEL FOR MECHANISMS

The development of a systematic approach for the topological synthesis of compliant mechanisms requires the formulation of a mathematical model to represent the structure of a compliant mechanism (Murphy, 1993). In addition to providing information concerning the connectivity of the segments, this model provides information on the nature of possible segment deformations and the connection types between segments. In rigid-body kinematics, graph theory provides a mathematically rigorous representation for link connectivity through the vertex-vertex adjacency matrix, and other matrices that represent the topology of a mechanism. The nature of possible deformations of a compliant segment can be specified by determining a segment's compliance content, sc (Norton, 1991; Midha et al., 1994b). Figure 1 provides examples of segment compliance content for various segment types. Therefore, by combining the concept of segment compliance content with the matrix representation for the segment connectivity, a model to represent the structure of a compliant mechanism can be formulated.

Vertex-vertex adjacency matrix for an RB mechanism *nth order matrix* -a(i,j) = 1 if *i* and *j* vertices are adjacence a(i,j) = 0 otherwise.

Noting that a(i,i) = 0, always For compliant mechanism, this fact can be gainfully

Set (new matrix) $b(i, i) = s_c$ for the *i*th link

New matrix called the *Compliance element matrix*, *CE*

Non-diagonal elements:

Segments *i* and *j* not connected, b(i, j) = 0

Segments *i* and *j* connected via kinematic pair, b(i, j) = 2Flexural pivot between segments *i* and *j*, b(i, j) = 2Segments *i* clamped to segment *j*, b(i, j) = 3

If two segments connected at more than one location one of these can be split into two and joined at fixed

Compliant Mechanisms (ME 851)

n cent;		DE-Vol. 71, Machine Elements and Machine Dyna ASME The vertex-vertex adjacency matrix corresponding to a rigid- body mechanism is an nth order matrix where the element a(i,j) is equal to one if the ith vertex is adjacent to the jth vertex, and
utilized		 zero otherwise. As a result of this definition, the diagonal elements a(i,i) are always equal to zero. Therefore, for compliant mechanisms, the a(i,i) element is gainfully utilized to convey the segment compliance information. This is done by making the a(i,i) element equal to the value of s_c for the segment represented by the ith vertex. The non-diagonal elements of the segment-compliance vertex-vertex adjacency matrix are used to provide information regarding the types of connection between segments. The resulting matrix (Murphy, 1993) is called the compliant element matrix (CE). For this representation, it segment i is not connected to segment j then its element b(i,j)=0. If segment i is connected to segment j with a kinematic pair, then b(i,j)=1. If a flexural pivot connects segment i to segment j, then b(i,j)=3. For a rigid-body mechanism, the compliant element matrix (CE) elements will have the same values as those for the standard vertex-vertex adjacency matrix. One consequence of this notation is that two segments can only be joined at one location. If two segments are connected at more than one location, one of the original segments can be divided into two segments, joined at a fixed connection. This will allow the topology of the chain to be described, while maintaining the integrity of the mathematical formulation. Murphy et al. (1993) also modified
j) = 1		
n, l connection		

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Compliant Mechanisms (ME 851)

DOFs for a Compliant Mechanism

Compute degenerate (Rigid-Body) DoFs Assume all flexible links to be rigid

 n_1 : number of links, j_1 :number of 1 DoF joints j_2 :number of 2 DoF joints

If only kinematic pairs present $n_1 = n_{seg}$: number of segments

For each kinematic pair replaced by a flexural joint or fixed connection

Number of links in the mechanism is reduced by 1

*n*_{*flp*}: Number of flexural pivots connecting segments

Compliant Mechanisms (ME 851)

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DEGREES OF FREEDOM FOR A COMPLIANT MECHANISM

Her and Midha (1987) defined the degenerate degrees of freedom, or rigid-body degrees of freedom, Fr, as those calculated by assuming all flexible links to be rigid. Thus using Grübler's criterion for a planar mechanism,

$$F_r = 3(n_1 - 1) - 2n_{j1} - n_{j2}$$

where n1 is the number of links, nj1 the number of singledegree-of-freedom pairs (joints) and nj2 the number of twodegree-of-freedom pairs (joints). If a compliant mechanism contains only kinematic pairs as connections between segments, then the number of links n1 is equal to the number of segments, nseg. For each kinematic pair that is replaced by a flexural pivot or a fixed connection, the number of links in the mechanism is reduced by one (Murphy, 1993). Therefore, the number of links in a compliant mechanism can be determined from:

$$n_1 = n_{seg} - n_{fp} - n_{fix}$$

DOFs for a Compliant Mechanism

 $F_r = 3(n_{seg} - n_{flp} - n_{fix} - 1) - 2j_1 - j_2$ Upon substitution

 F_T can be computed from the Compliance Element Matrix, CE

 F_T is usually less than ONE for a compliant mechanism, else, mechanism will be operations without deformation of its members.

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One benefit of this formulation is that F_r can be calculated directly from information provided in the compliant element matrix (Murphy, 1993). Moreover, the rigid-body degrees of freedom (Fr) are now related directly to a segment formulation for compliant mechanisms. Howell (1991) noted that the rigid-body degrees of freedom are typically less than unity for a compliant mechanism, otherwise the mechanism could have motion without using the deflections of its members.

FC: Fixed Connection **KP: Kinematic Pair** FP: Flexural Pivot 0 2 0 3 0

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Examples

$$n_{seg} = 4; n_{fix} = 1; n_{j_1} = 2; n_{nfp} = 1; n_{j_2} = 0$$

$$F_r = 3(n_{seg} - n_{flp} - n_{fix} - 1) - 2j_1 - j_2$$

$$F_r = 3(4 - 1 - 1 - 1) - 2(2) - 0 = -1$$

$$n_{seg} = 4; n_{fix} = 3; n_{j_1} = 1; n_{nfp} = 0; n_{j_2} = 0$$

$$F_r = 3(n_{seg} - n_{flp} - n_{fix} - 1) - 2j_1 - j_1$$

$$F_r = 3(4 - 3 - 1) - 2(1) - 0 = -2$$

COMPLIANCE NUMBER — C

C represents the DoFs gained by adding compliance to the mechanism

If only kinematic pairs present as connections between segments, $C = \sum$ Segment (link) Compliances

Introduction of Flexural Pivots also *increases* Compliance of a mechanism

Fixed connections *offer no change* to the value of the Compliance number

 $C = n_{flp} + n_{sc_1} + 2n_{sc_2} + 3n_{sc_3}$

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*n*_{*flp*}: Number of flexural pivots

Compliant Mechanisms (ME 851)

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The compliance number, C, represents the degrees of freedom gained by adding compliance to a mechanism (Her, 1986; Her and Midha, 1987). For a compliant mechanism that contains only kinematic pairs as connections between segments, the compliance number is equal to the summation of the segment (link) compliances (Howell, 1991). In addition to segment compliance, introduction of flexural pivots also increases the compliance of a mechanism. When a kinematic pair is replaced by a flexural pivot, the compliance number of the mechanism is increased by one. Fixed connections neither add to nor subtract from the value of the compliance number. Therefore, for mechanisms containing only binary compliant segments, the compliance number can be calculated as

$$C = n_{fp} + n_{sc1} + 2n_{sc2} + 3n_{sc3}$$
 (

where nfp is the number of flexural pivots, and nsc1. nsc2 and nsc3 are the number of segments with a segment compliance (sc) of one, two and three, respectively. For a mechanism containing k-nary segments (k>2), the segment compliance may be greater than three. In general, the compliance number can be determined by

$$C = n_{fp} + \sum_{i=1}^{q} i\{n_{sci}\}$$

COMPLIANCE NUMBER — C

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Examples

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$$F_r = 3(n_{seg} - n_{flp} - n_{fix} - 1) - 2j_1 - 2j$$

$$n_{seg} = 4; n_{fix} = 3; n_{j_1} = 1; n_{nfp} = 0; n_{j_2} = 0$$

$$F_r = 3(n_{seg} - n_{flp} - n_{fix} - 1) - 2j_1 - j_2$$

$$F_r = 3(4 - 3 - 1) - 2(1) - 0 = -2$$

$$C = n_{flp} + \text{trace}(CE) = 0 + 6 = 6$$

$$F_c$$

