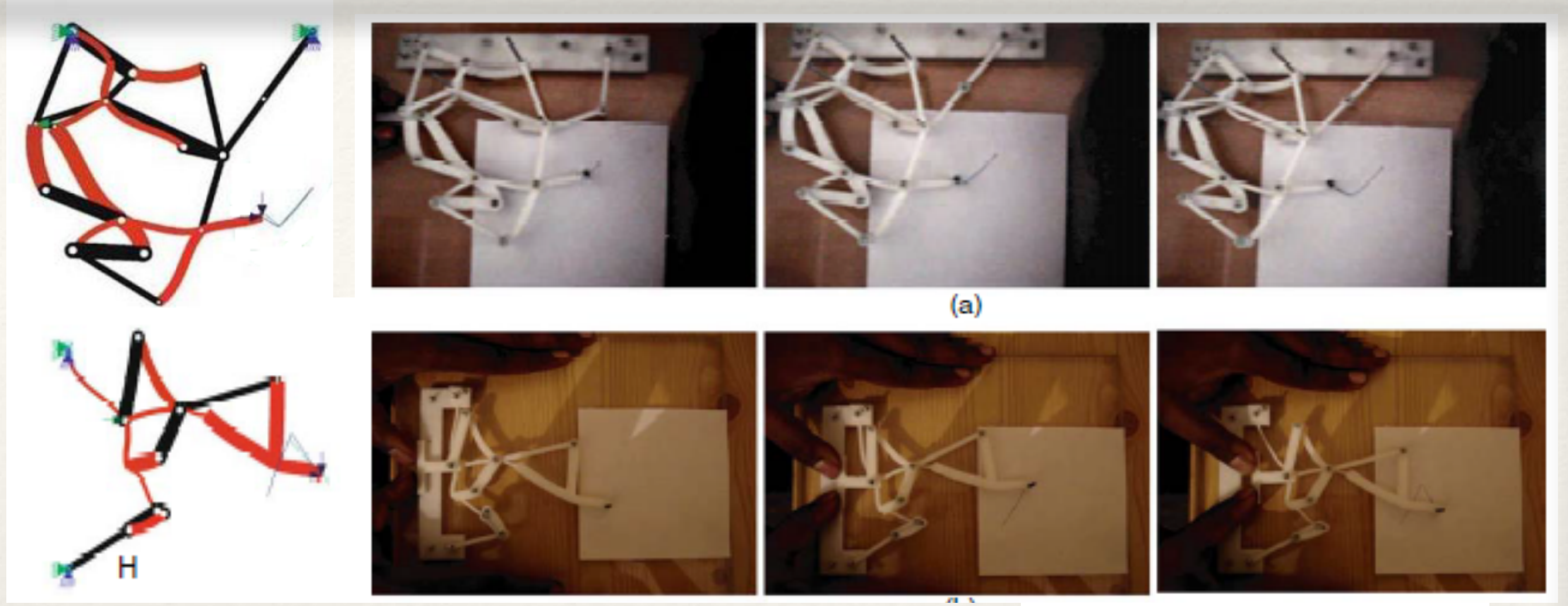

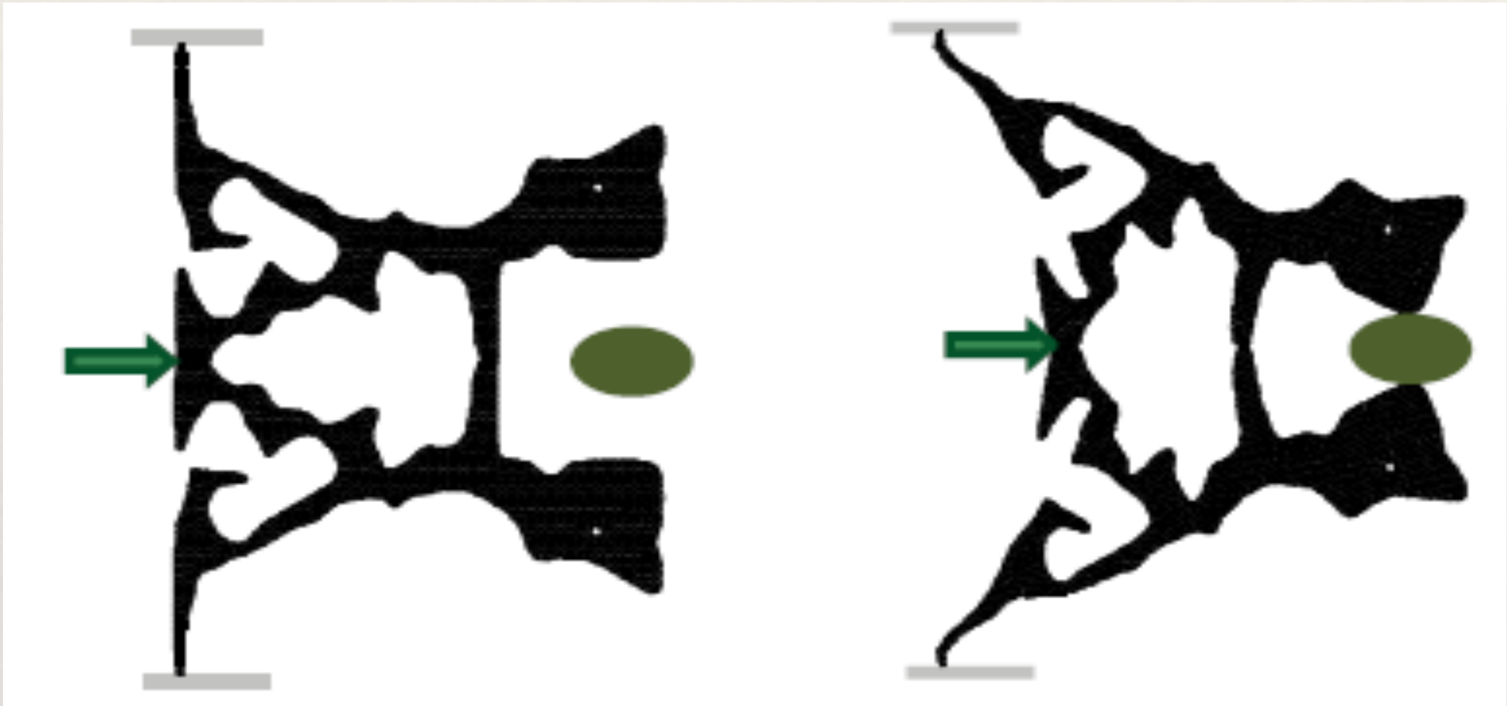

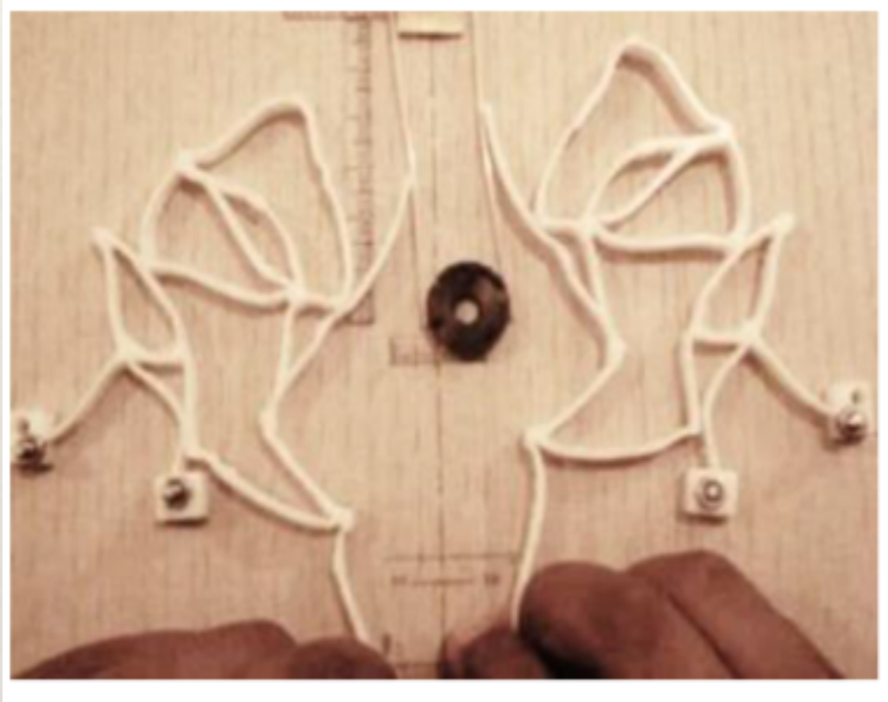

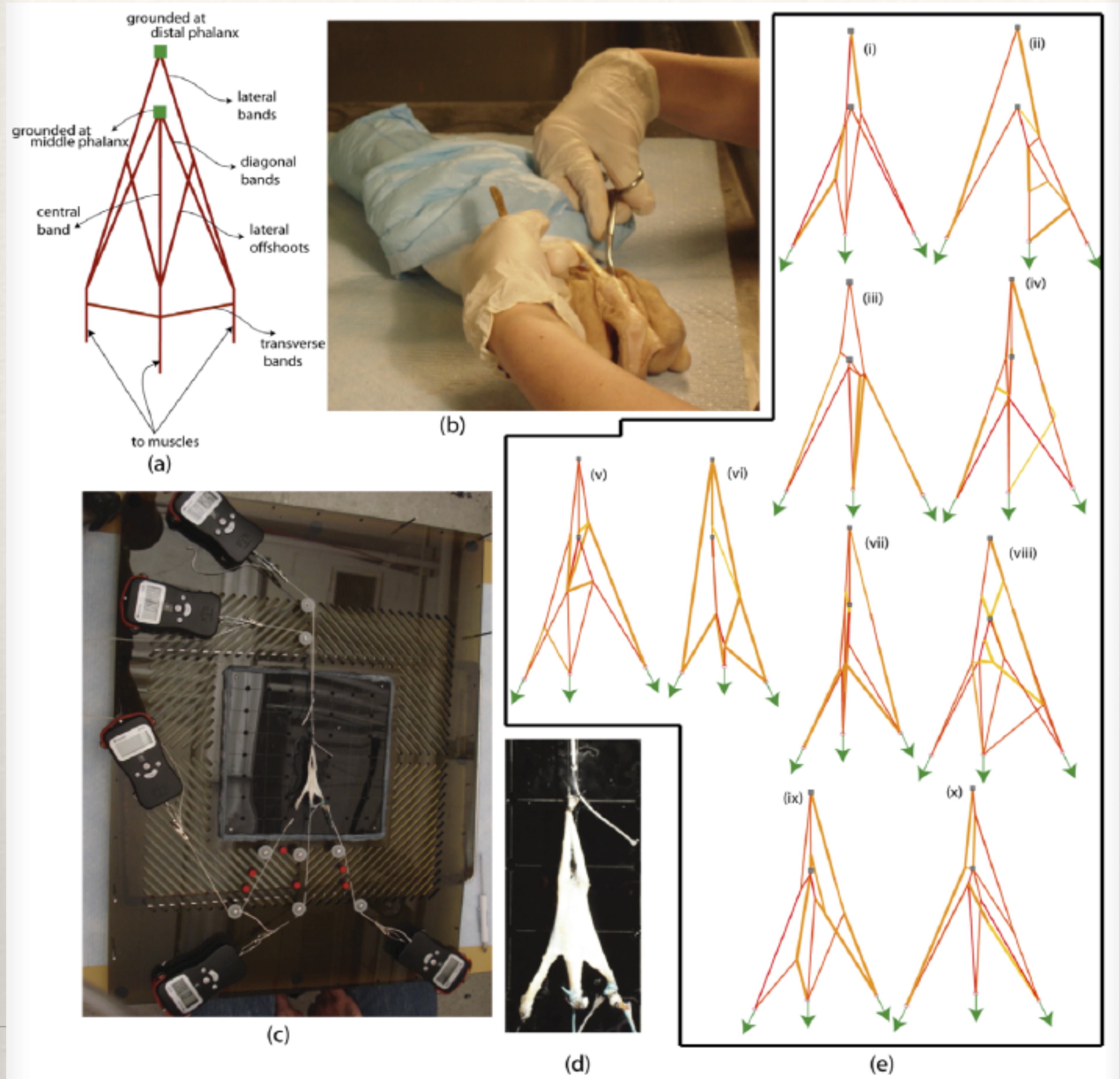

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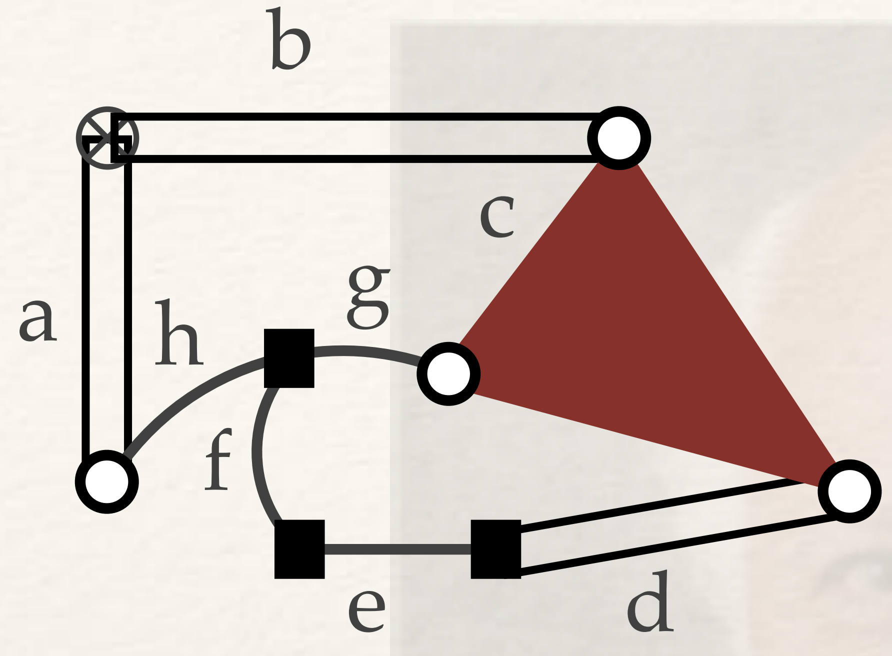

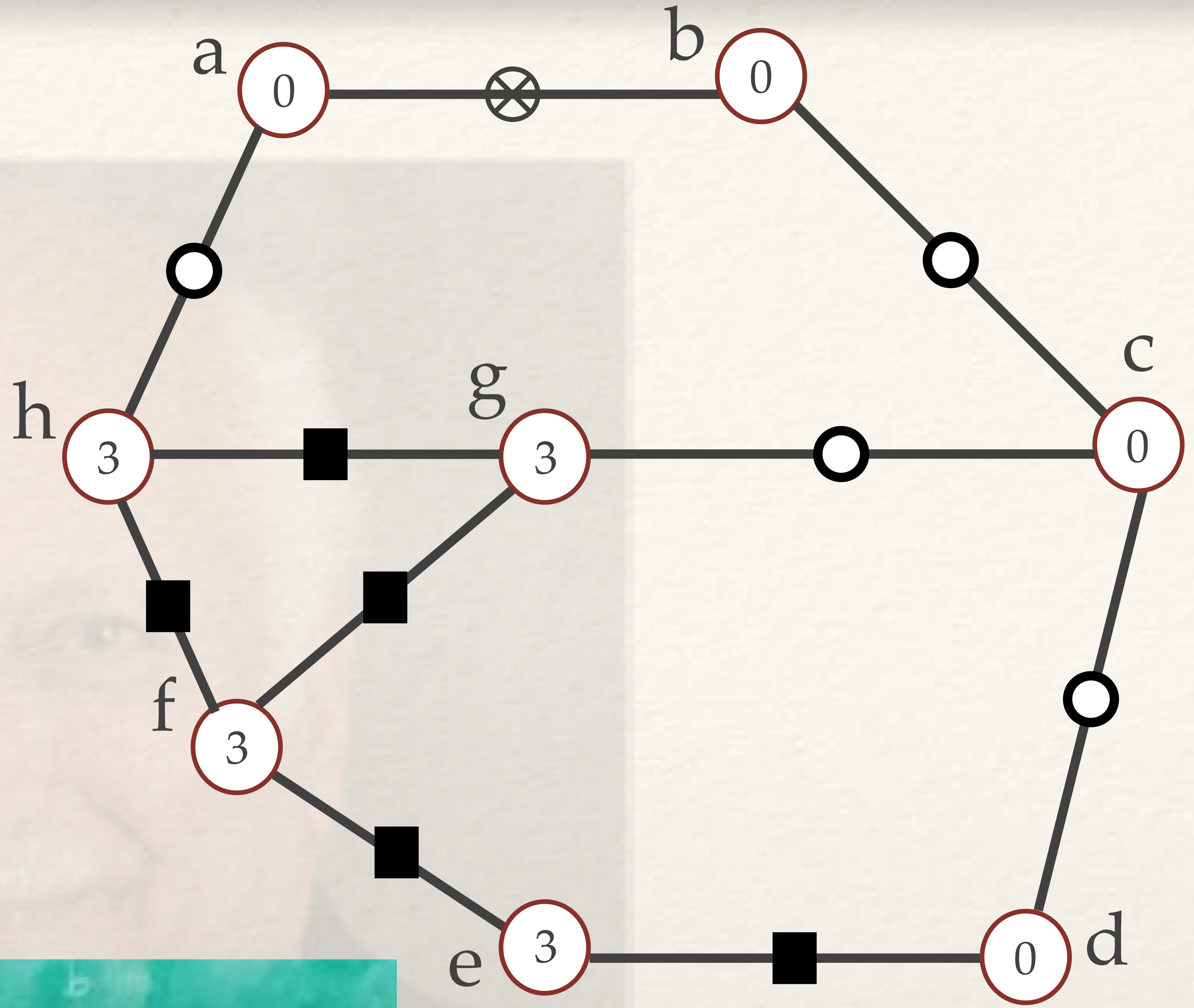

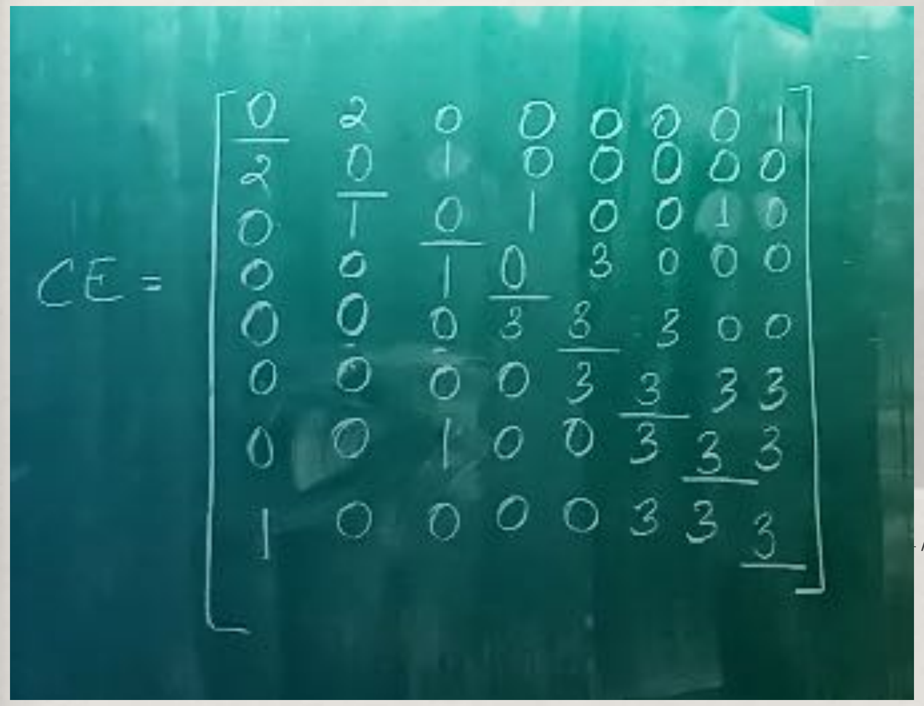

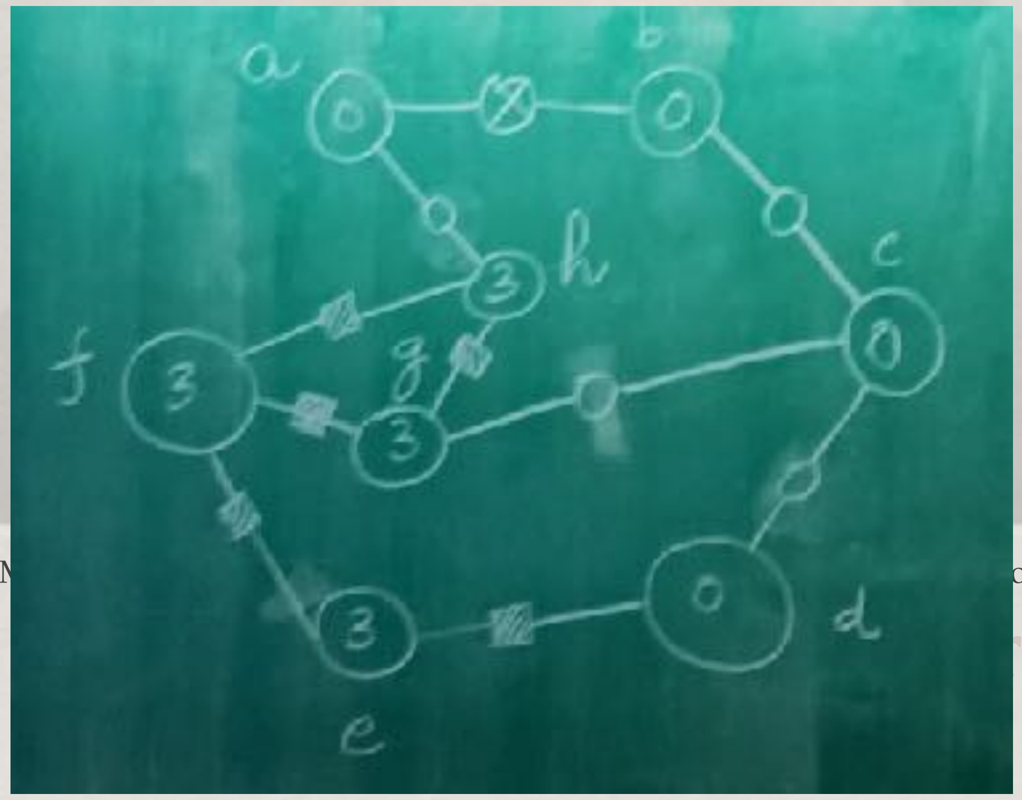

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DE-Vol. 71, Machine Elements and Machine Dynamics
ASME 1994

Examples

$$CE = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 3 & 3 & 3 \\ 0 & 0 & 1 & 0 & 0 & 3 & 3 & 3 \\ 1 & 0 & 0 & 0 & 0 & 3 & 3 & 3 \end{bmatrix}$$

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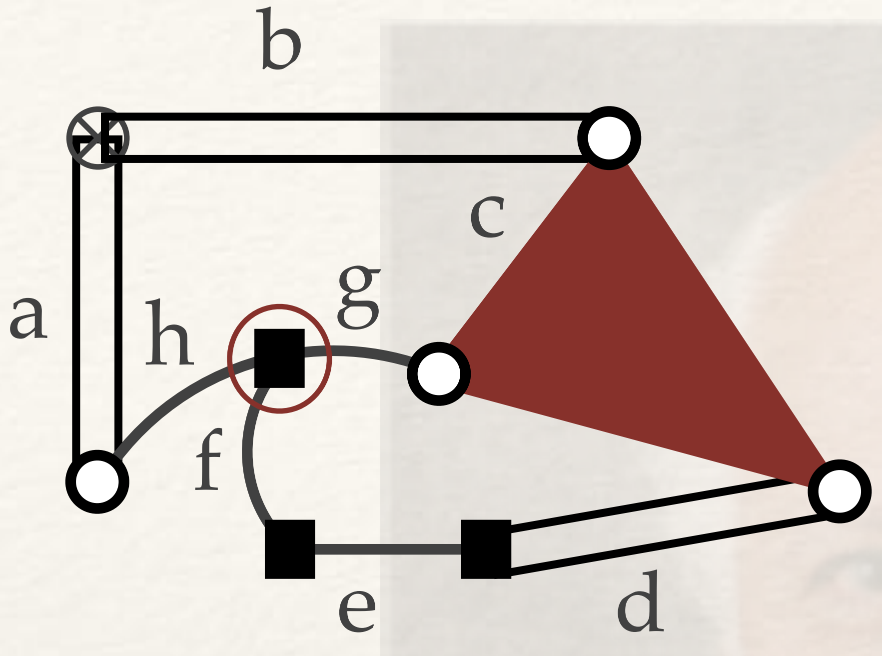

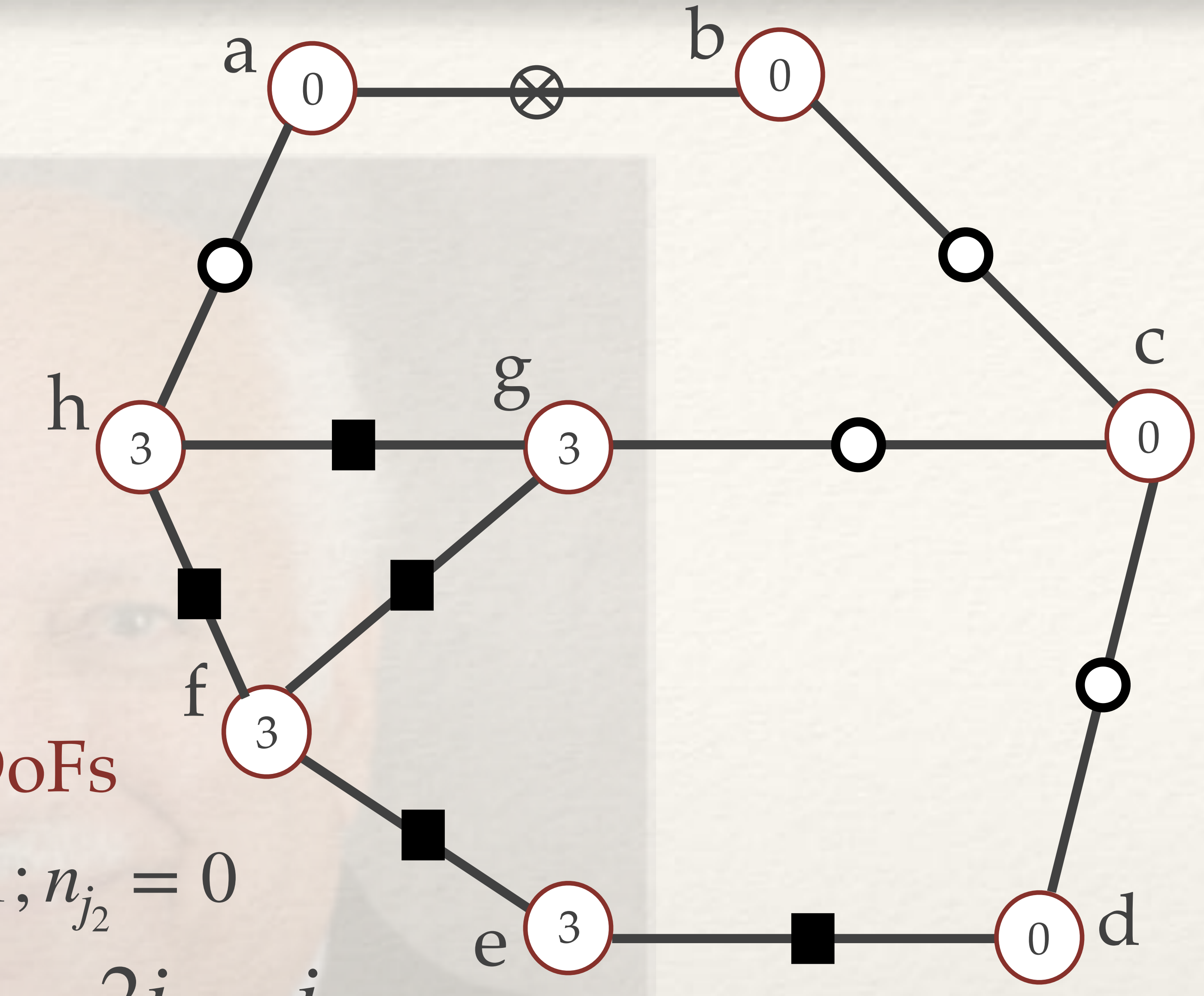

First way of computing degenerate DoFs

$$n_{seg} = 8; n_{fix} = 4; n_{j_1} = 4; n_{nfp} = 1; n_{j_2} = 0$$

$$F_r = 3(n_{seg} - n_{flp} - n_{fix} - 1) - 2j_1 - j_2$$

$$F_r = 3(8 - 1 - 4 - 1) - 2(4) - 0 = -2$$

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Examples

$$CE = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 3 & 3 & 3 \\ 0 & 0 & 1 & 0 & 0 & 3 & 3 & 3 \\ 1 & 0 & 0 & 0 & 0 & 3 & 3 & 3 \end{bmatrix}$$

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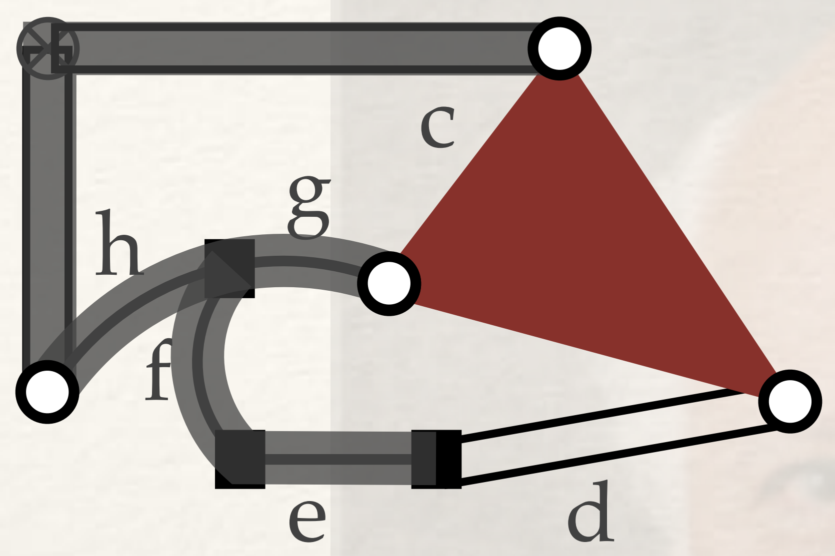

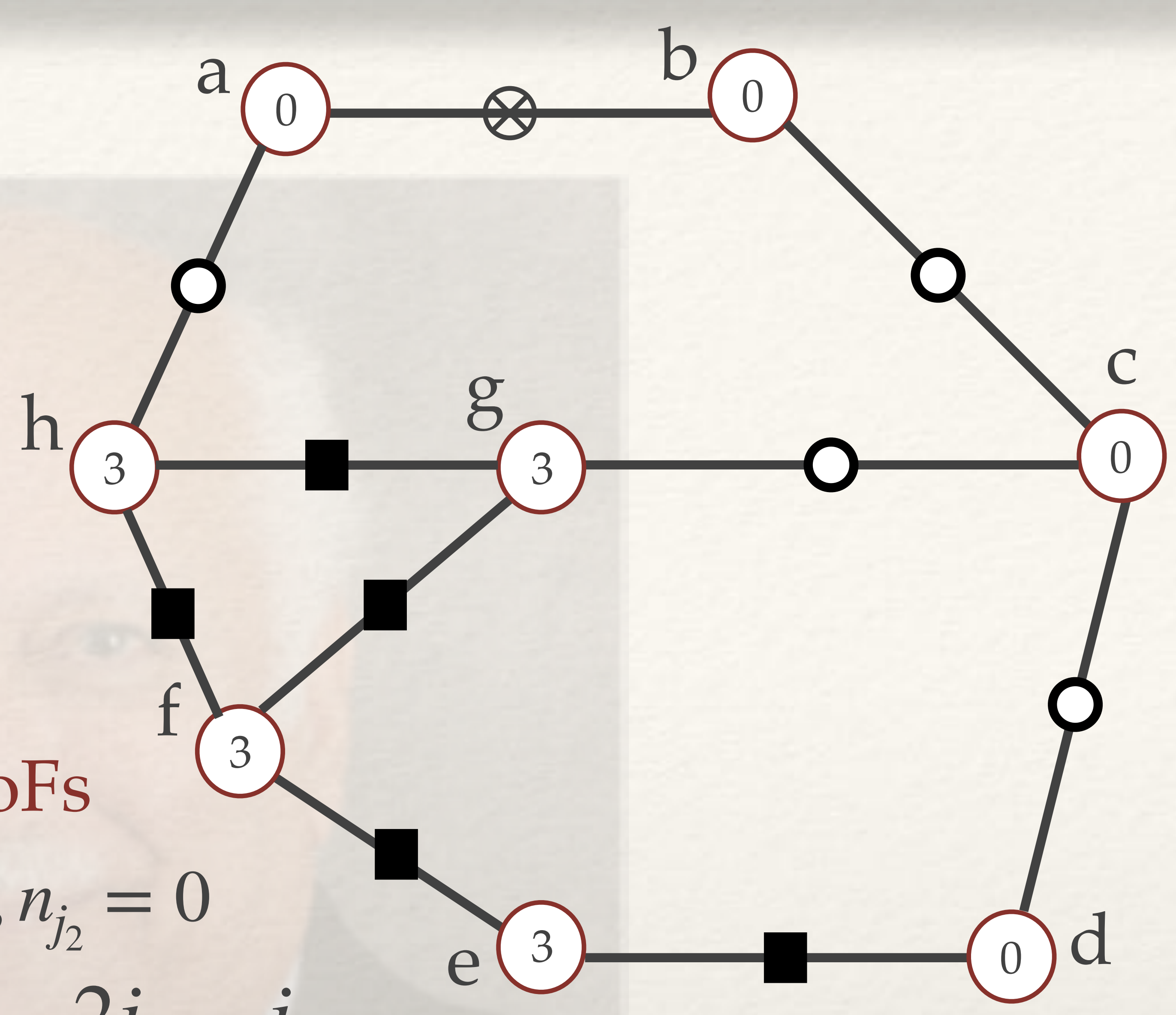

First way of computing degenerate DoFs

$$n_{seg} = 8; n_{fix} = 3; n_{j_1} = 4; n_{nfp} = 1; n_{j_2} = 0$$

$$F_r = 3(n_{seg} - n_{flp} - n_{fix} - 1) - 2j_1 - j_2$$

$$F_r = 3(8 - 1 - 4 - 1) - 2(4) - 0 = -2$$

Second way of computing degenerate DoFs

$$F_r = 3(n_l - 1) - 2j_1 - j_2 = 3(3-1) - 2(4) = -2$$

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Compliance number

$$C = n_{flp} + \text{trace}(CE) = 1 + 12 = 13$$

$$\text{DoFs} = -2 + 13 = 11$$

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Examples

$$CE = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 3 & 3 & 3 \\ 0 & 0 & 1 & 0 & 0 & 3 & 3 & 3 \\ 1 & 0 & 0 & 0 & 0 & 3 & 3 & 3 \end{bmatrix}$$

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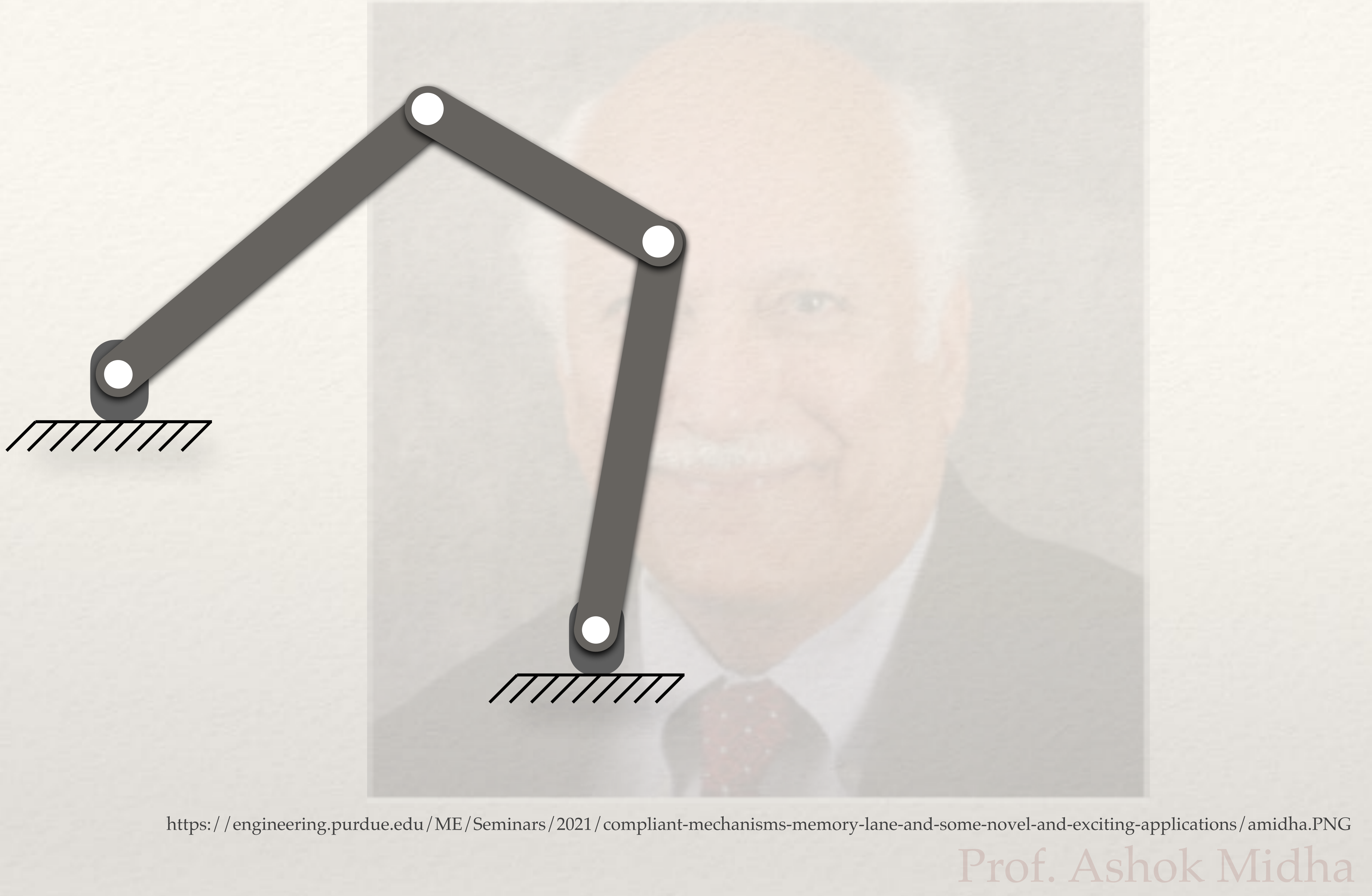
More on Mobility



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Grübler's Criterion

$$F_r = 3(4-1) - 2(2) = 1$$

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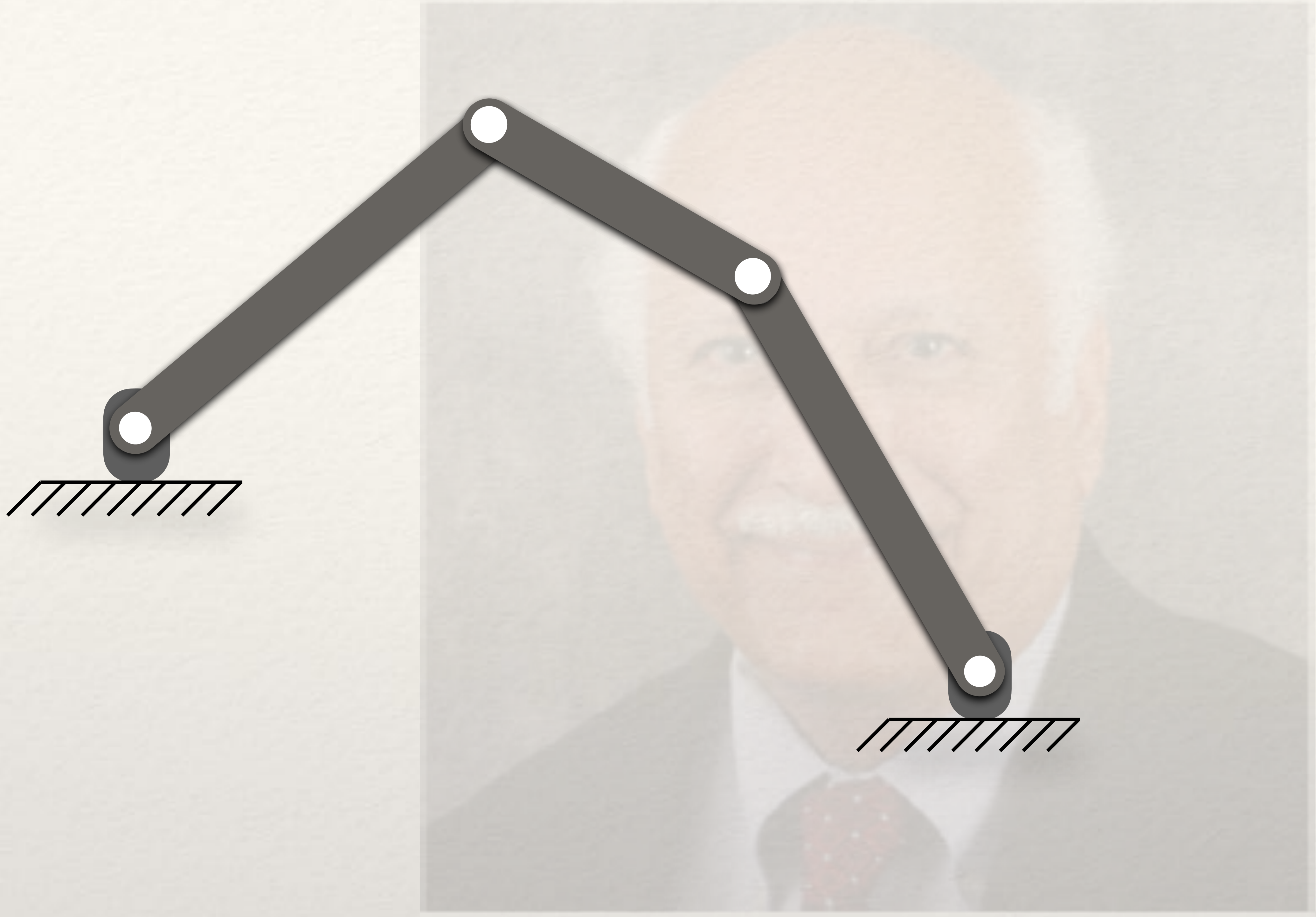
More on Mobility



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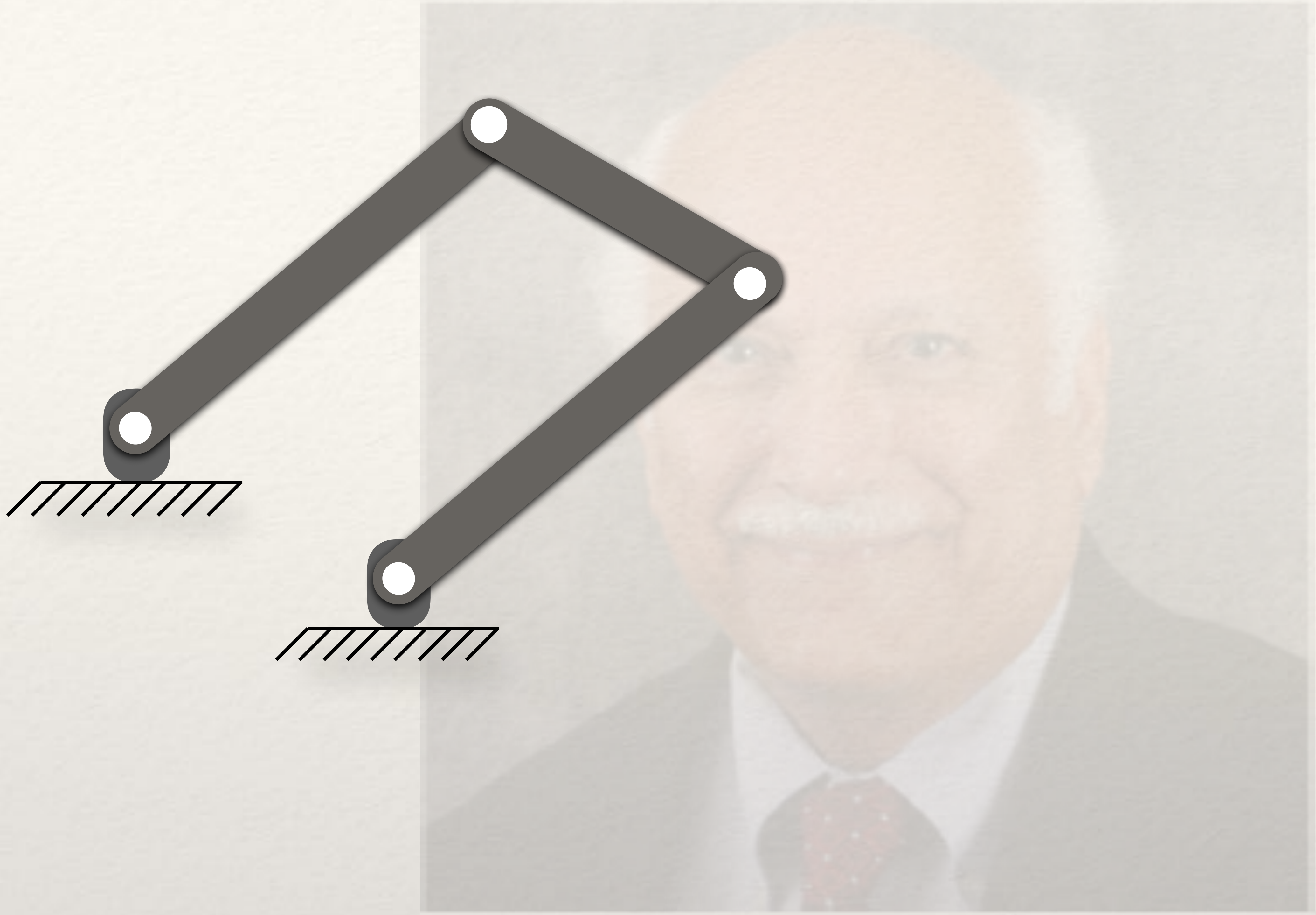

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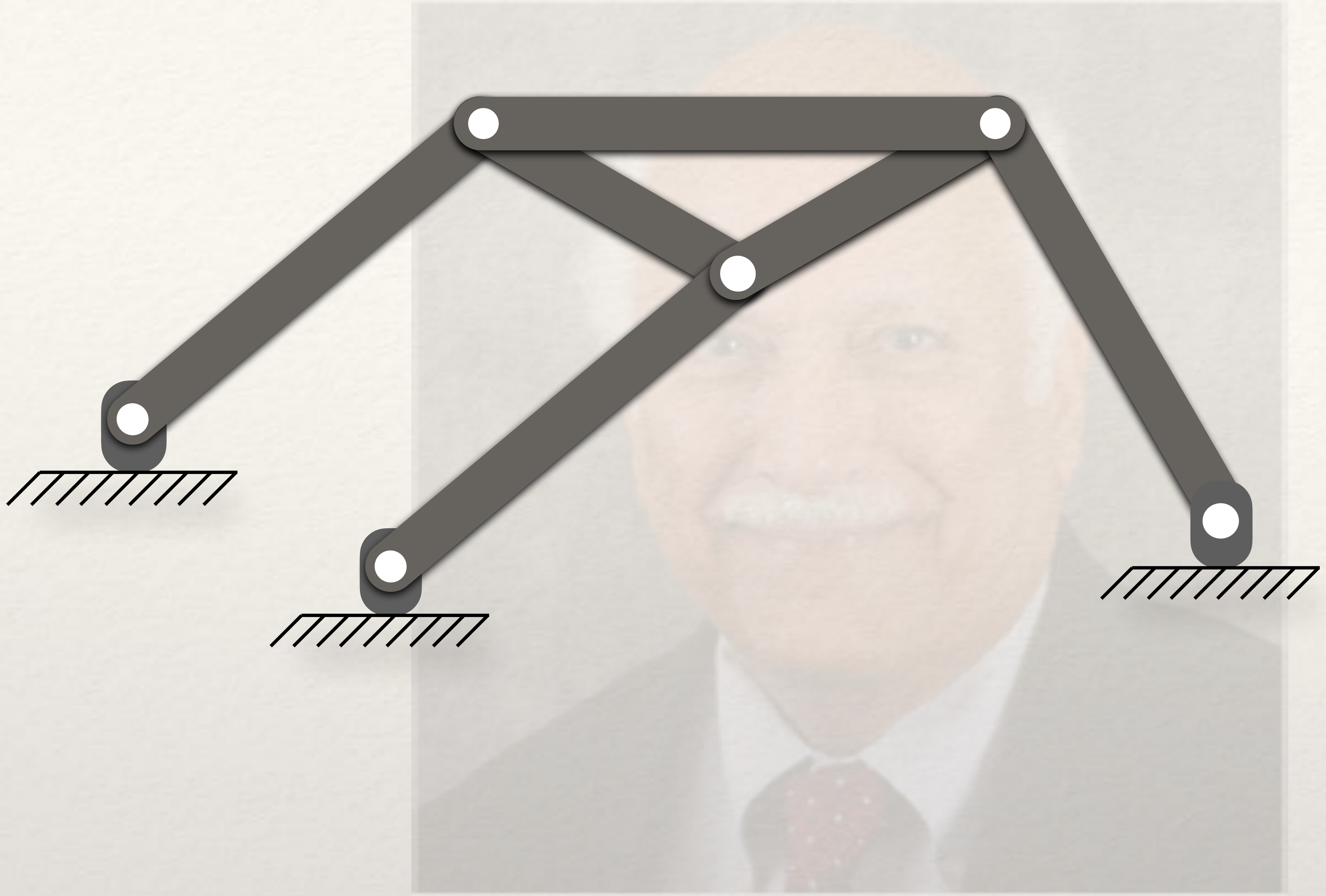

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Grübler's Criterion

$$F_r = 3(6-1) - 2(7) = 1$$

$$F_r = 3(7-1) - 2(9) = 0$$

Does one have confidence ?

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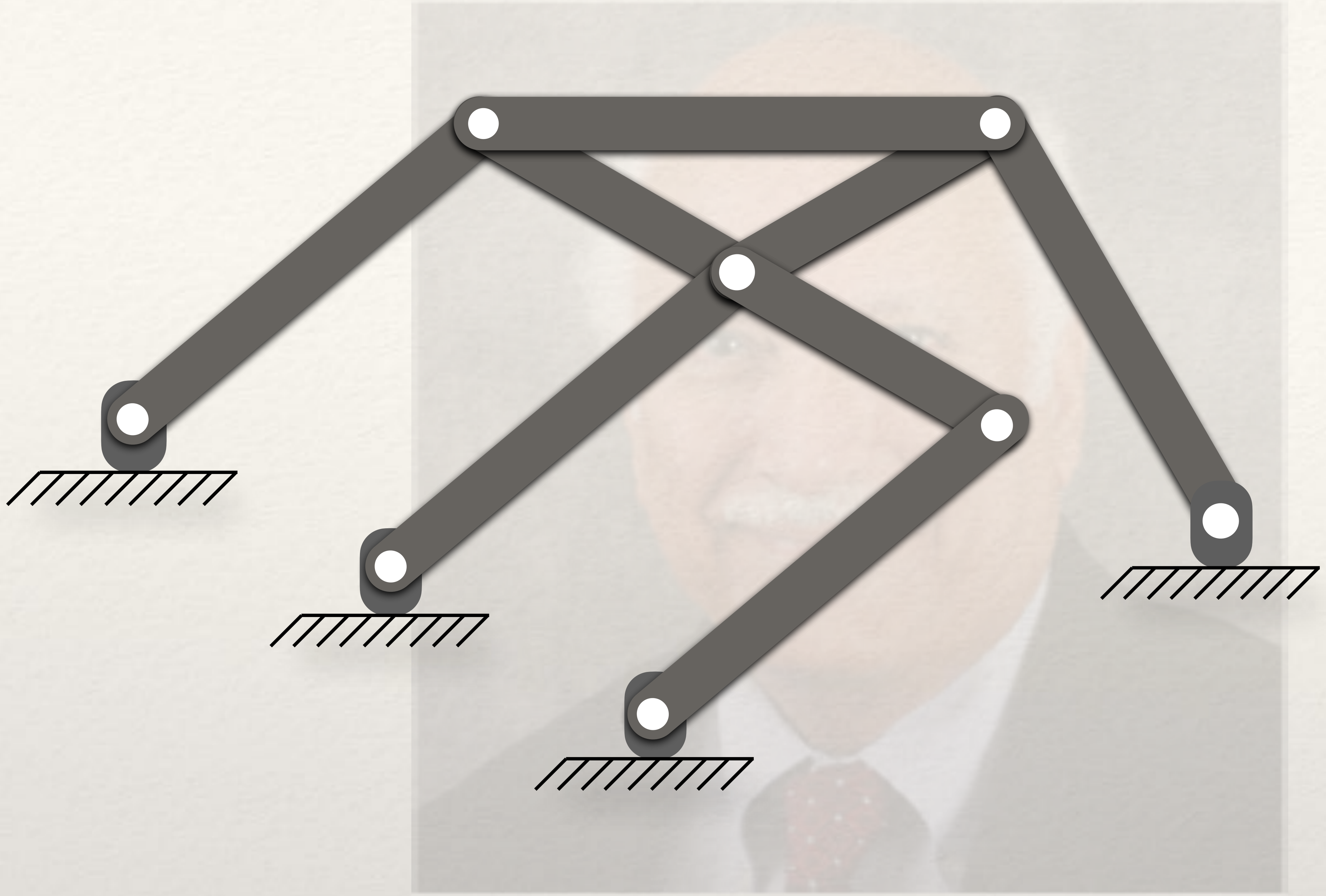

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Grübler's Criterion

$$F_r = 3(6-1) - 2(7) = 1$$

$$F_r = 3(7-1) - 2(9) = 0$$

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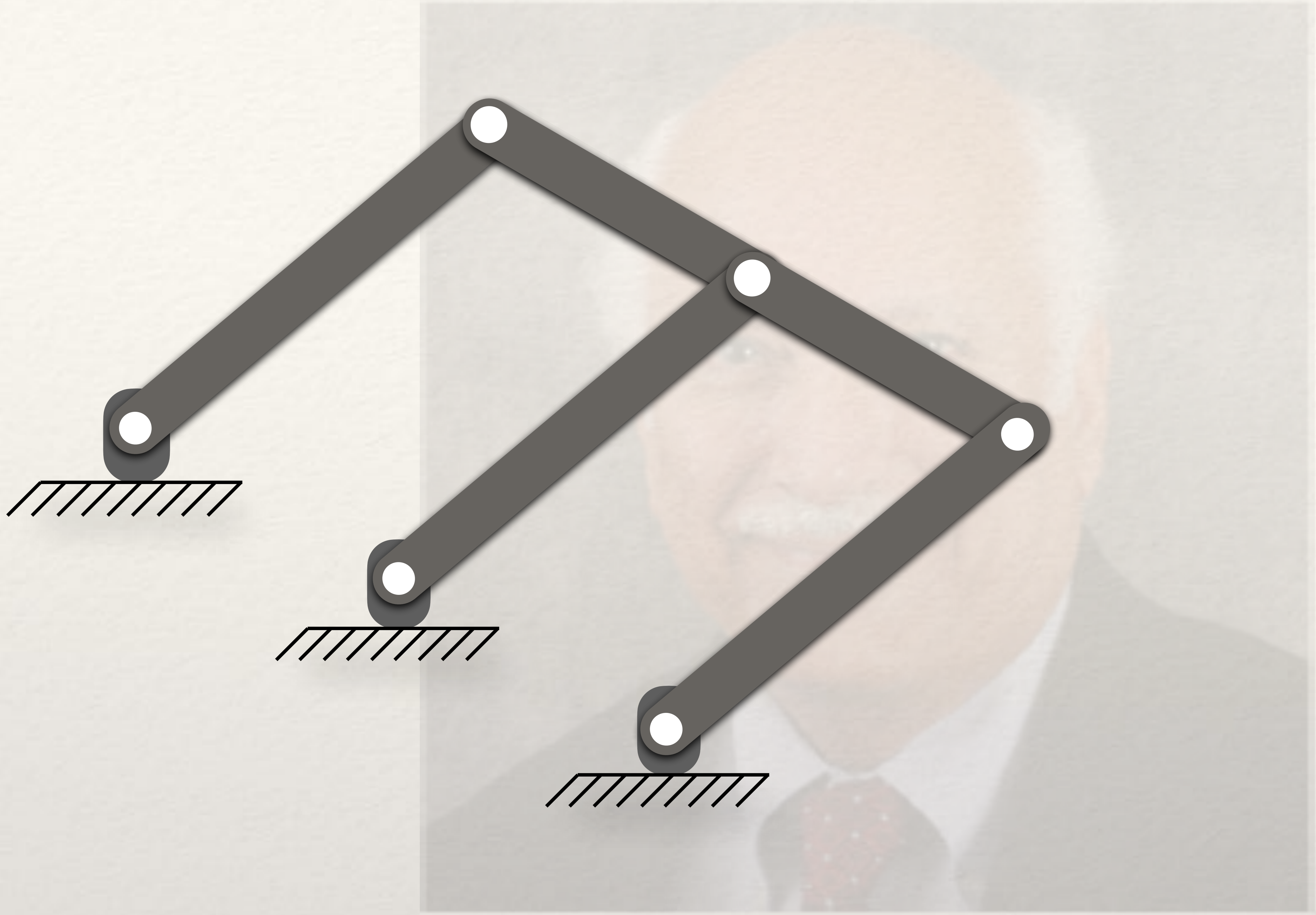

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Grübler's Criterion

$$F_r = 3(6 - 1) - 2(7) = 1$$

Does one have confidence ?

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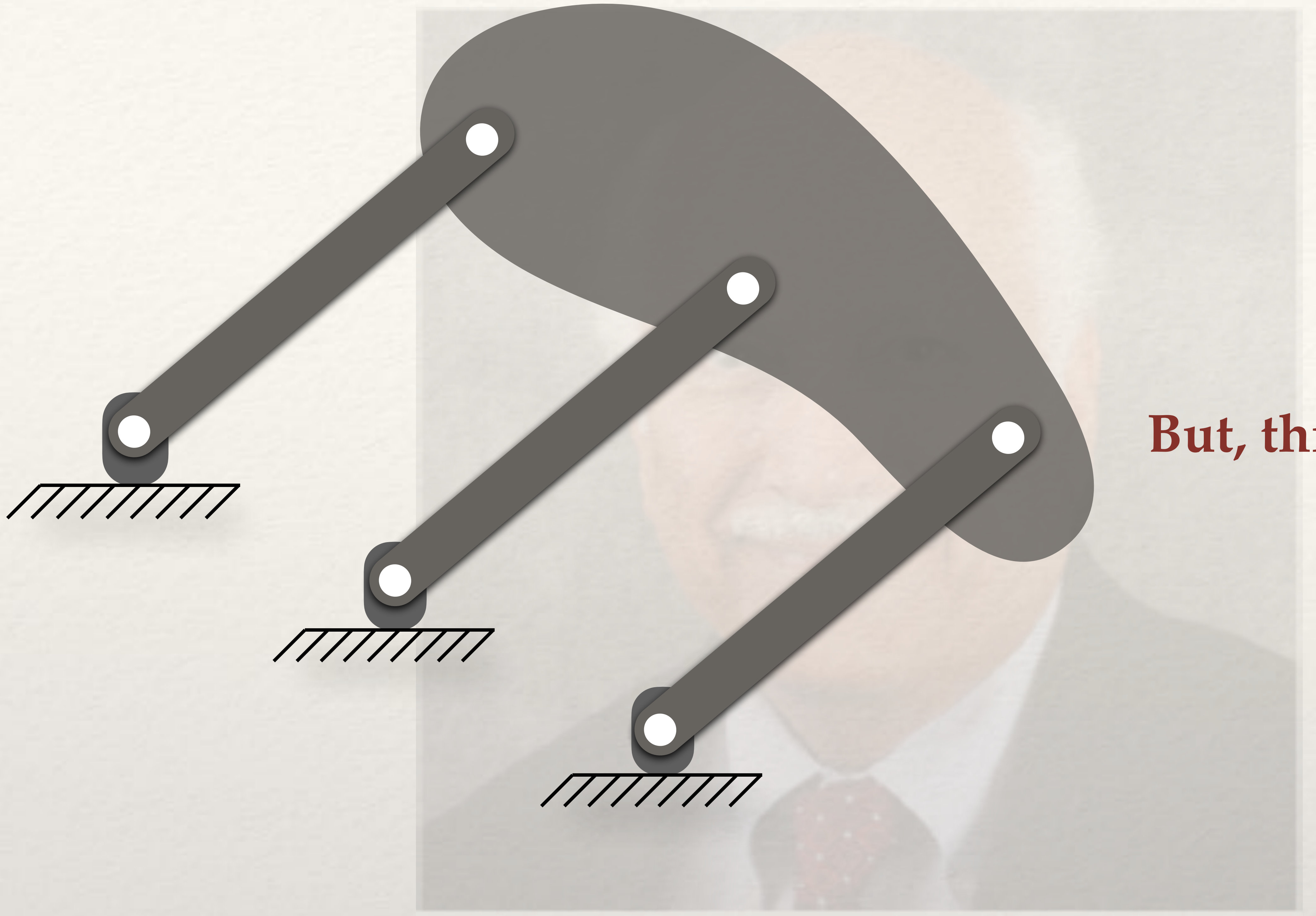

But, this linkage moves…

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Grübler's Criterion

$$F_r = 3(6-1) - 2(7) = 1$$

Does one have confidence ?

$$F_r = 3(5-1) - 2(6) = 0$$

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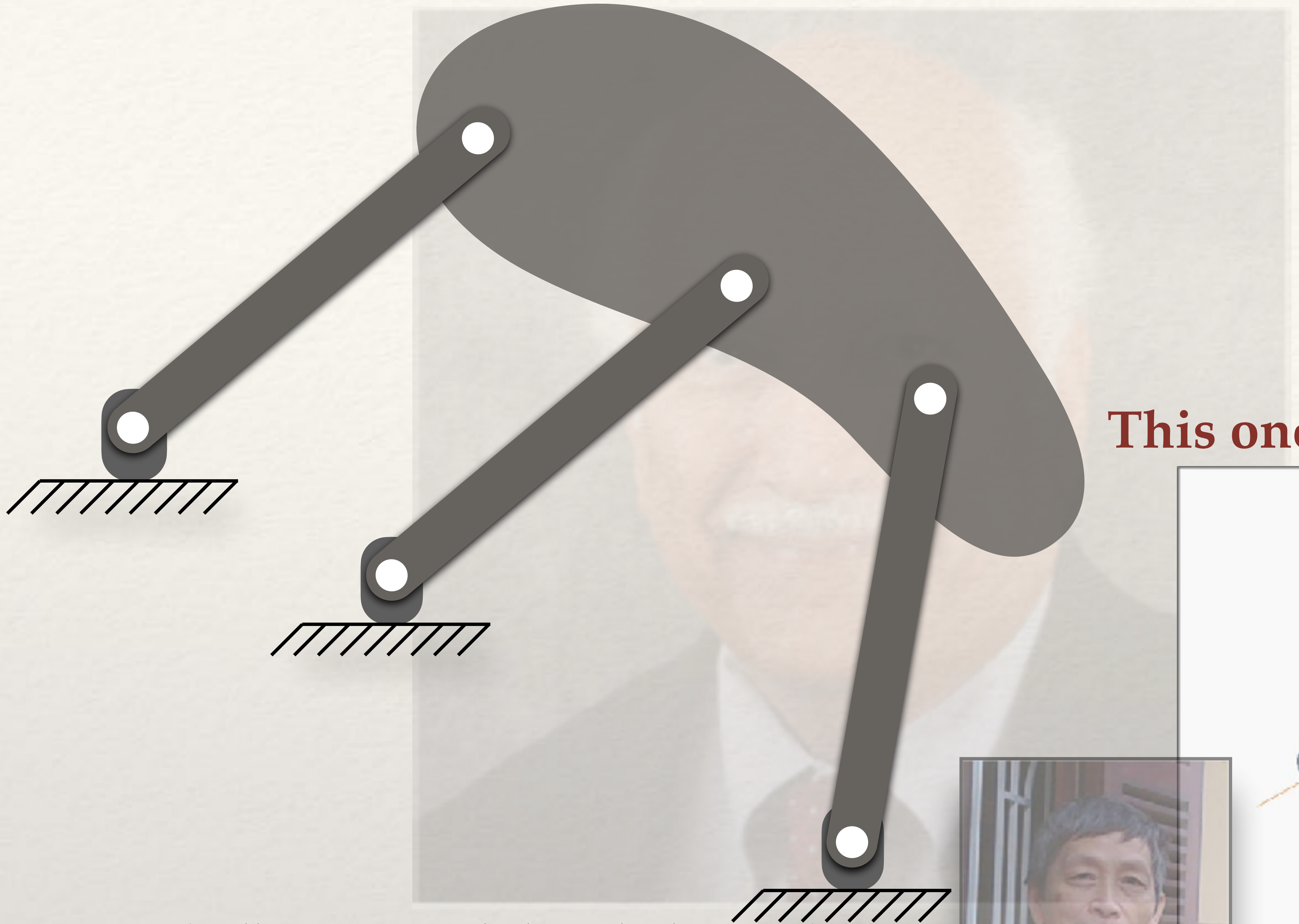
More on Mobility



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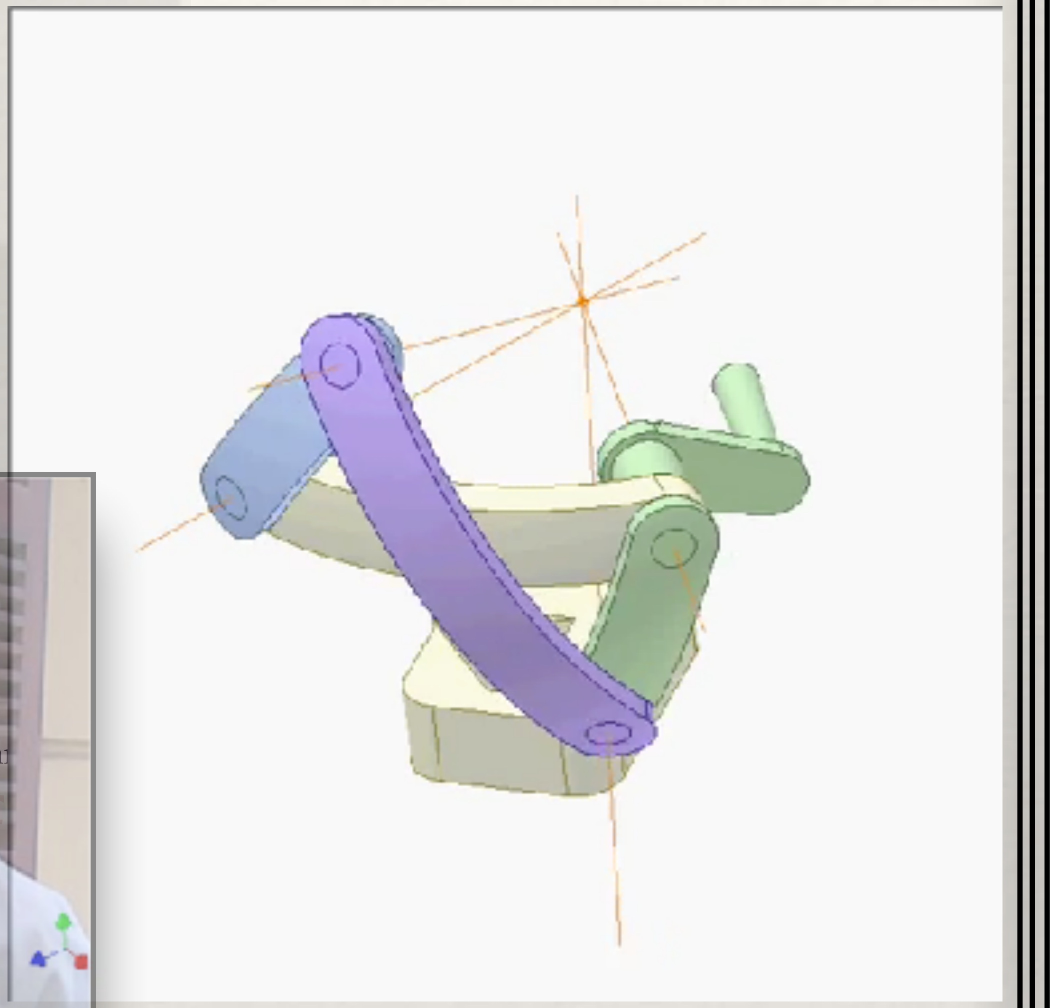
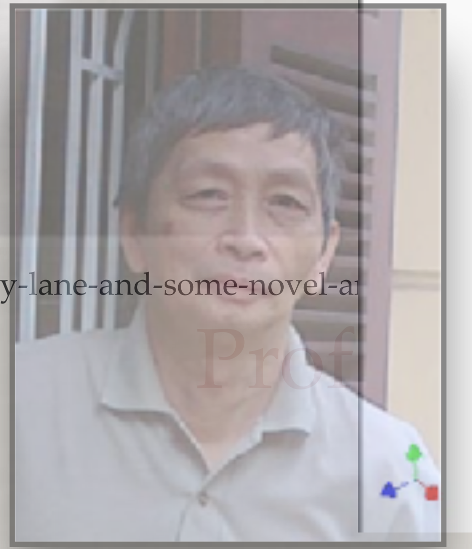
This one does not





Grübler's Criterion

$$F_r = 3(6-1) - 2(7) = 1$$

Does one have confidence ?

$$F_r = 3(5-1) - 2(6) = 0$$

Kutzbach's Criterion

4L, $4j_1$: $DOF = 6(3) - 5(4) = -2$

Only Physics can explain !

Merely counting to gauge mobility seems insufficient

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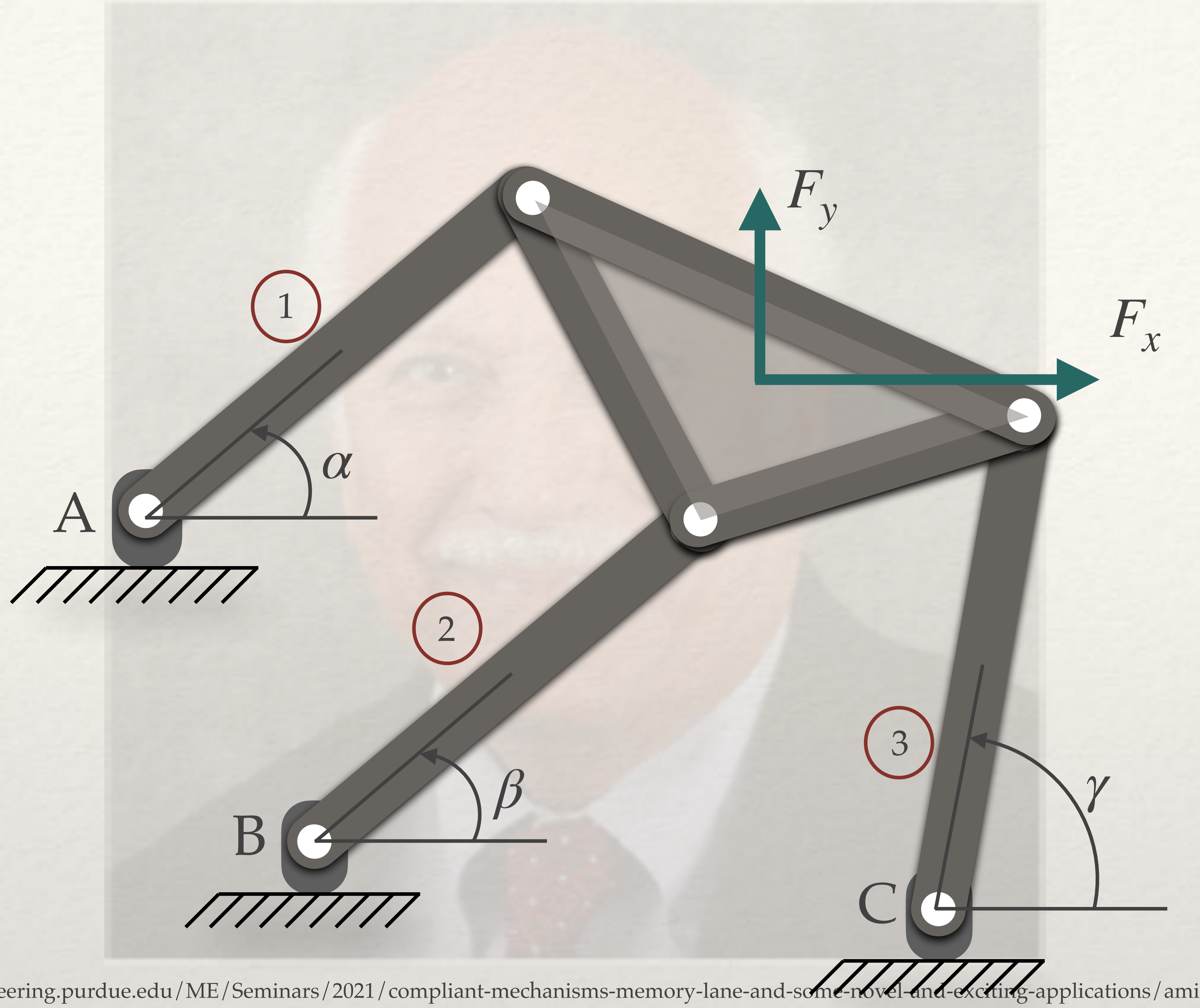
Force Equilibrium Equations



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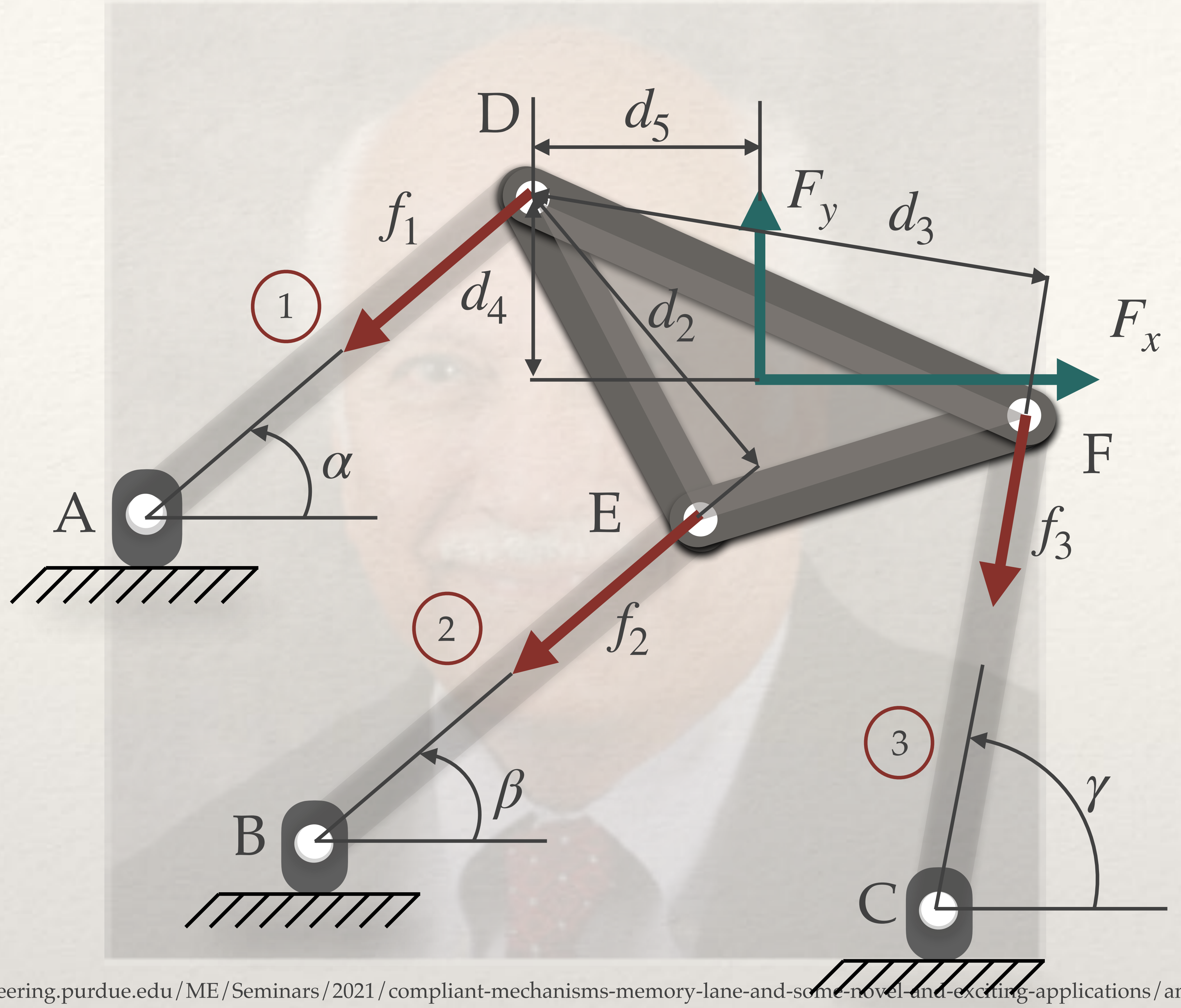
Force Equilibrium Equations



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Force Balance Horizontal

$$f_1 \cos\alpha + f_2 \cos\beta + f_3 \cos\gamma = F_x$$

Force Balance Vertical

$$f_1 \sin\alpha + f_2 \sin\beta + f_3 \sin\gamma = F_y$$

Moment Balance about Z, at D

$$f_2 d_2 + f_3 d_3 = F_x d_4 + F_y d_5$$

$$\begin{bmatrix} \cos\alpha & \cos\beta & \cos\gamma \\ \sin\alpha & \sin\beta & \sin\gamma \\ 0 & d_2 & d_3 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_x d_4 + F_y d_5 \end{bmatrix}$$

$$\mathbf{P}\,\mathbf{f}_{truss} = \mathbf{F}_{ext}$$

If $\alpha = \beta = \gamma$

P becomes singular, rank deficient

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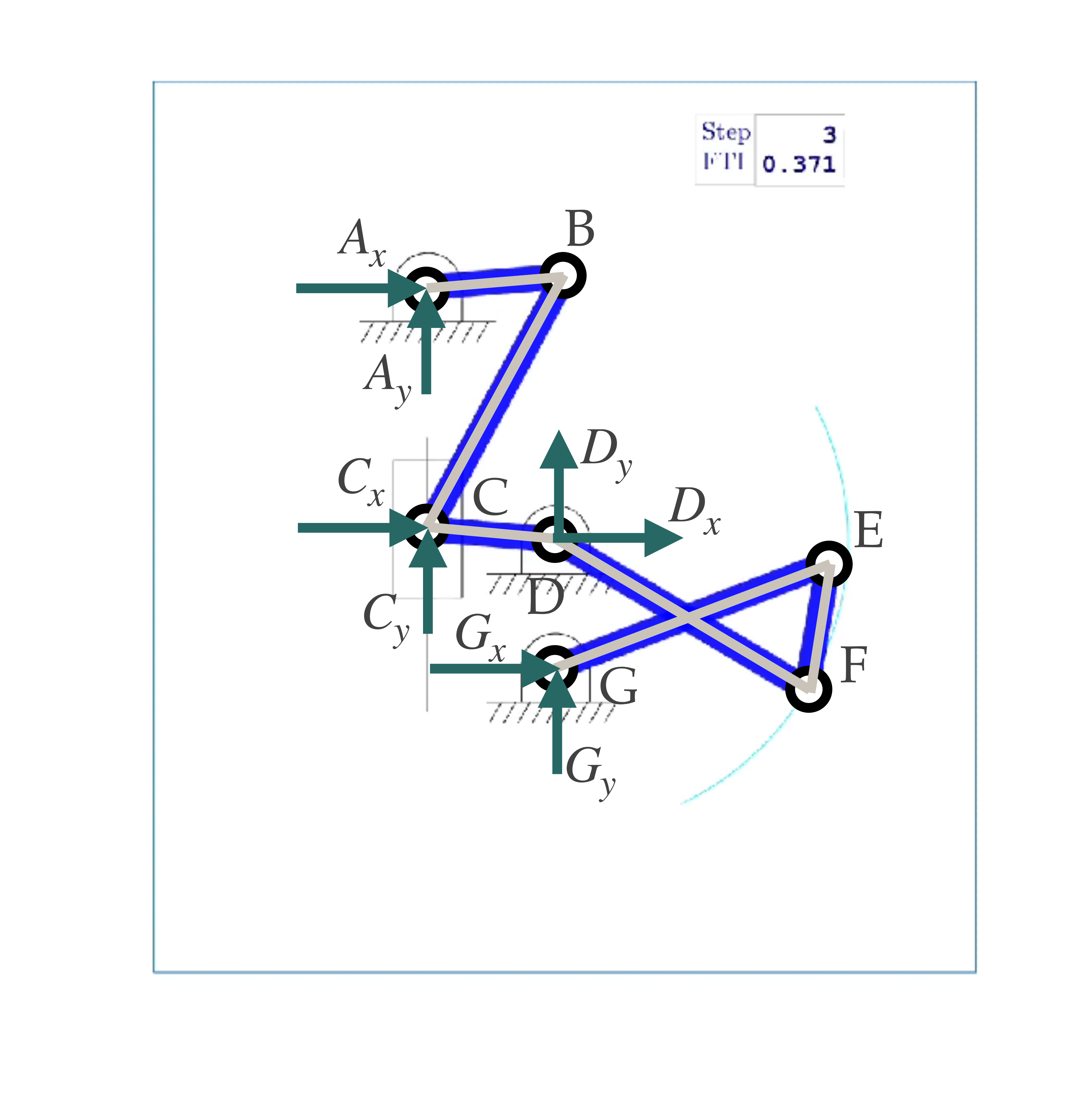
Force Equilibrium Equations



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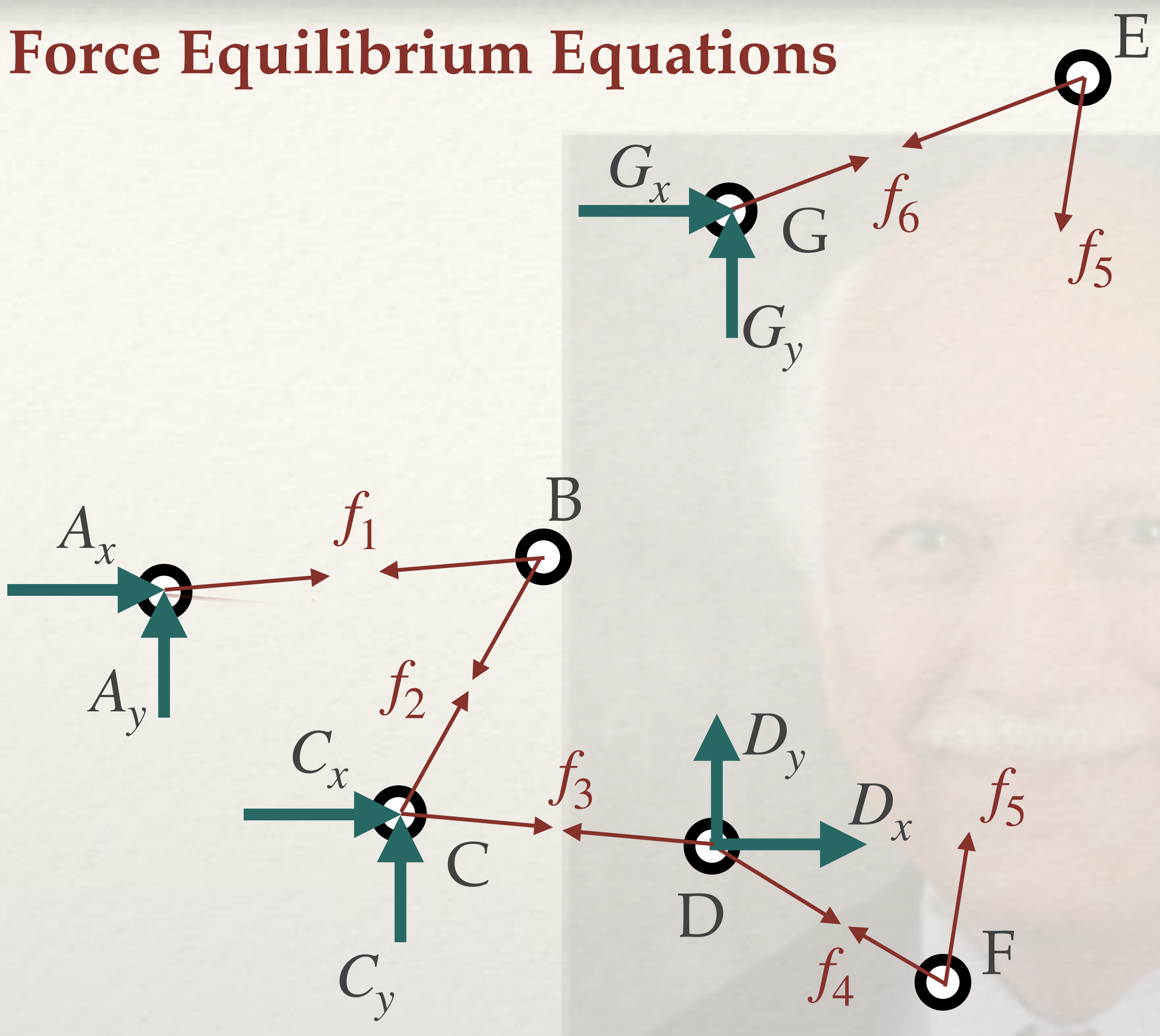
Force Equilibrium Equations



$$
\begin{aligned}
p_{11}f_1 + 0f_2 + 0f_3 + 0f_4 + 0f_5 + 0f_6 &= A_x \\
p_{21}f_1 + 0f_2 + 0f_3 + 0f_4 + 0f_5 + 0f_6 &= A_y \\
p_{31}f_1 + p_{32}f_2 + 0f_3 + 0f_4 + 0f_5 + 0f_6 &= 0 \\
p_{41}f_1 + p_{42}f_2 + 0f_3 + 0f_4 + 0f_5 + 0f_6 &= 0 \\
0f_1 + p_{52}f_2 + p_{53}f_3 + 0f_4 + 0f_5 + 0f_6 &= C_x \\
0f_1 + p_{62}f_2 + p_{63}f_3 + 0f_4 + 0f_5 + 0f_6 &= C_y \\
0f_1 + 0f_2 + p_{73}f_3 + p_{74}f_4 + 0f_5 + 0f_6 &= D_x \\
0f_1 + 0f_2 + p_{83}f_3 + p_{84}f_4 + 0f_5 + 0f_6 &= D_y \\
0f_1 + 0f_2 + 0f_3 + 0f_4 + p_{95}f_5 + p_{96}f_6 &= 0 \\
0f_1 + 0f_2 + 0f_3 + 0f_4 + p_{10,5}f_5 + p_{10,6}f_6 &= 0
\end{aligned}
$$

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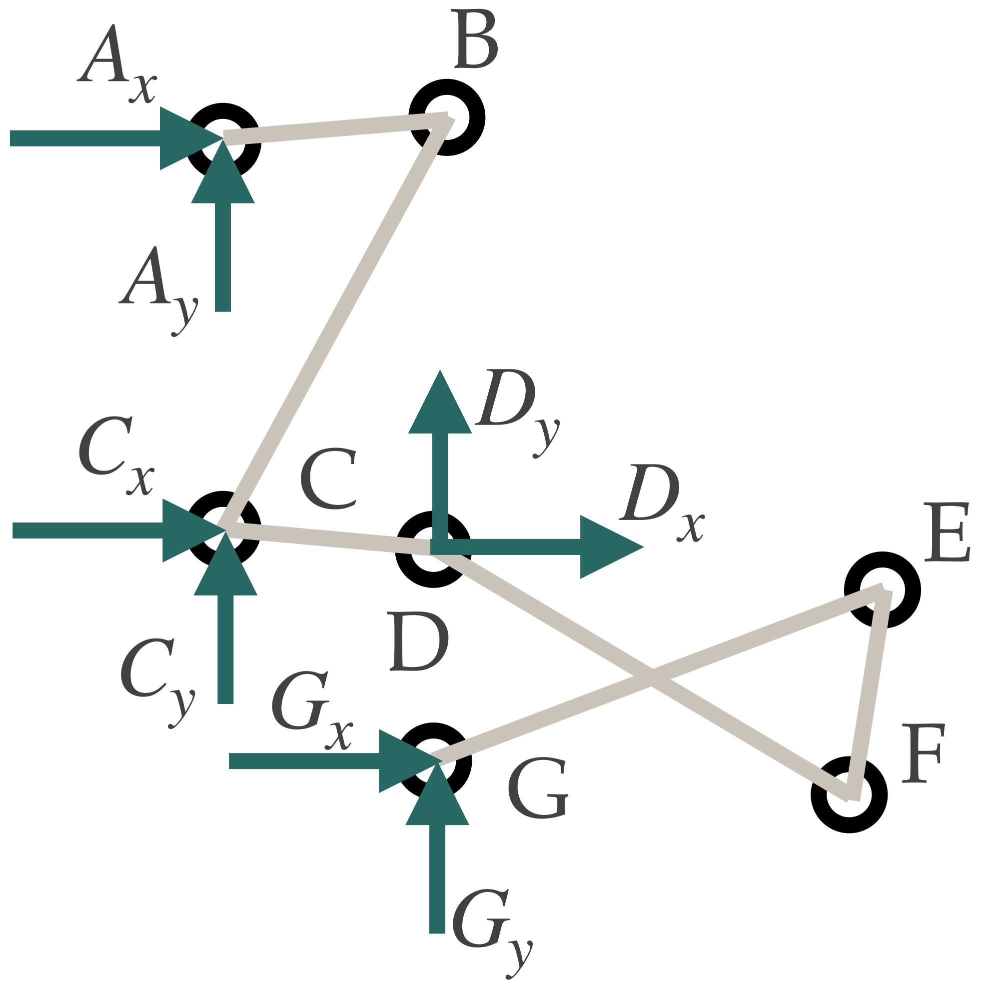
$$
\begin{aligned}
0f_1 + 0f_2 + 0f_3 + p_{11,4}f_4 + p_{11,5}f_5 + 0f_6 &= 0 \\
0f_1 + 0f_2 + 0f_3 + p_{12,4}f_4 + p_{12,5}f_5 + 0f_6 &= 0 \\
0f_1 + 0f_2 + 0f_3 + 0f_4 + 0f_5 + p_{13,6}f_6 &= G_x \\
0f_1 + 0f_2 + 0f_3 + 0f_4 + 0f_5 + p_{14,6}f_6 &= G_y
\end{aligned}
$$

$$\mathbf{P}_{14\times 6}\ \mathbf{f}_{truss\ \ 6\times 1} = \mathbf{F}_{ext\,14\times 1}$$



$\mathbf{P}_{2\ hinges\ \times\ trusses}$

$\mathbf{P}$ need not be square

Can still be rank deficient

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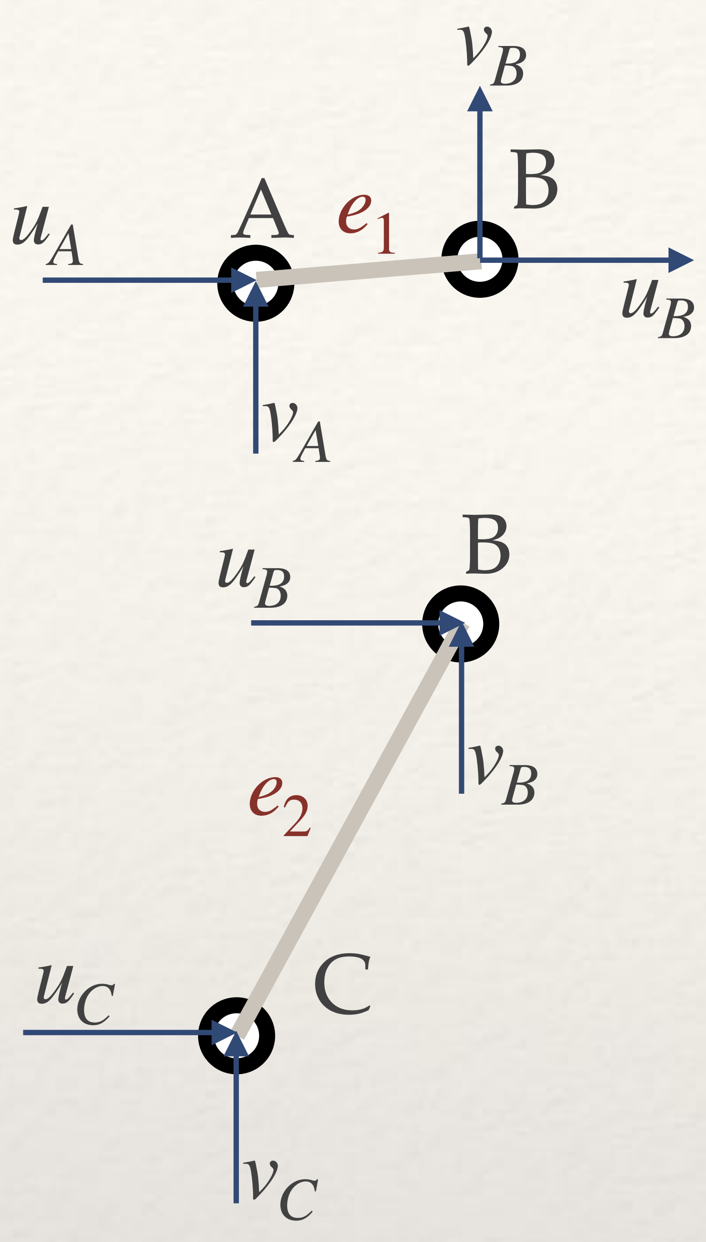
Elongation-Displacement Equations



$e_1 = linear\_fun_1(u_A, v_A, u_B, v_B)$

$e_2 = linear\_fun_1(u_B, v_B, u_C, v_C)$

$\dots$

$e_3 = linear\_fun_1(u_C, v_C, u_D, v_D)$

$e_4 = linear\_fun_1(u_D, v_D, u_F, v_F)$

$e_5 = linear\_fun_1(u_E, v_E, u_F, v_F)$

$e_6 = linear\_fun_1(u_F, v_F, u_G, v_G)$

$$\mathbf{C}_{6\times14}\ \mathbf{U}_{truss\ \ 14\times1} = \mathbf{e}_{6\times1}$$

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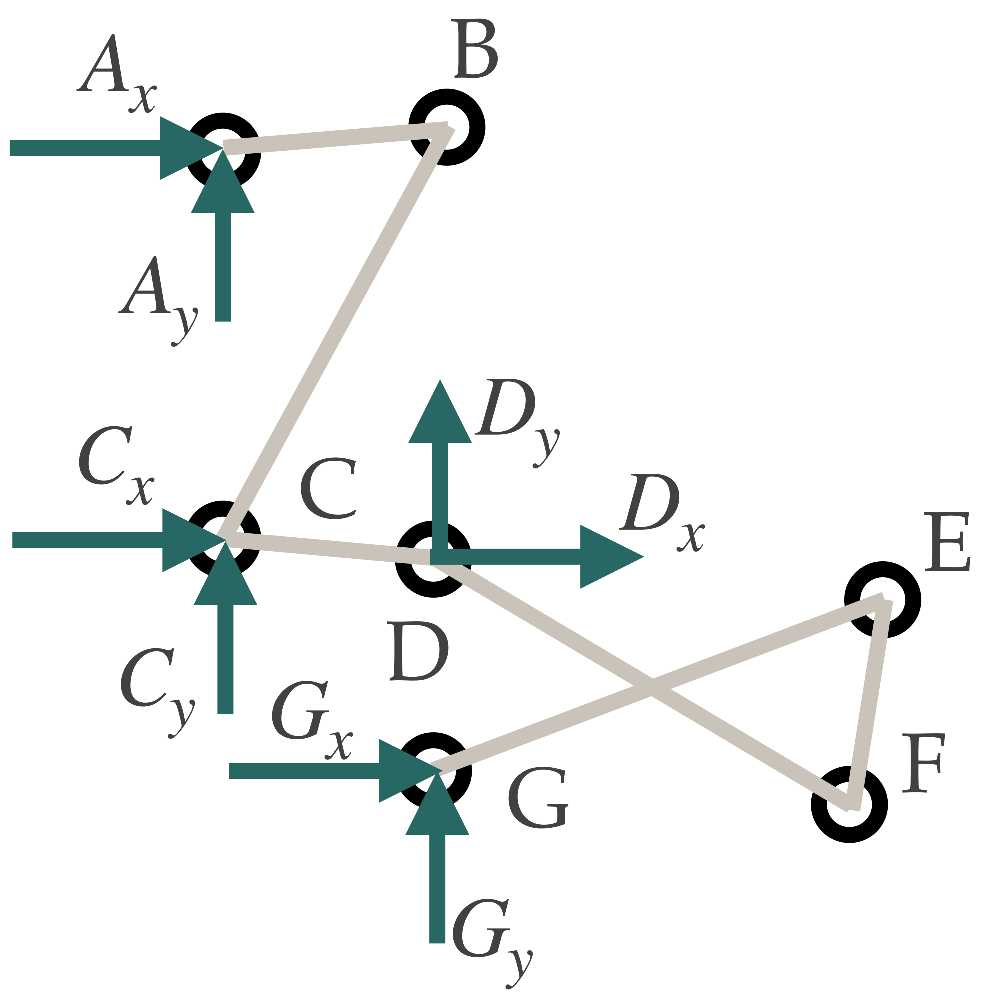
C Compliance matrix

$\mathbf{C}_{trusses \times 2\ hinges}$



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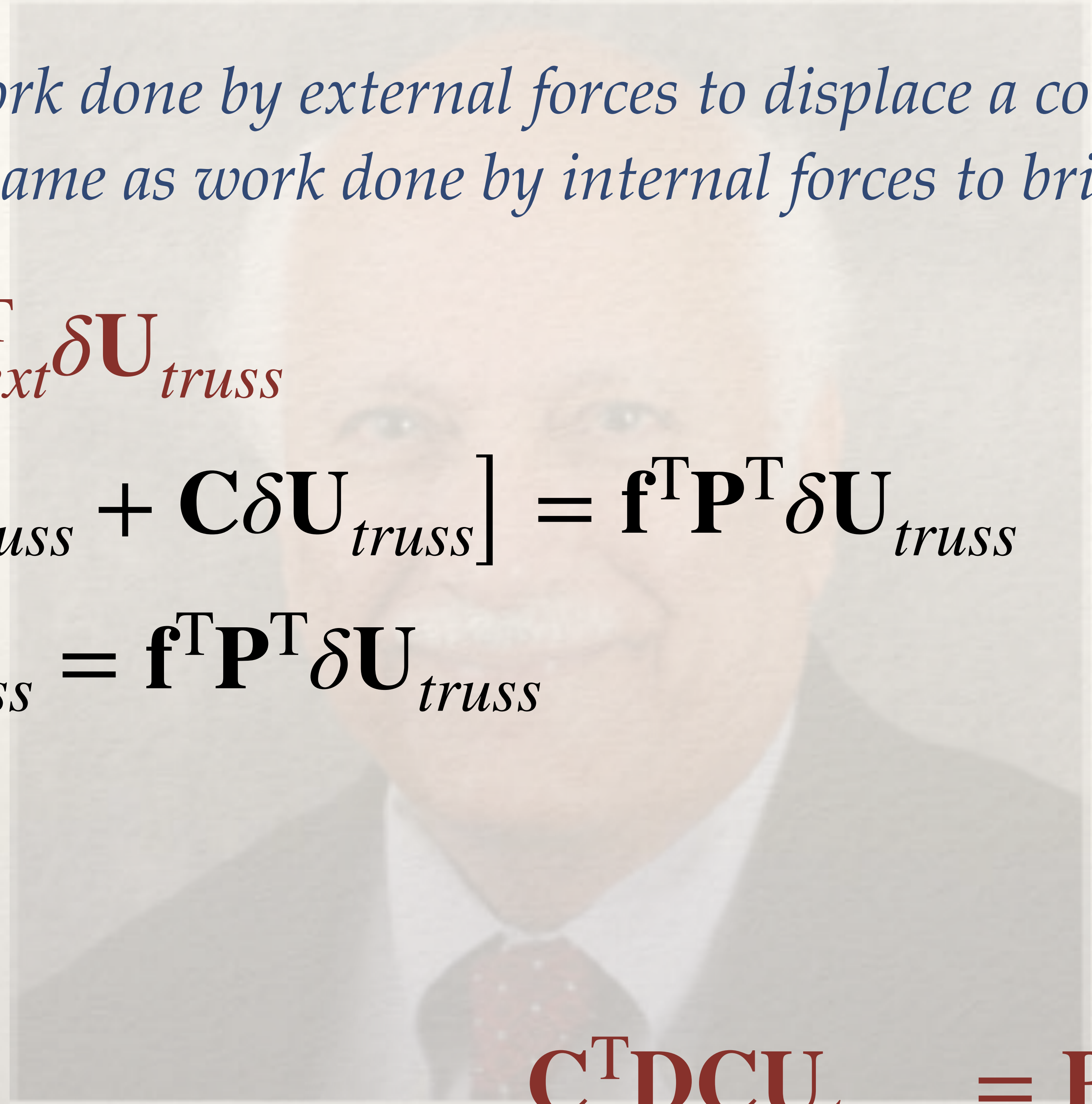
Principle of VIRTUAL WORK…

At equilibrium, work done by external forces to displace a configuration arbitrarily is the same as work done by internal forces to bring it back.



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$$\mathbf{f}^T \delta \mathbf{e} = \mathbf{F}_{ext}^T \delta \mathbf{U}_{truss}$$

$$\mathbf{f}^T \left[\delta \mathbf{C} \mathbf{U}_{truss} + \mathbf{C} \delta \mathbf{U}_{truss}\right] = \mathbf{f}^T \mathbf{P}^T \delta \mathbf{U}_{truss}$$

$$\mathbf{f}^T \mathbf{C} \delta \mathbf{U}_{truss} = \mathbf{f}^T \mathbf{P}^T \delta \mathbf{U}_{truss} \qquad \mathbf{C} = \mathbf{P}^T$$

$$\mathbf{P}\mathbf{f} = \mathbf{F}_{ext}$$

$$\mathbf{P}\mathbf{D}\mathbf{e} = \mathbf{F}_{ext}$$

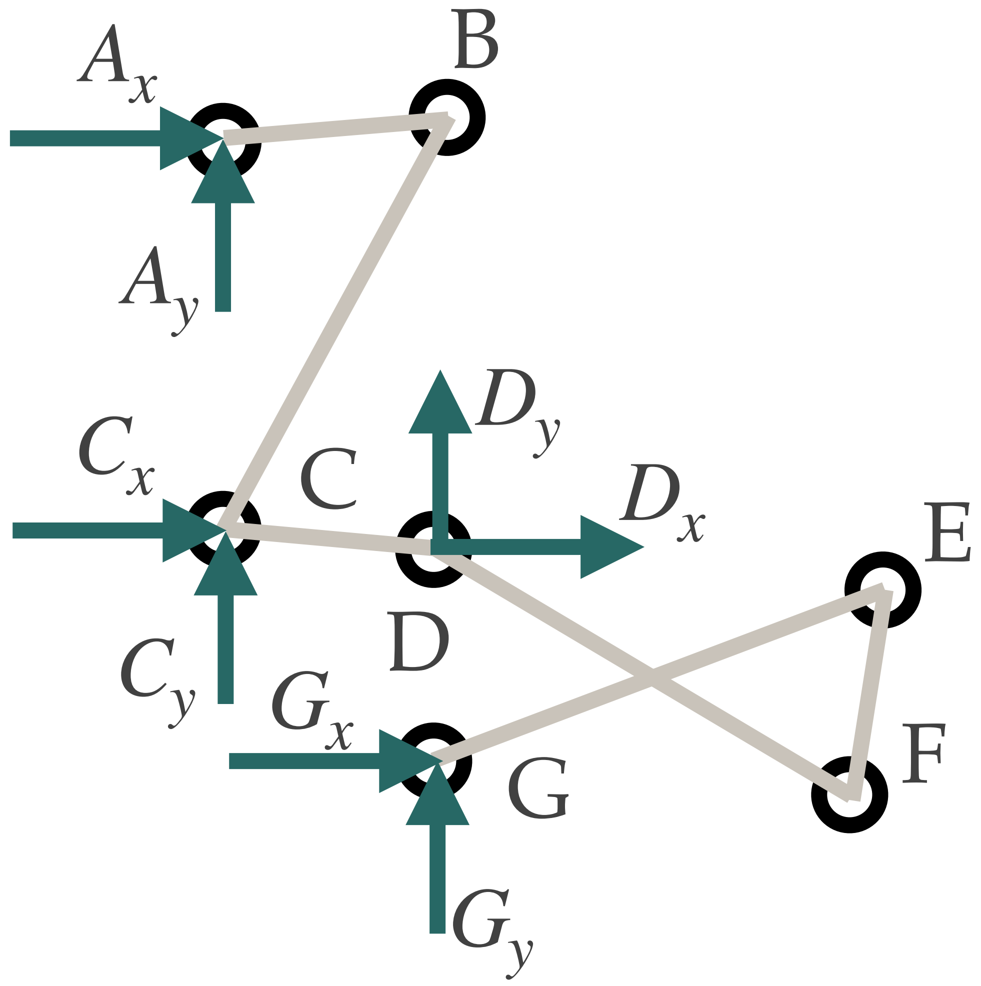
$$\mathbf{P}\mathbf{D}\mathbf{C}\mathbf{U}_{truss} = \mathbf{F}_{ext}$$

$$\mathbf{C}^T\mathbf{D}\mathbf{C}\mathbf{U}_{truss} = \mathbf{P}\mathbf{D}\mathbf{P}^T\mathbf{U}_{truss} = \mathbf{F}_{ext}$$

$$\mathbf{K}\mathbf{U}_{truss} = \mathbf{F}_{ext}$$

$\mathbf{P}\,\mathbf{f} = \mathbf{F}_{ext}$ — Force equilibrium

$\mathbf{C}\,\mathbf{U}_{truss} = \mathbf{e}$ — Elongation-displacement equation



$\mathbf{f} = \mathbf{D}\mathbf{e}$ — Constitutive relation

$\mathbf{K}$ — Stiffness Matrix

DoF: rank deficiency of $\mathbf{K}$

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