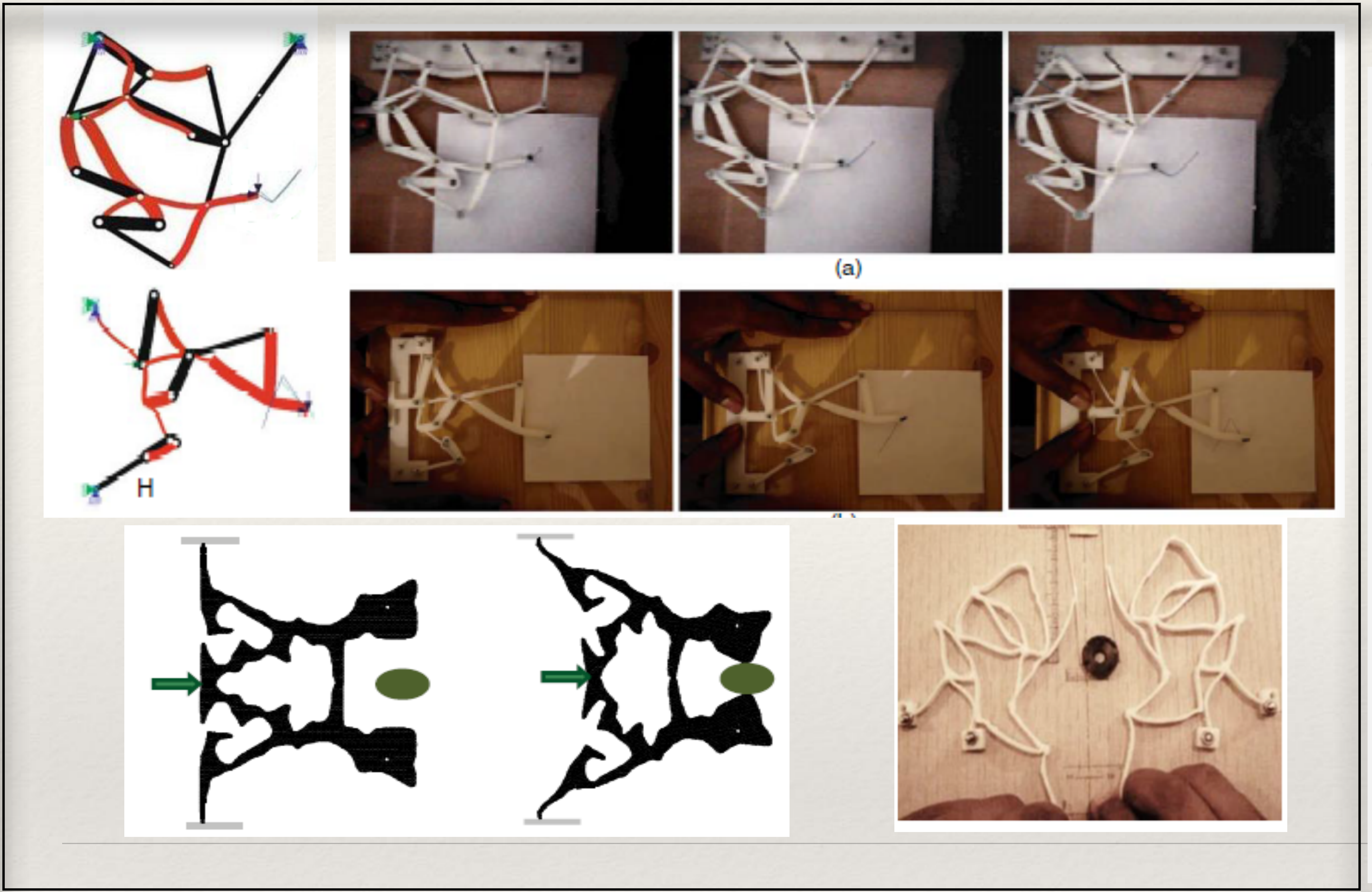

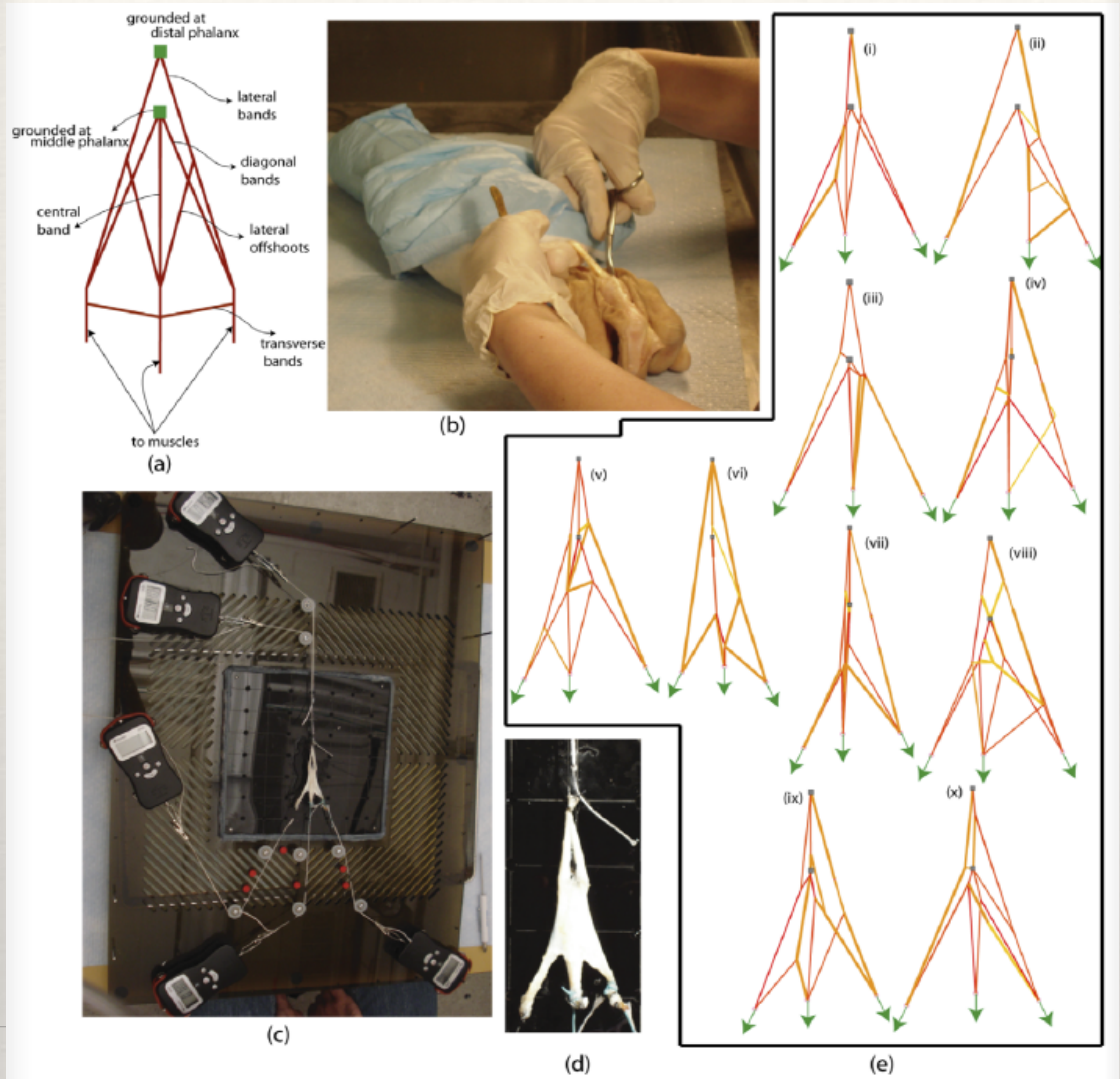

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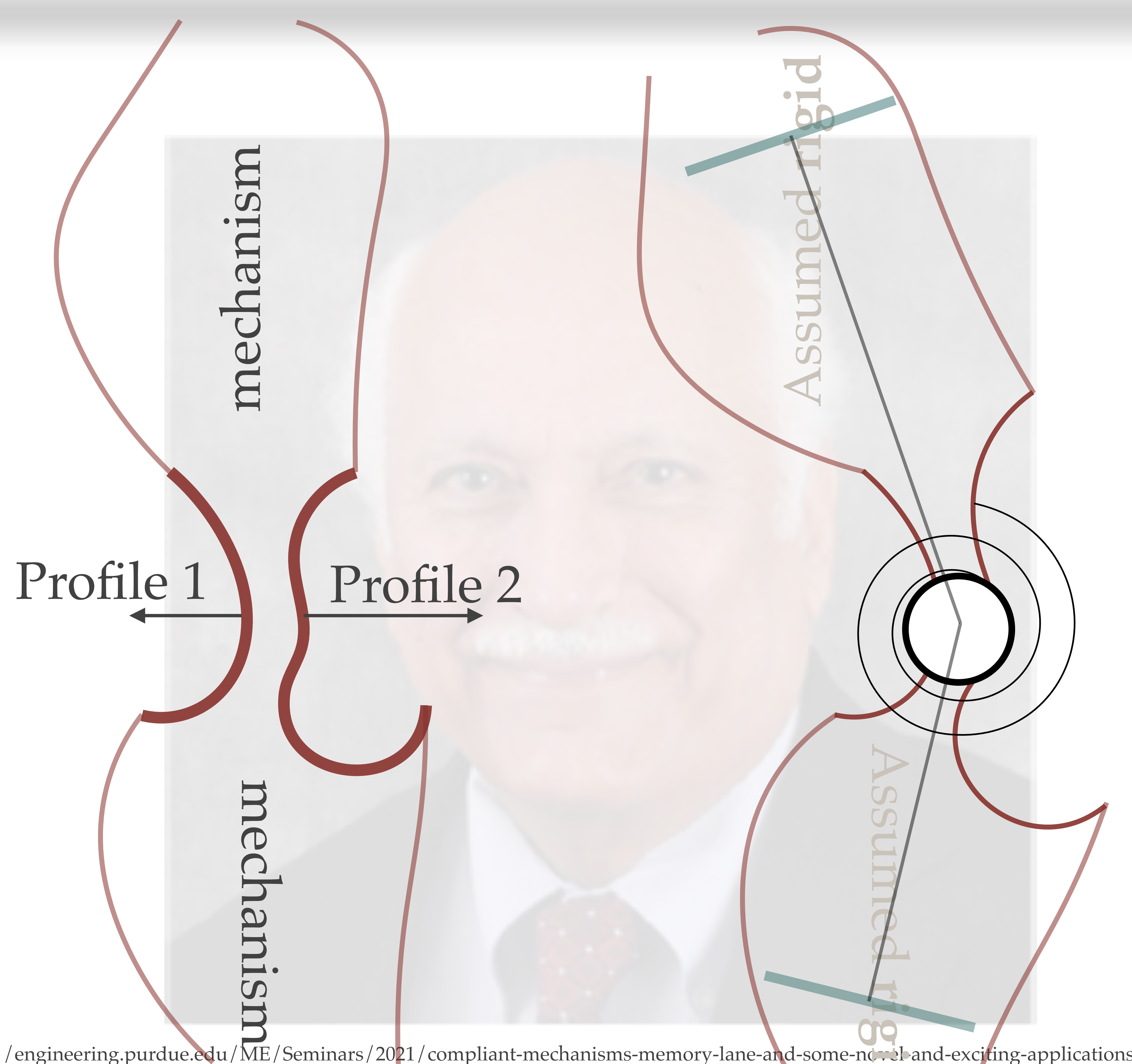

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Sets up a nice inverse, optimization problem — design profiles 1 and 2, s. t.

Summary

Got an overview about **flexures**

Lumped compliant joint in a monolithic mechanism that behaves like a **rigid body hinge** with a torsional spring

Design queries…

Where should the **hinge** be?

What is the torsional stiffness?

How much should/does the **hinge** displace?

How does the **precision** get influenced?

What should be the stress levels?

Small or large deformation?

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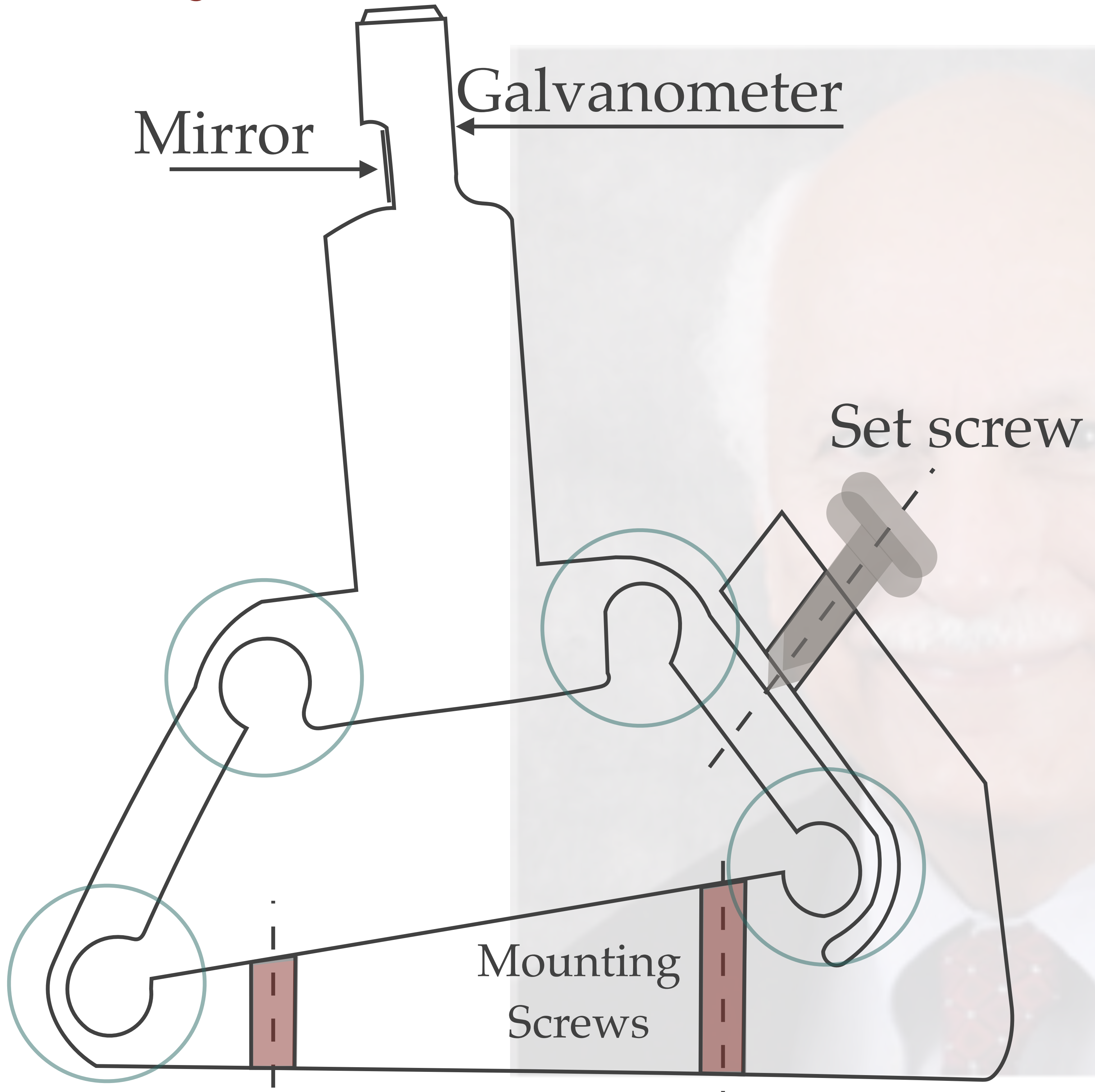
History, current status of flexures



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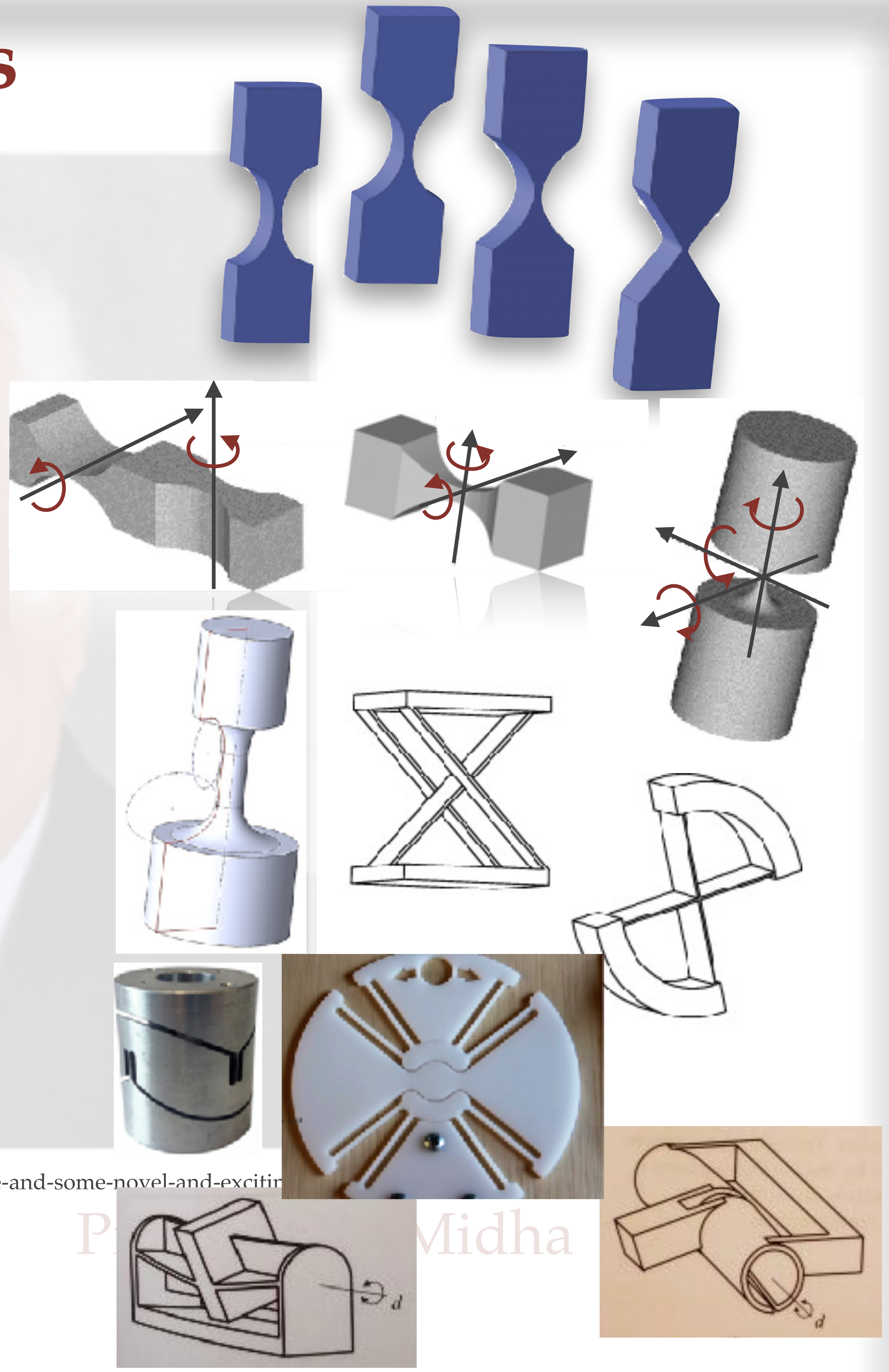
Towfigh's adjusting mechanism



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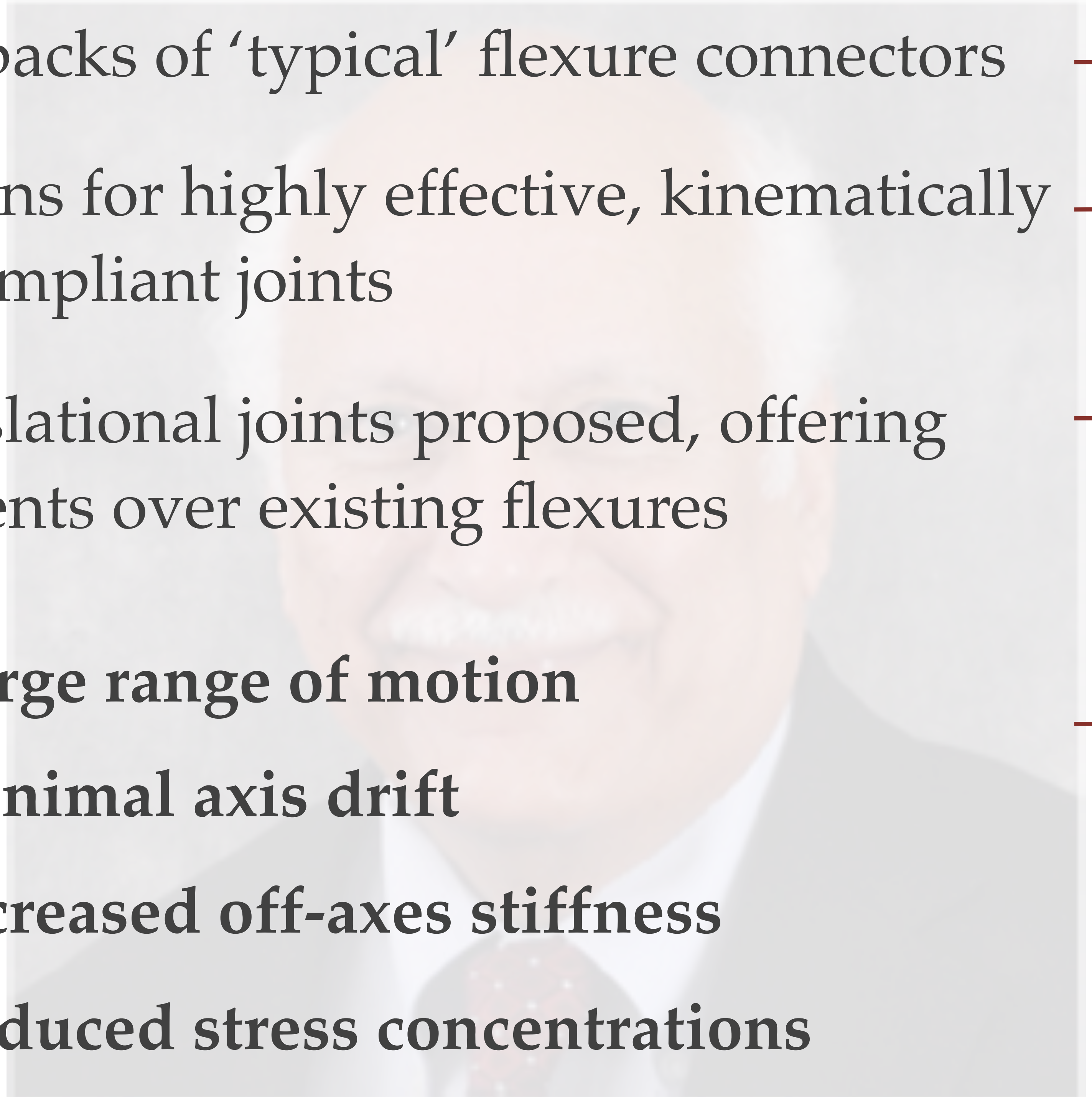
Classify, Discuss and Evaluate Flexures



Investigates drawbacks of ‘typical’ flexure connectors

Presents new designs for highly effective, kinematically well-behaved compliant joints

Revolute and translational joints proposed, offering great improvements over existing flexures

Large range of motion

Minimal axis drift

Increased off-axes stiffness

Reduced stress concentrations

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Design of Large-Displacement Compliant Joints

This paper investigates the drawbacks of typical flexure connectors and presents several new designs for highly effective, kinematically well-behaved compliant joints. A revolute and a translational compliant joint are proposed, both of which offer great improvements over existing flexures in the qualities of (1) a large range of motion, (2) minimal "axis drift," (3) increased off-axis stiffness, and (4) a reduced stress-concentrations. Analytic stiffness equations are developed for each joint and parametric computer models are used to verify their superior stiffness properties. A catalog of design charts based on the parametric models is also presented, allowing for rapid sizing of the joints for custom performance. A joint range of motion has been calculated with finite element analysis, including stress concentration effects. [DOI: 10.1115/1.1900149]

Compliant Mechanisms (ME 851)

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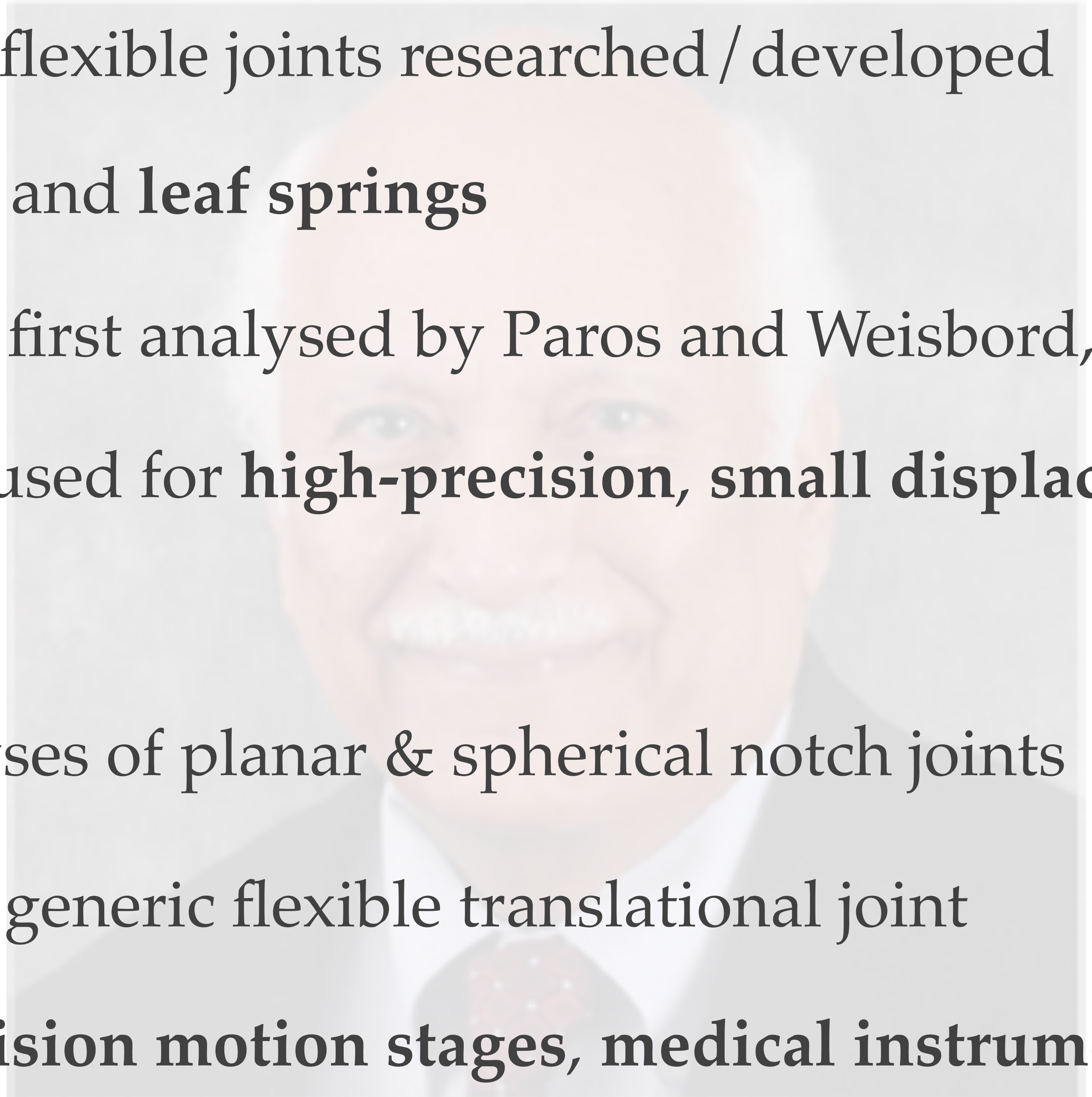
Classify, Discuss and Evaluate Flexures



In 50 years, many flexible joints researched / developed

Notch-type joints and **leaf springs**

Notch-type joints, first analysed by Paros and Weisbord, 1965

Notch-type joints used for **high-precision**, **small displacement** mechanisms

Lobontiu for analyses of planar & spherical notch joints

Leaf springs, most generic flexible translational joint

Used in **high-precision motion stages**, **medical instruments** and **MEMS**

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In the last 50 years, many flexible joints have been researched and developed, most of which are considered one of two varieties: Notch-type joints [Figs. 2(*a*) and 2(*b*)] [also see Tables 1(*b*)–(*d*) and 2(*a*)] and leaf springs [Fig. 2(*c*)] [also see Tables 1(*a*) and 2(*b*)–(*i*)]. Notch-type flexible joints (a.k.a. fillet joints) were first analyzed by Paros and Weisbord in 1965 [1] and have since become well understood by many researchers and designers. Today, notch-type joint assemblies are widely used for high-precision, small-displacement mechanisms [2]. These joints have also been applied by Howell and Midha [3] to develop the field of pseudo-rigid-body compliant mechanisms. See Lobontiu for the most current analyses of planar [4] and spherical three-dimensional (3D) [5] *filleted* notch joints. For the inverse static analysis of a planar system with flexural pivots, please see Carricato et al. [6].

Leaf springs provide the most generic flexible translational joint, composed of sets of parallel flexible beams [Fig. 2(*c*)]. In addition to high-precision motion stages, leaf spring joints are also widely used in medical instrumentation [7] and MEMS devices [8].

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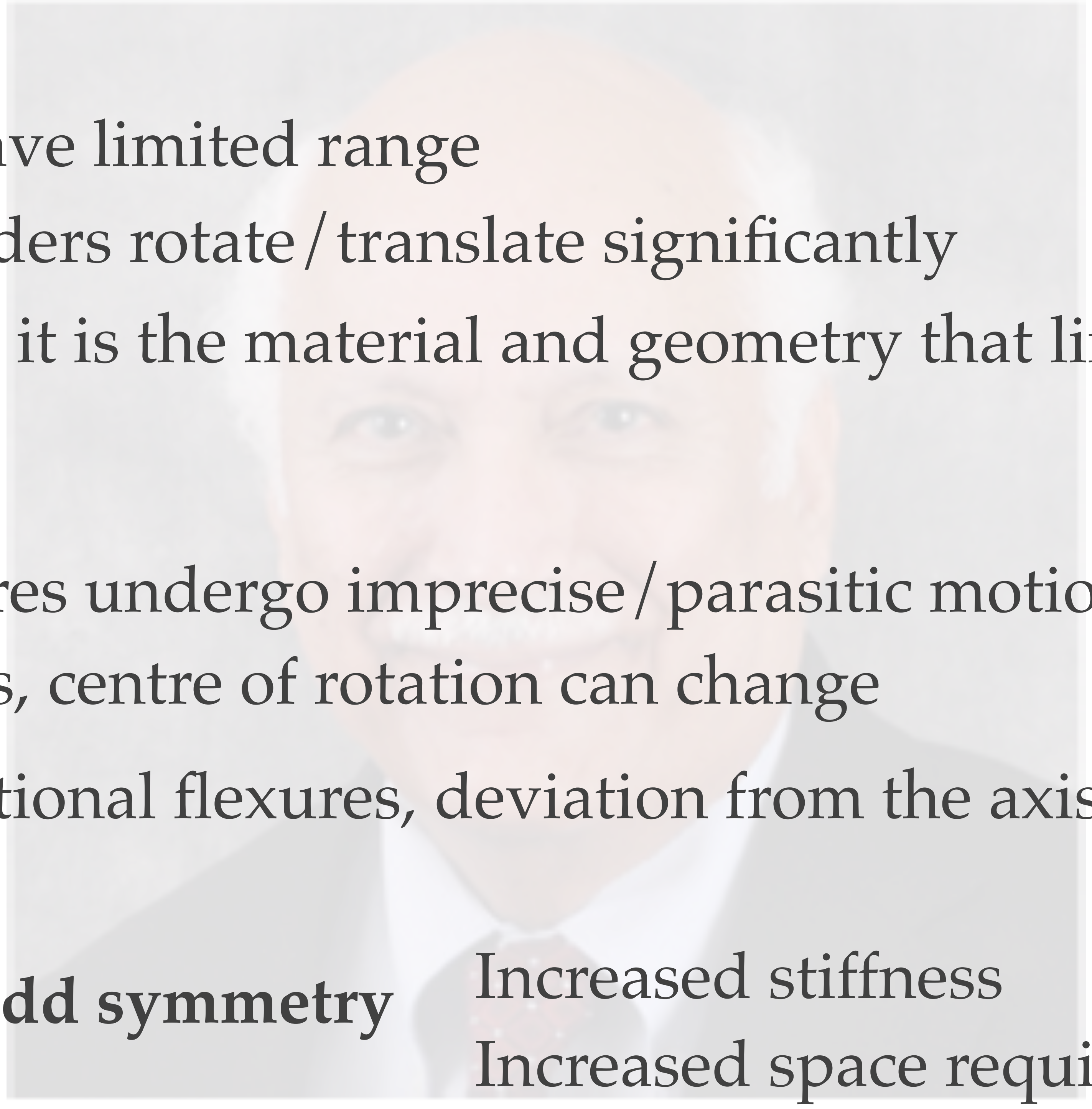
Classify, Discuss and Evaluate Flexures



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Range of motion

- Flexures have limited range
- Hinges/sliders rotate/translate significantly
- In flexures, it is the material and geometry that limits the range

Axis Drift

- Most flexures undergo imprecise/parasitic motion
- For notches, centre of rotation can change
- For translational flexures, deviation from the axis of straight line motion

Remedy: Add symmetry

- Increased stiffness
- Increased space requirements

Off-Axis Stiffness

- Low stiffness in most flexures in undesired (other) directions

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Range of Motion. All flexures are limited to a finite range of motion, while their rigid counterparts rotate infinitely or translate long distances. The range of motion of a flexible joint is limited by the permissible stresses and strains in the material. When the yield stress is reached, elastic deformation becomes plastic, after which, joint behavior is unstable and unpredictable. Therefore, the range of motion is determined by both the material and geometry of the joint.

Axis Drift. In addition to a limited range of motion, most flexure joints also undergo imprecise motion referred to as axis drift or parasitic motion. For notch-type joints, the center of rotation does not remain fixed with respect to the links it connects. With translational flexures, there can be considerable deviation from the axis of straight-line motion. For example, a simple four-bar leaf spring experiences curvilinear motion.

The axis drift can be improved by adding symmetry to the design of a joint. However, this often increases the stiffness of the joint in the desired direction of motion. Further, more space is required to accommodate any symmetric joint components.

Off-Axis Stiffness. While most flexure joints deliver some degree of compliance in the desired direction, they typically suffer from low rotational and translational stiffness in other directions. A high ratio of off-axis to axial stiffness is considered a key characteristic of an effective compliant joint.

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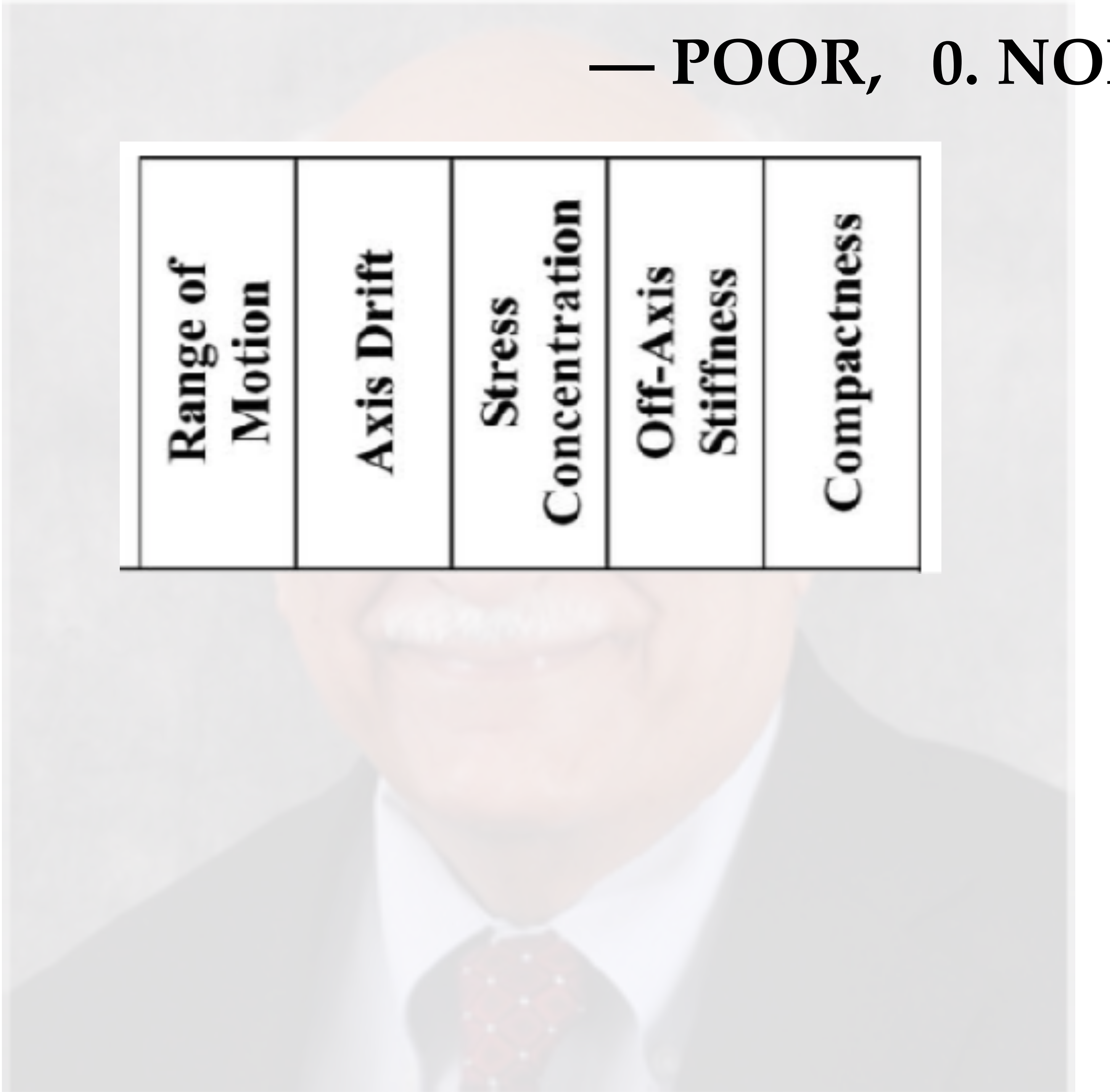
Classify, Discuss and Evaluate Flexures

— POOR, 0. NORMAL, + GOOD



Range of Motion	Axis Drift	Stress Concentration	Off-Axis Stiffness	Compactness

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Benchmarking Translational Joints

		Range of Motion	Axis Drift	Stress Concentration	Off-Axis Stiffness	Compactness
(a)		0	–	0	0	+
(b)		–	–	–	0	+
(c)		–	0	–	0	+
(d)		–	+	–	0	+
(e)		+	+	+	+	+

Compliant Mechanisms (ME 851)

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Classify, Discuss and Evaluate Flexures

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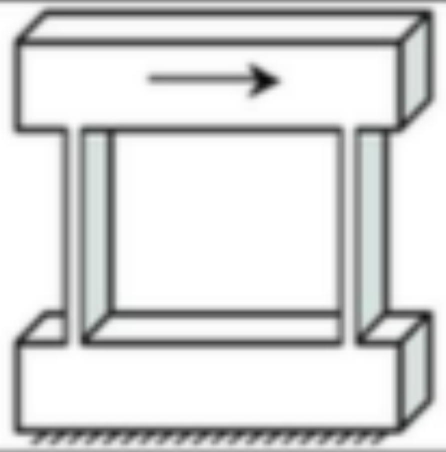	0	–	0	0	+

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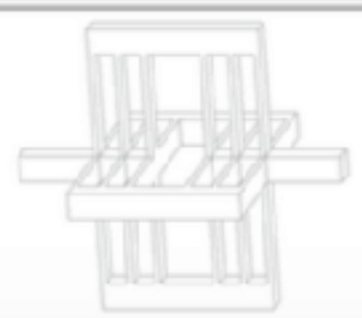
(e)		+	+	+	+	+

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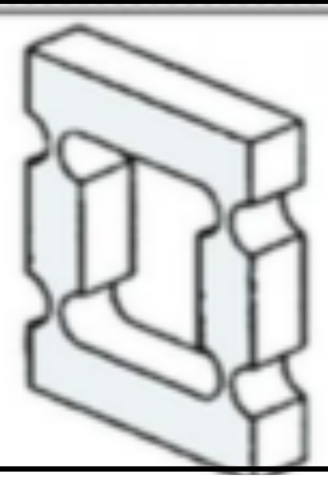	–	–	–	0	+

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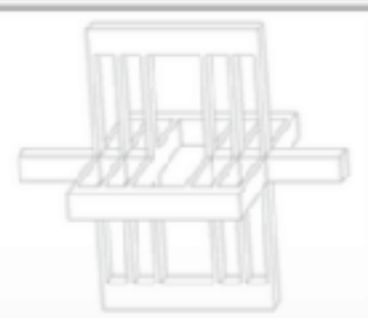
(e)		+	+	+	+	+

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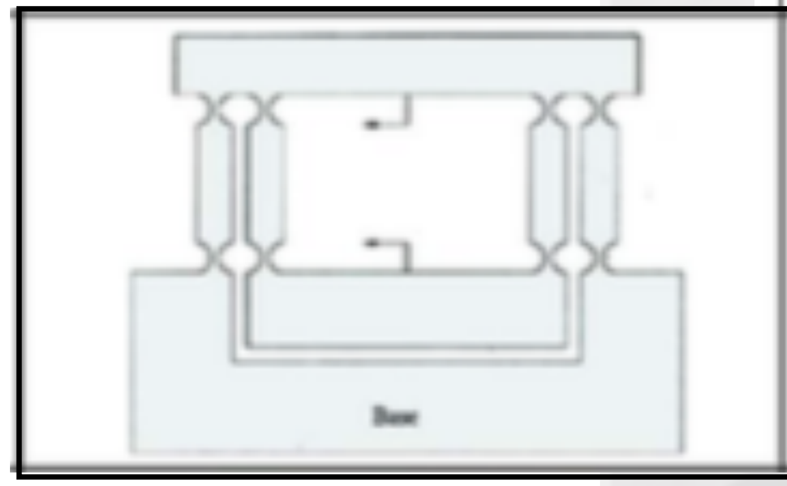	–	0	–	0	+

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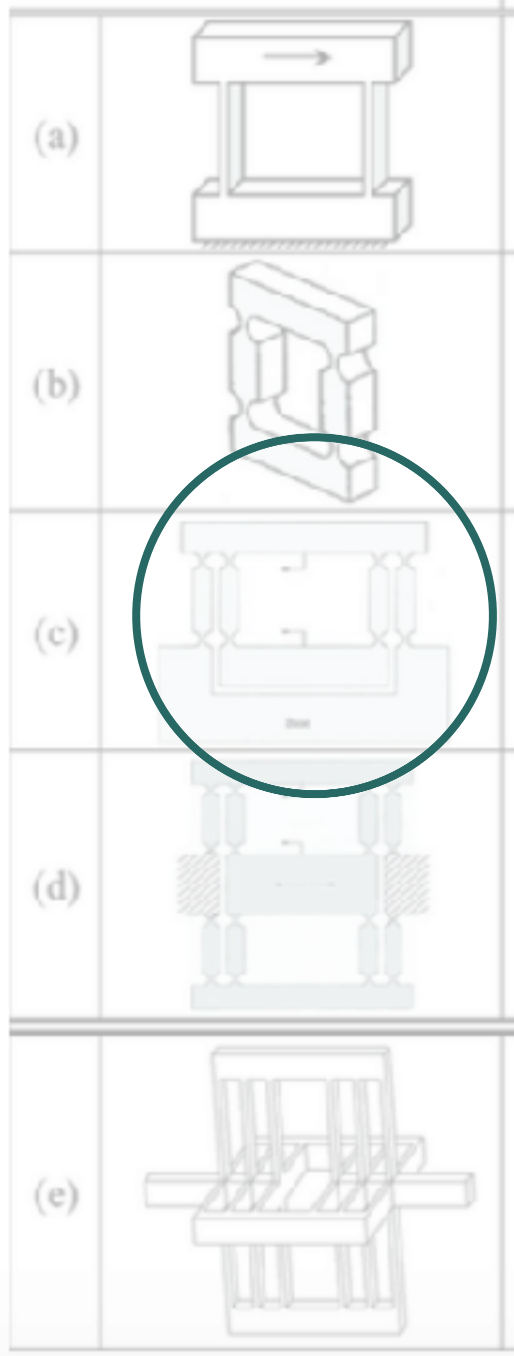

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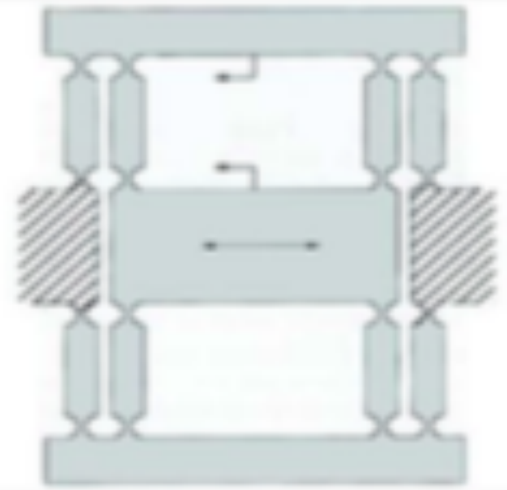	–	+	–	0	+

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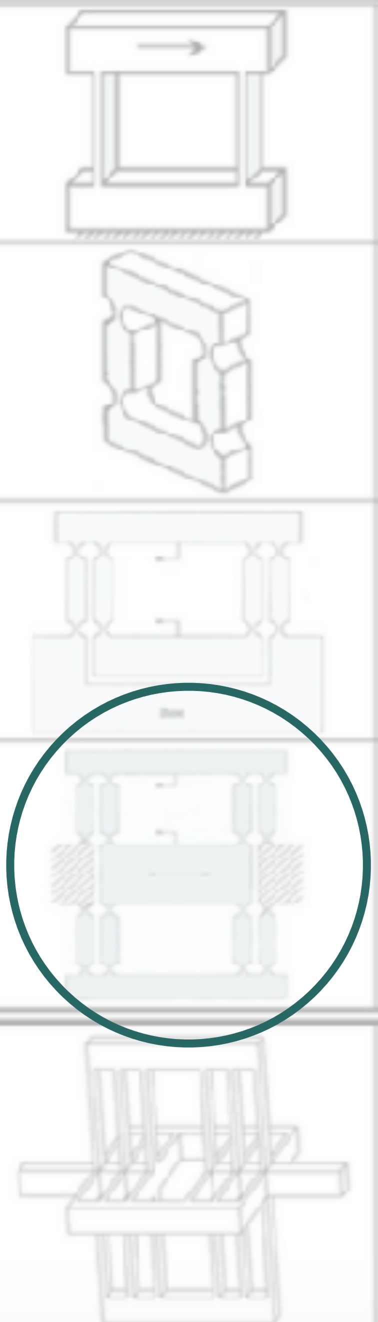

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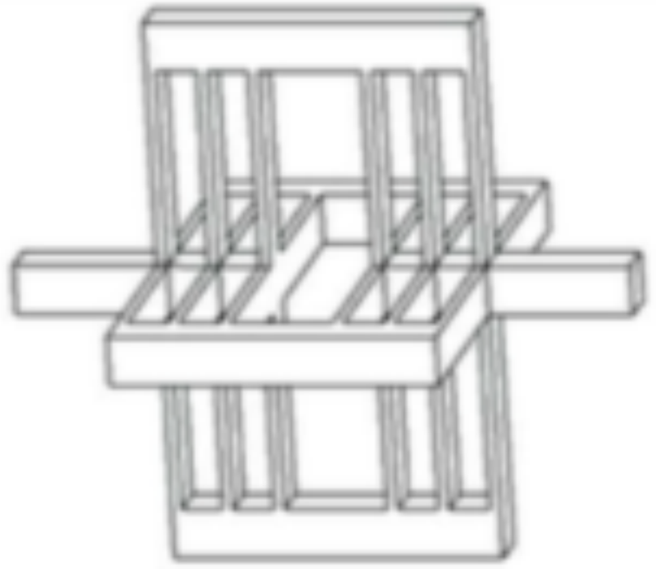	+	+	+	+	+

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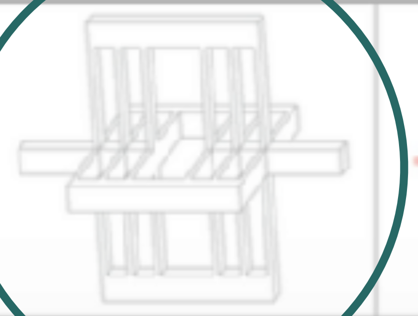
(e)		+	+	+	+	+

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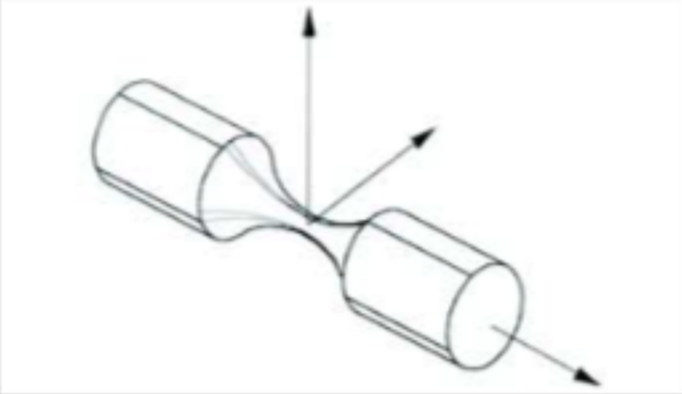	−	−	−	−	+

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Benchmarking ROTATIONAL Joints

Table 2 Benchmarked flexible revolute joints

	Range of Motion	Axis Drift	Stress Concentration	Off-Axis Stiffness	Compactness
(a)	−	−	−	−	+
(b)	0	−	+	−	0
(c)	+	−	+	−	−
(d)	−	−	0	−	+
(e)	−	0	−	0	0
(f)	−	+	0	−	−
(g)	+	+	+	−	−
(h)	−	+	−	−	−
(i)	−	0	−	−	0
(j)	+	0	+	+	0
(k)	+	+	+	+	0

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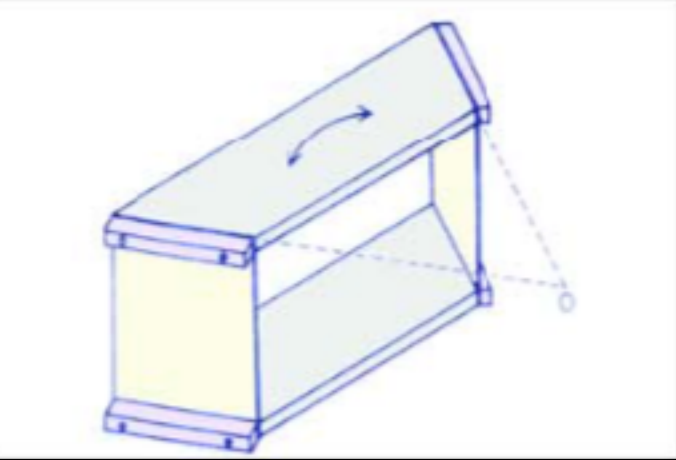	0	–	+	–	0

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(d)	–	–	0	–	+
(e)	–	0	–	0	0
(f)	–	+	0	–	–
(g)	+	+	+	–	–
(h)	–	+	–	–	–
(i)	–	0	–	–	0
(j)	+	0	+	+	0
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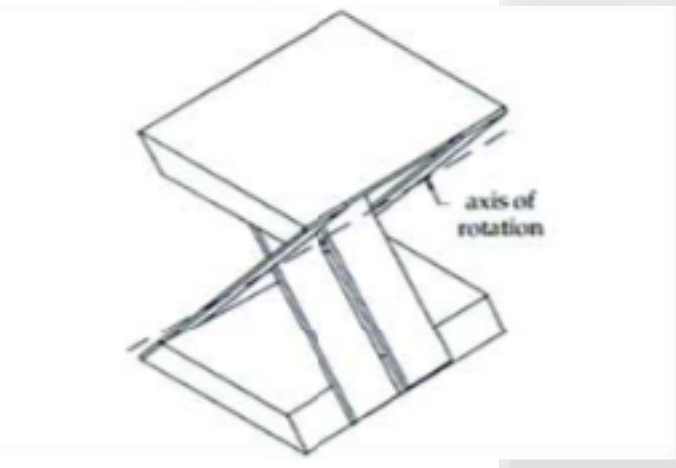	+	–	+	–	–

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(d)	–	–	0	–	+
(e)	–	0	–	0	0
(f)	–	+	0	–	–
(g)	+	+	+	–	–
(h)	–	+	–	–	–
(i)	–	0	–	–	0
(j)	+	0	+	+	0
(k)	+	+	+	+	0

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	Range of Motion	Axis Drift	Stress Concentration	Off-Axis Stiffness	Compactness
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(c)	+	–	+	–	–
(d)	–	–	0	–	+
(e)	–	0	–	0	0
(f)	–	+	0	–	–
(g)	+	+	+	–	–
(h)	–	+	–	–	–
(i)	–	0	–	–	0
(j)	+	0	+	+	0
(k)	+	+	+	+	0

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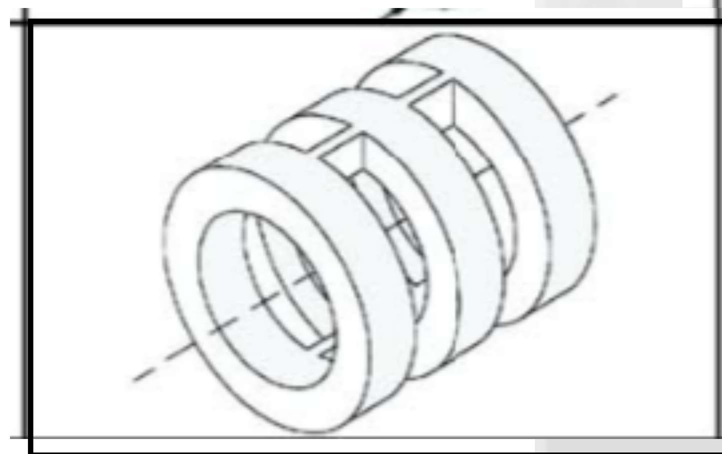	–	0	–	0	0

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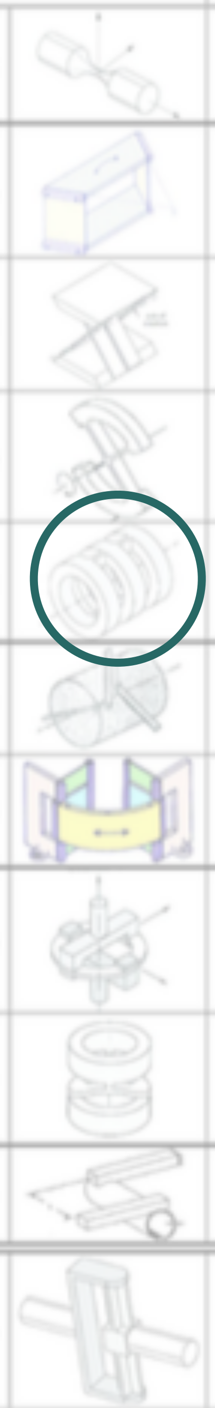

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(c)	+	–	+	–	–
(d)	–	–	0	–	+
(e)	–	0	–	0	0
(f)	–	+	0	–	–
(g)	+	+	+	–	–
(h)	–	+	–	–	–
(i)	–	0	–	–	0
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Classify, Discuss and Evaluate Flexures

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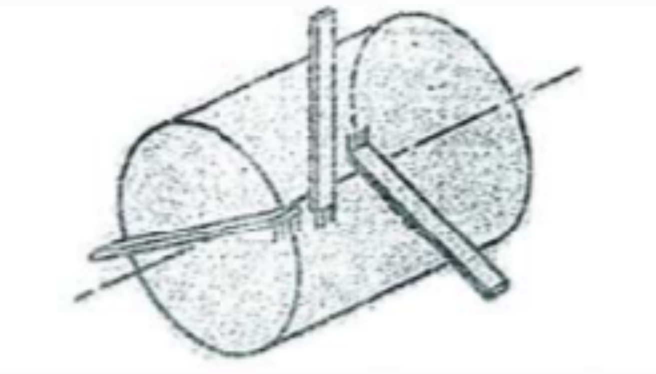
	Range of Motion	Axis Drift	Stress Concentration	Off-Axis Stiffness	Compactness
	–	+	0	–	–

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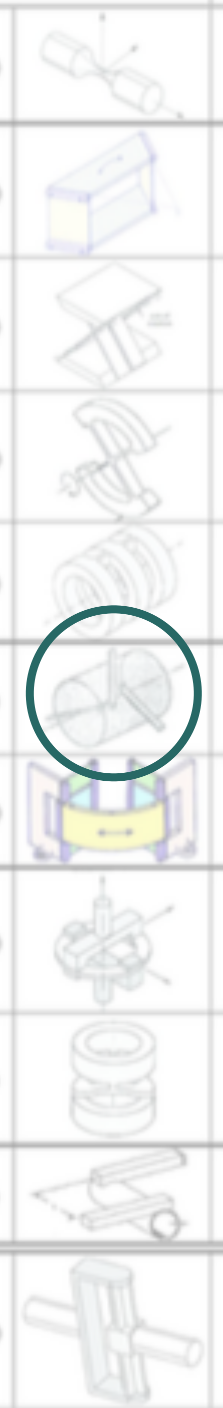
Benchmarking ROTATIONAL Joints

Table 2 Benchmarked flexible revolute joints



	Range of Motion	Axis Drift	Stress Concentration	Off-Axis Stiffness	Compactness
(a)	–	–	–	–	+
(b)	0	–	+	–	0
(c)	+	–	+	–	–
(d)	–	–	0	–	+
(e)	–	0	–	0	0
(f)	–	+	0	–	–
(g)	+	+	+	–	–
(h)	–	+	–	–	–
(i)	–	0	–	–	0
(j)	+	0	+	+	0
(k)	+	+	+	+	0

Compliant Mechanisms (ME 851)

Anupam Saxena
Professor
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Classify, Discuss and Evaluate Flexures

— POOR, 0. NORMAL, + GOOD

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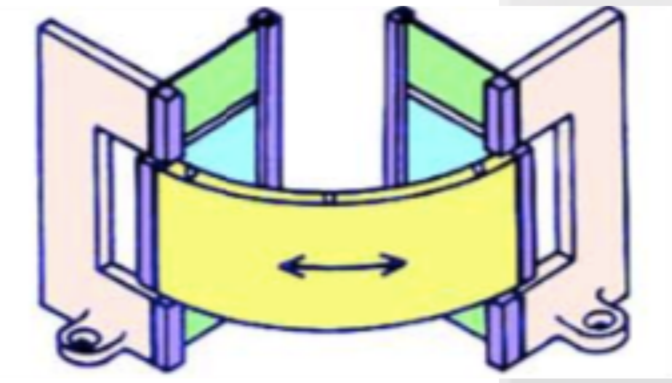	+	+	+	–	–

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(d)		–	–	0	–	+
(e)		–	0	–	0	0
(f)		–	+	0	–	–
(g)		+	+	+	–	–
(h)		–	+	–	–	–
(i)		–	0	–	–	0
(j)		+	0	+	+	0
(k)		+	+	+	+	0

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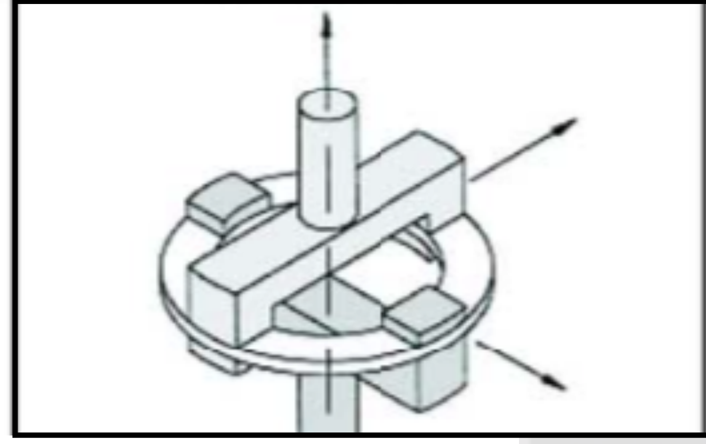	–	+	–	–	–

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(d)	–	–	0	–	+
(e)	–	0	–	0	0
(f)	–	+	0	–	–
(g)	+	+	+	–	–
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(i)	–	0	–	–	0
(j)	+	0	+	+	0
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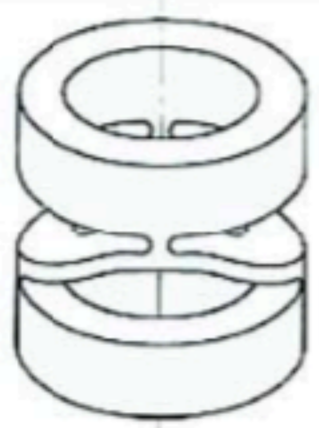	–	0	–	–	0

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(f)	–	+	0	–	–
(g)	+	+	+	–	–
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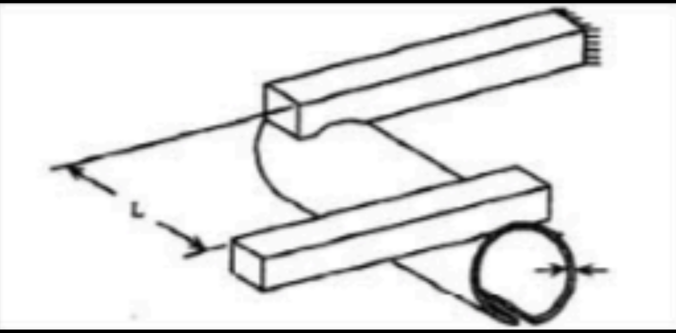	+	0	+	+	0

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(f)	–	+	0	–	–
(g)	+	+	+	–	–
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(k)	+	+	+	+	0

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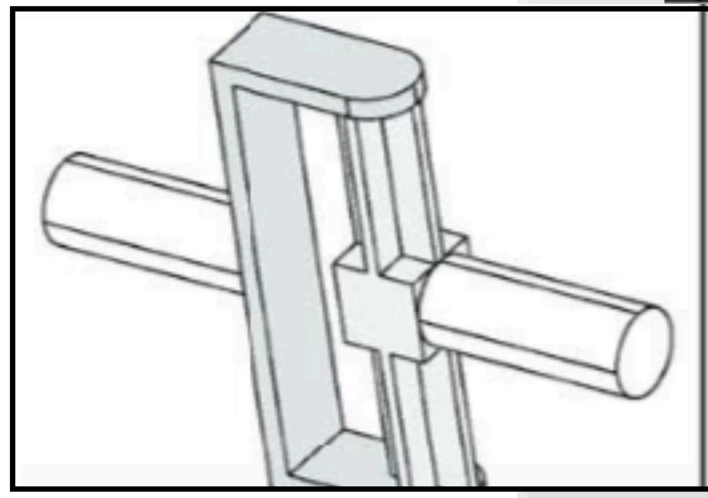

	Range of Motion	Axis Drift	Stress Concentration	Off-Axis Stiffness	Compactness
	+	+	+	+	0

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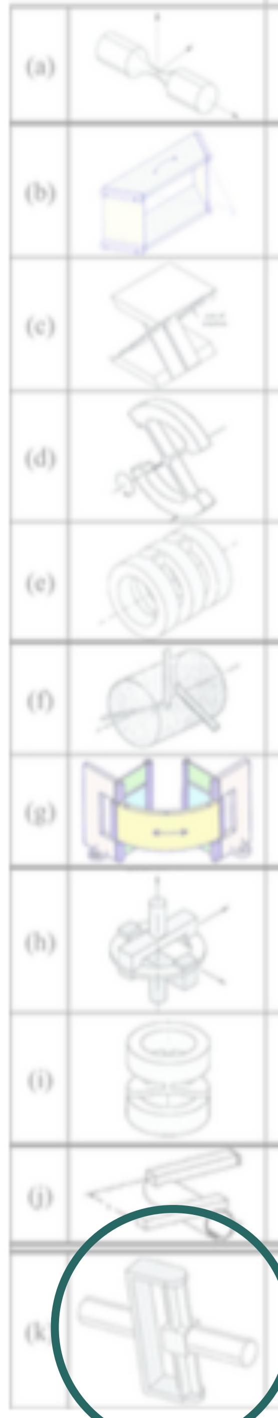

	Range of Motion	Axis Drift	Stress Concentration	Off-Axis Stiffness	Compactness
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(d)	–	–	0	–	+
(e)	–	0	–	0	0
(f)	–	+	0	–	–
(g)	+	+	+	–	–
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(j)	+	0	+	+	0
(k)	+	+	+	+	0

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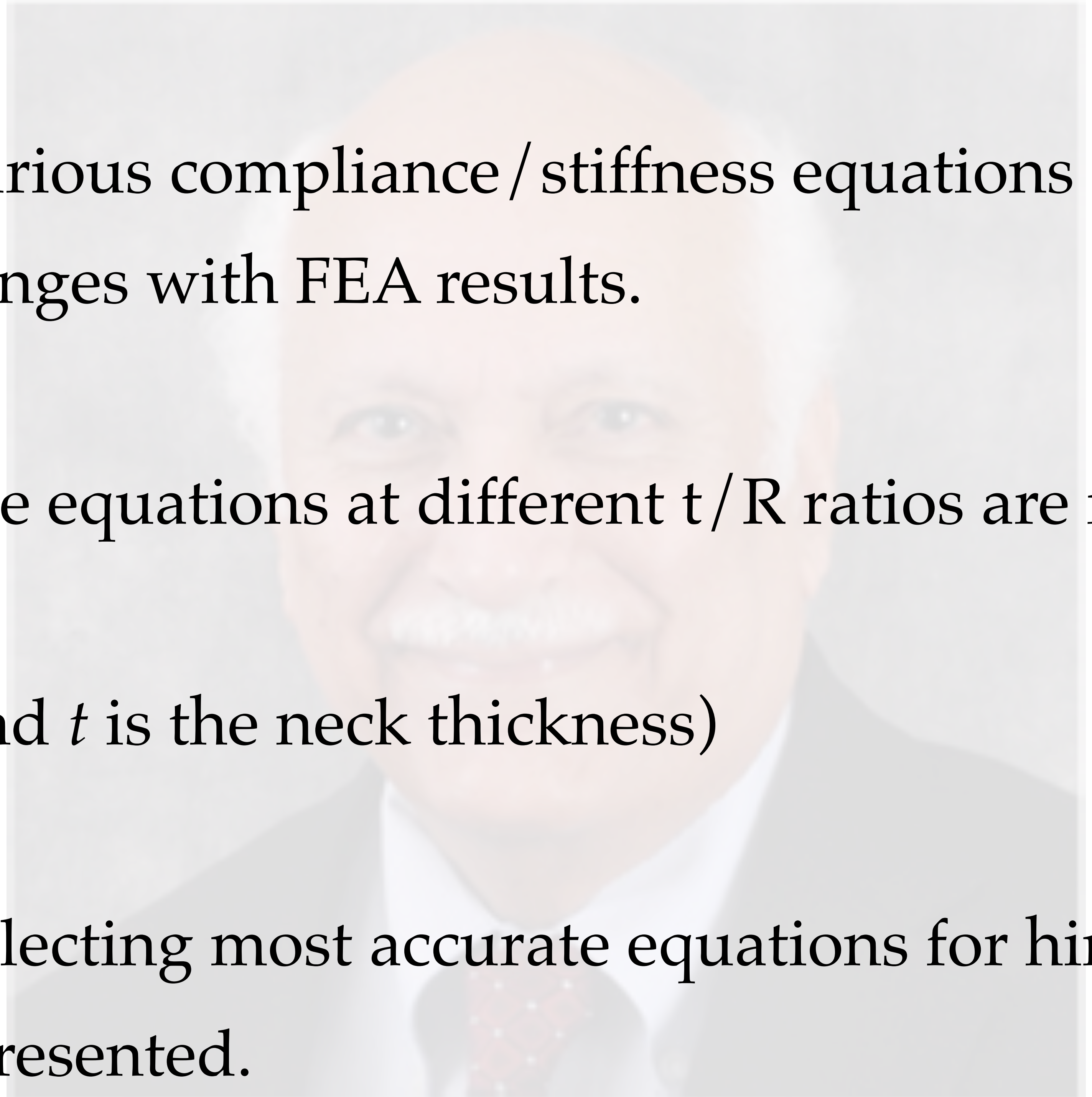
Review of Circular Flexure Hinges



Comparison of various compliance/stiffness equations of circular flexure hinges with FEA results.

Limitation of these equations at different t/R ratios are revealed.

(R is the radius and t is the neck thickness)

A guideline for selecting most accurate equations for hinge design calculations are presented.

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Review

Review of circular flexure hinge design equations and derivation of empirical formulations

Yuen Kuan Yong*, Tien-Fu Lu, Daniel C. Handley
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Abstract

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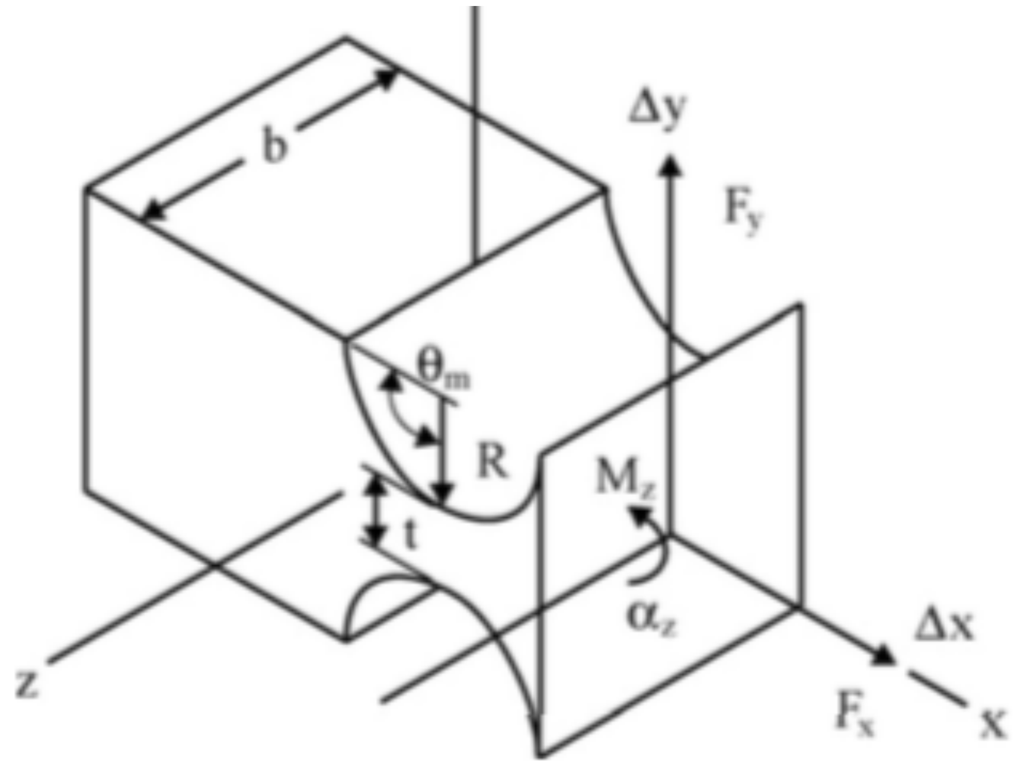

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Review of Circular Flexure Hinges

rotation where their center of rotations do not displace as much as other flexure hinges such as the left-type [8] and the corner-fillet [9]. There have been many methods adopted to derive satisfactory compliance/stiffness equations of flexure hinges, including the integration of linear differential equations of a beam [1,10,11], Castigliano's second theorem [11], inverse conformal mapping [8] and empirical equations formed from FEA (finite element analysis) results [12,13]. However, some of these methods provide better accuracies than the others depending on the t/R ratios of circular flexure hinges (see Fig. 1 for dimensions). Paros and Weisbord [1] were the first research group to introduce right circular flexure hinges. They formulated design equations, including both the full and simplified, to calculate compliances of flexure hinges. The error of the simplified equation relative to full equation was within 1% for hinges with t/R in the range 0.02–0.1, and within 5–12% for thicker hinge with t/R in the range 0.2–0.6 [8]. However, both the full and simplified rotational compliance equations (α_z/M_z) show a large difference of up to 25% or more for $t/R = 0.6$ when compared with FEA results [8].

$$\frac{M}{I} = \frac{E}{\rho}\left(\approx E\frac{d^2y}{dx^2}\right) = -\frac{\sigma_x}{y_c}$$

$$u = \frac{\partial E}{\partial F}$$

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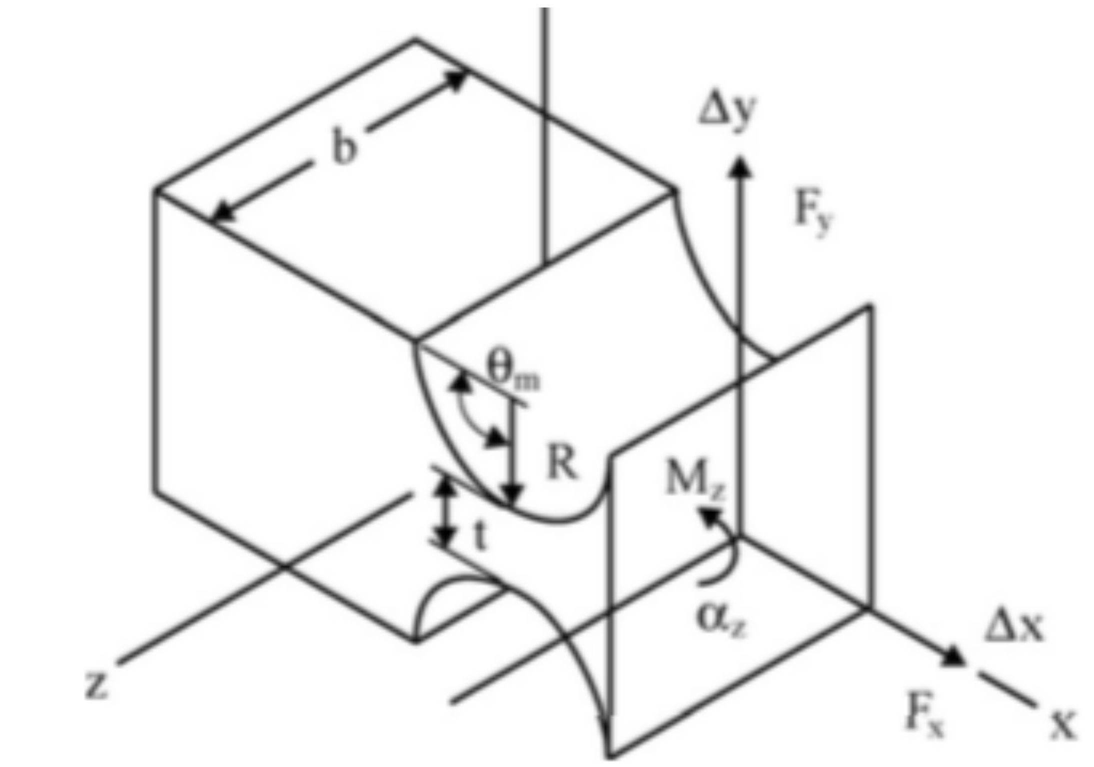

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Review of Circular Flexure Hinges

A.1.1. Full equations

$\beta = t/2R$, $\gamma = 1+\beta$, $\theta_m = \pi/2$ for right circular flexure hinge

$$\frac{\alpha_z}{M_z} = \frac{3}{2EbR^2}\left[\frac{1}{2\beta+\beta^2}\right]\left\{\left[\frac{1+\beta}{\gamma^2} + \frac{3+2\beta+\beta^2}{\gamma(2\beta+\beta^2)}\right]\right.$$
$$\times\left[\sqrt{1-(1+\beta-\gamma)^2}\right] + \left[\frac{6(1+\beta)}{(2\beta+\beta^2)^{3/2}}\right]$$
$$\left.\times\left[\tan^{-1}\left(\sqrt{\frac{2+\beta}{\beta}} \times \frac{(\gamma-\beta)}{\sqrt{1-(1+\beta-\gamma)^2}}\right)\right]\right\}$$

$$\frac{\Delta y}{F_y} = R^2 \sin^2\theta_m\left(\frac{\alpha_z}{M_z}\right)$$
$$-\frac{3}{2Eb}\left\{\left[\frac{1+\beta}{(1+\beta-\cos\theta_m)^2} - \frac{2+(1+\beta)^2/(2\beta+\beta^2)}{(1+\beta-\cos\theta_m)}\right]\right.$$
$$\times \sin\theta_m + \left[\frac{4(1+\beta)}{\sqrt{2\beta+\beta^2}} - \frac{2(1+\beta)}{(2\beta+\beta^2)^{3/2}}\right]$$
$$\left.\times \tan^{-1}\sqrt{\frac{2+\beta}{\beta}}\tan\frac{\theta_m}{2} - (2\theta_m)\right\}$$

$$\frac{\Delta x}{F_x} = \frac{1}{Eb}\left[-2\tan^{-1}\frac{\gamma-\beta}{\sqrt{1-(1+\beta-\gamma)^2}}\right.$$
$$\left.+\frac{2(1+\beta)}{\sqrt{2\beta+\beta^2}}\tan^{-1}\left(\sqrt{\frac{2+\beta}{\beta}} \times \frac{\gamma-\beta}{\sqrt{1-(1+\beta-\gamma)^2}}\right)\right] \quad \text{(A.3)}$$

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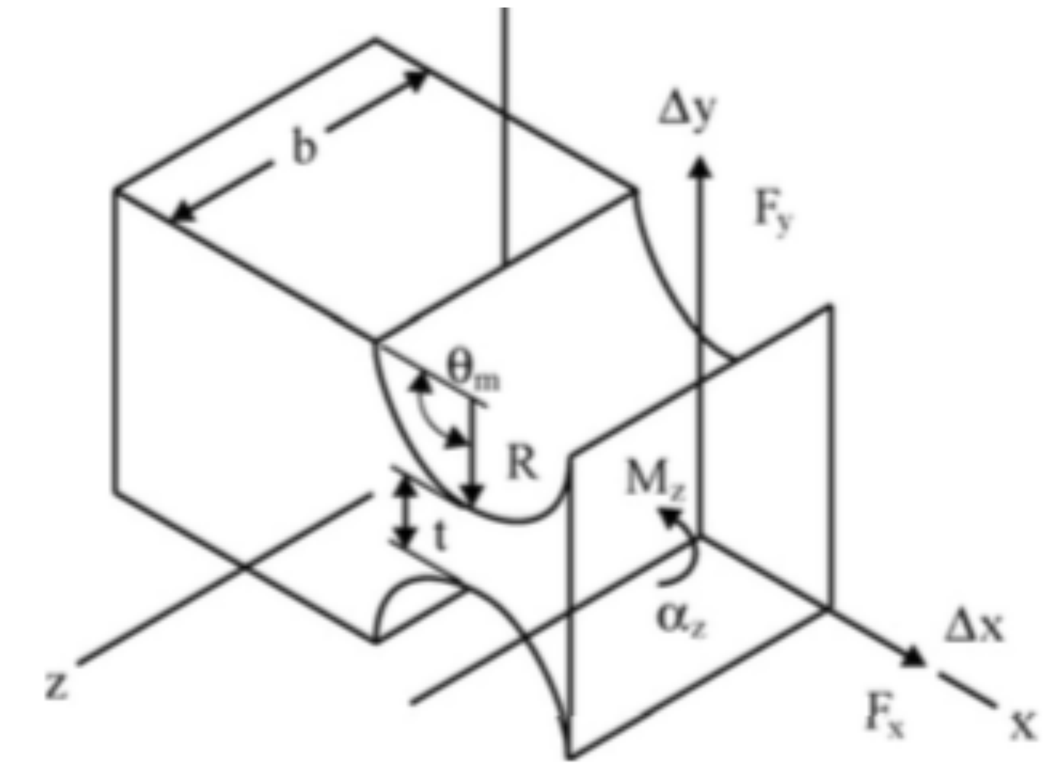

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Review of Circular Flexure Hinges

A.1.2. Simplified equations

$$\frac{\alpha_z}{M_z} = \frac{9\pi R^{1/2}}{2Ebt^{5/2}}$$

$$\frac{\Delta y}{F_y} = \frac{9\pi}{2Eb}\left(\frac{R}{t}\right)^{5/2}$$

$$\frac{\Delta x}{F_x} = \frac{1}{Eb}[\pi(R/t)^{1/2} - 2.57]$$

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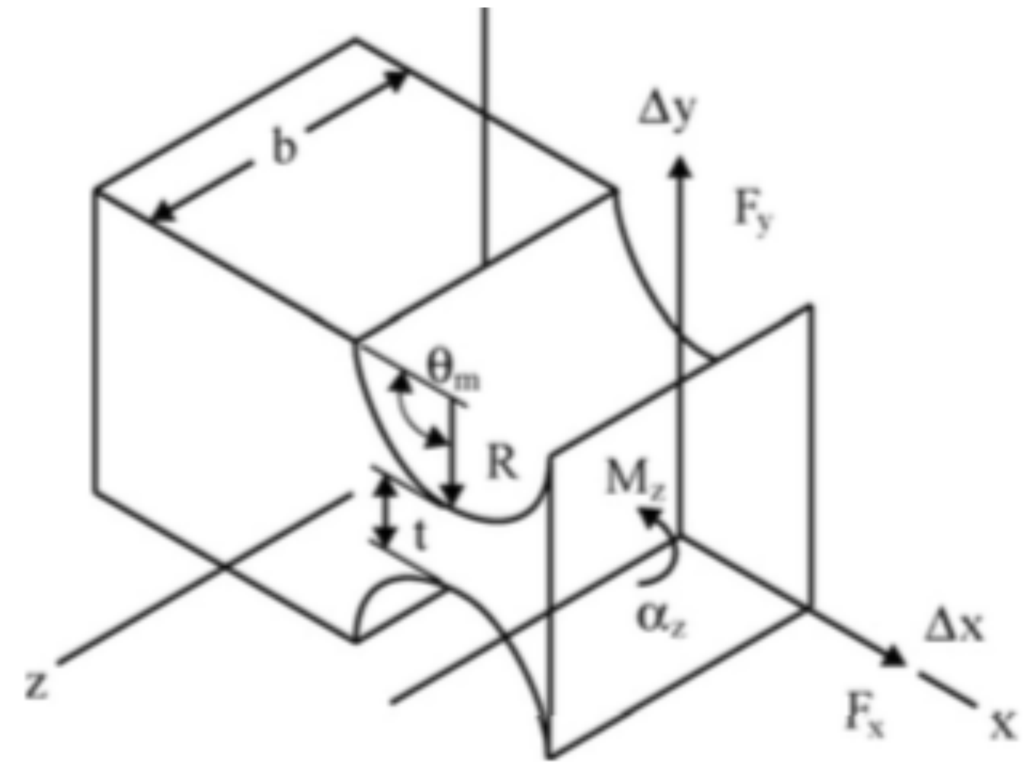

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Review of Circular Flexure Hinges

$$\frac{\alpha_z}{M_z} = \frac{24R}{Ebt^3(2R+t)(4R+t)^3}\left[t(4R+t)(6R^2+4Rt+t^2) + 6R(2R+t)^2\sqrt{t(4R+t)}\arctan\left(\sqrt{1+\frac{4R}{t}}\right)\right]$$

$$\frac{\Delta x}{F_x} = \frac{1}{Eb}\left[\frac{2(2R+t)}{\sqrt{t(4R+t)}}\left(\arctan\sqrt{1+\frac{4R}{t}} - \frac{\pi}{2}\right)\right]$$

$$\frac{\Delta y}{F_y} = \frac{3}{4Eb(2R+t)}\left\{2(2+\pi)R + \pi t + \frac{8R^3(44R^2+28Rt+5t^2)}{t^2(4R+t)^2} + \frac{(2R+t)\sqrt{t(4R+t)}}{\sqrt{t^5(4R+t)^5}} \times \left[-80R^4+24R^3t+8(3+2\pi)R^2t^2 + 4(1+2\pi)Rt^3+\pi t^4\right] - \frac{8(2R+t)^4(-6R^2+4Rt+t^2)}{\sqrt{t^5(4R+t)^5}} \times \left(\arctan\sqrt{1+\frac{4R}{t}}\right)\right\} \quad (A.10)$$

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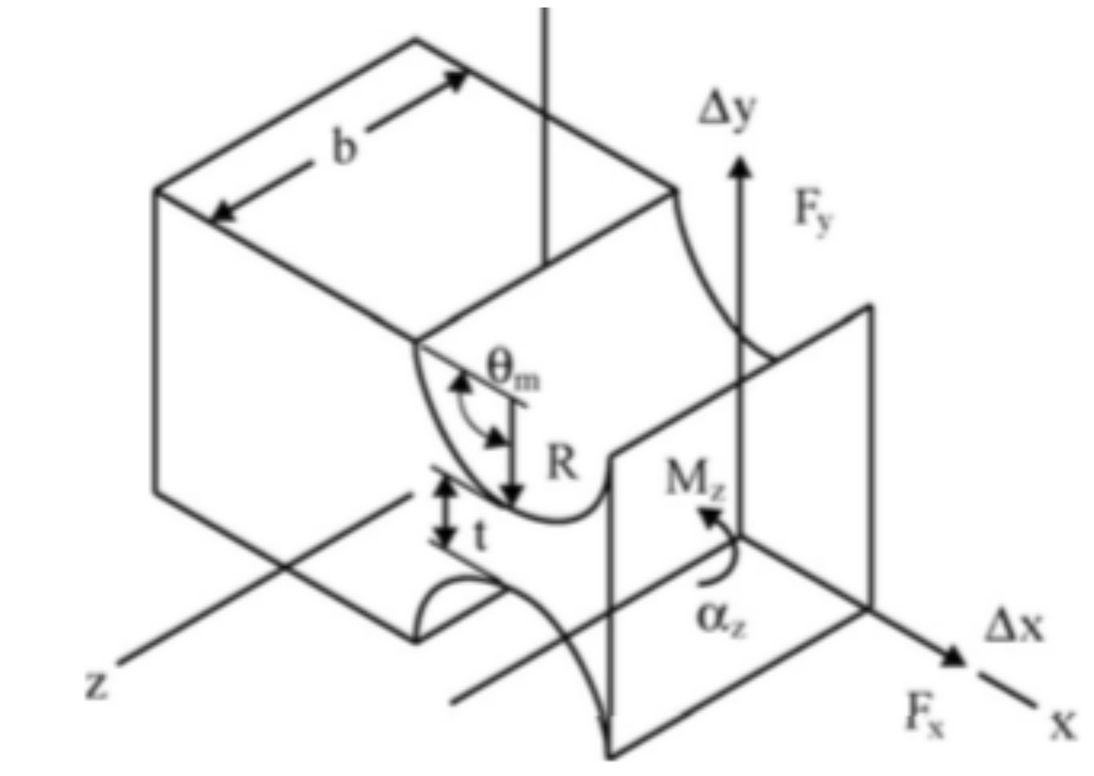

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Review of Circular Flexure Hinges

A.3. Wu and Zhou [10]

$$s = R/t$$

$$\frac{\alpha_z}{M_z} = \frac{12}{EbR^2}\left[\frac{2s^3(6s^2+4s+1)}{(2s+1)(4s+1)^2} + \frac{12s^4(2s+1)}{(4s+1)^{5/2}}\arctan\sqrt{4s+1}\right]$$

$$\frac{\Delta y}{F_y} = \frac{12}{Eb}\left[\frac{s(24s^4+24s^3+22s^2+8s+1)}{2(2s+1)(4s+1)^2} + \frac{(2s+1)(24s^4+8s^3-14s^2-8s-1)}{2(4s+1)^{5/2}} \times \left(\arctan\sqrt{4s+1}+\frac{\pi}{8}\right)\right]$$

$$\frac{\Delta x}{F_x} = \frac{1}{Eb}\left[\frac{2(2s+1)}{\sqrt{4s+1}}\arctan\sqrt{4s+1} - \frac{\pi}{2}\right]$$

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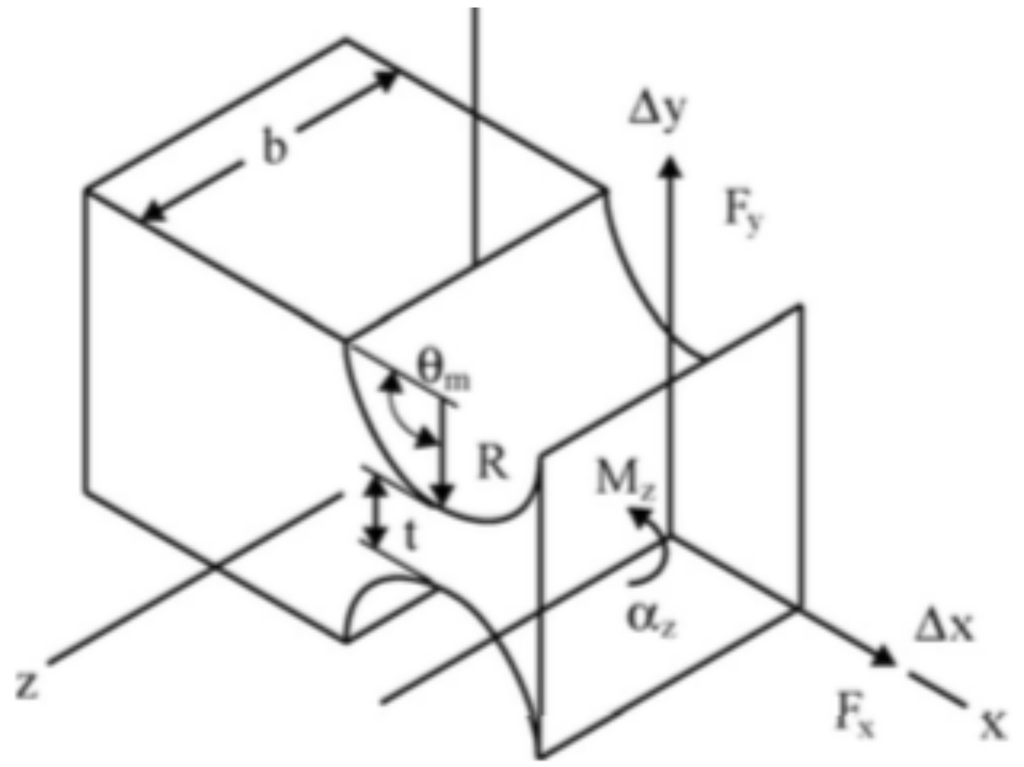

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Review of Circular Flexure Hinges

For thin circular hinges, $t/R \leq 0.07$

$$\frac{\alpha_z}{M_z} = 4\left\{1+\left[1+0.1986\left(\frac{2R}{t}\right)\right]^{1/2}\right\} \Big/ \left[Eb\left(\frac{t}{2}\right)^2\right] \quad \text{(A.17)}$$

The coefficient 0.1984 may be changed to 0.215 at angle $\theta_m \subseteq \pm 0.9$.

For intermediate circular hinges, $0.07 < t/R \leq 0.2$

$$\frac{\alpha_z}{M_z} = 4\left\{1+\left[1+0.373\left(\frac{2R}{t}\right)\right]^{1/2}\right\} \Big/ \left[1.45Eb\left(\frac{t}{2}\right)^2\right] \quad \text{(A.18)}$$

For thick circular hinges, $0.2 < t/R \leq 0.6$

$$\frac{\alpha_z}{M_z} = 4\left\{1+\left[1+0.5573\left(\frac{2R}{t}\right)\right]^{1/2}\right\} \Big/ \left[2Eb\left(\frac{t}{2}\right)^2\right] \quad \text{(A.19)}$$

If Poisson's ration $\nu \neq 0.333$, multiply α_z/M_z by the factor $(1-\nu^2)/0.889$

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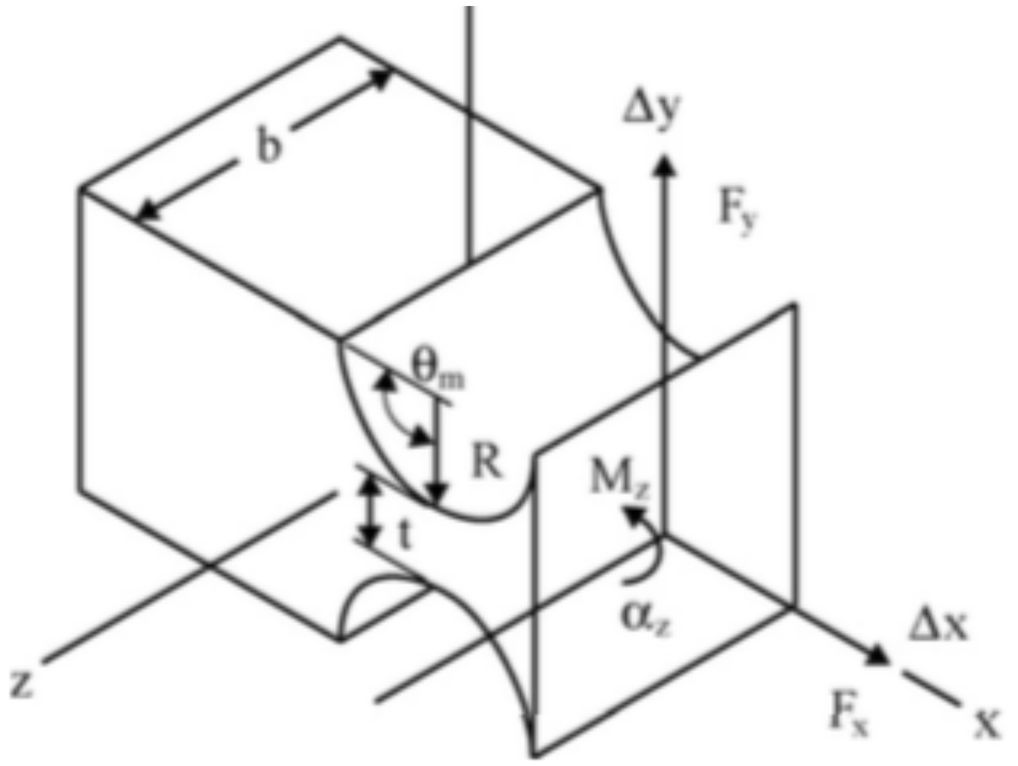
Abstract

This article presents the comparison of various compliance/stiffness equations of circular flexure hinges with FEA results. The limitation of these equations at different t/R (R is the radius and t is the neck thickness) ratios are revealed. Based on the limitations of these design equations, a guideline for selecting the most accurate equations for hinge design calculations are presented. In addition to the review and comparisons, general empirical stiffness equations in the x- and y-direction were formulated in this study (with errors less than 3% when compared to FEA results) for a wide range of t/R ratios ($0.05 \leq t/R \leq 0.8$).



Compliant Mechanisms (ME 851)

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Review of Circular Flexure Hinges

A.5. Smith et al. [12]

$$I_{zz} = 1/12bt^3$$

$$\frac{\alpha_z}{M_z} = \frac{(1.13t/R + 0.332)R}{EI_{zz}} \quad \text{(A.20)}$$

A.6. Schotborgh et al. [13]

$$\frac{\alpha_z}{M_z} = \left\{ \frac{Ebt^2}{12} \left[-0.0089 + 1.3556\sqrt{\frac{t}{2R}} - 0.5227\left(\sqrt{\frac{t}{2R}}\right)^2 \right] \right\}^{-1} \quad \text{(A.21)}$$

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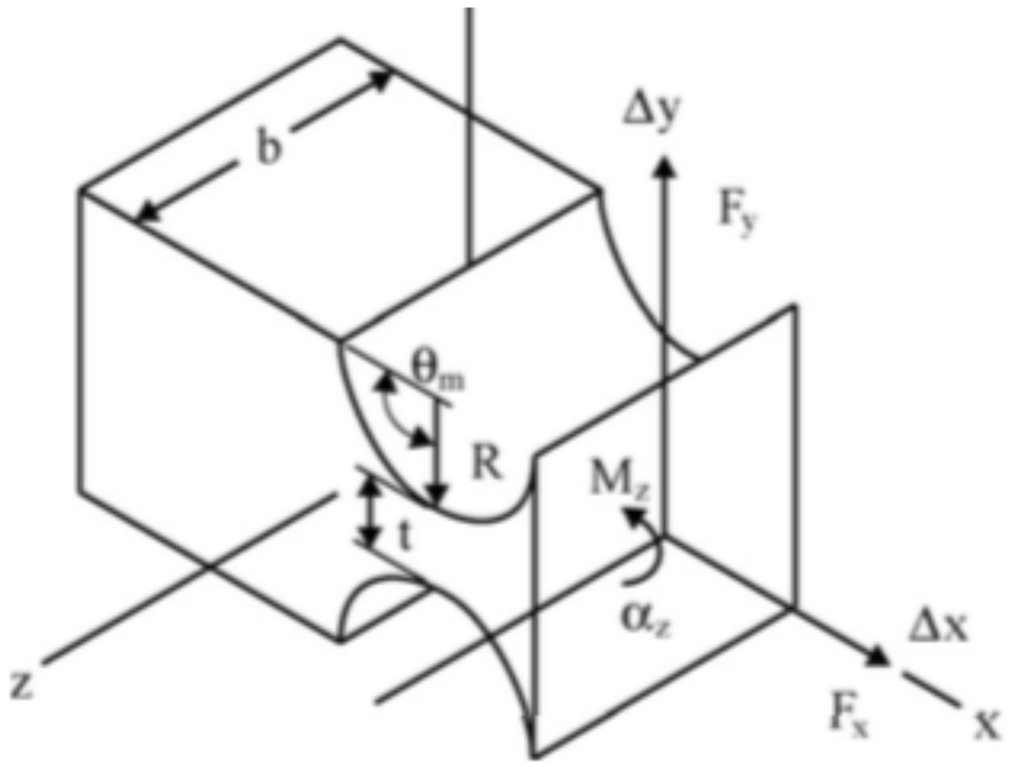

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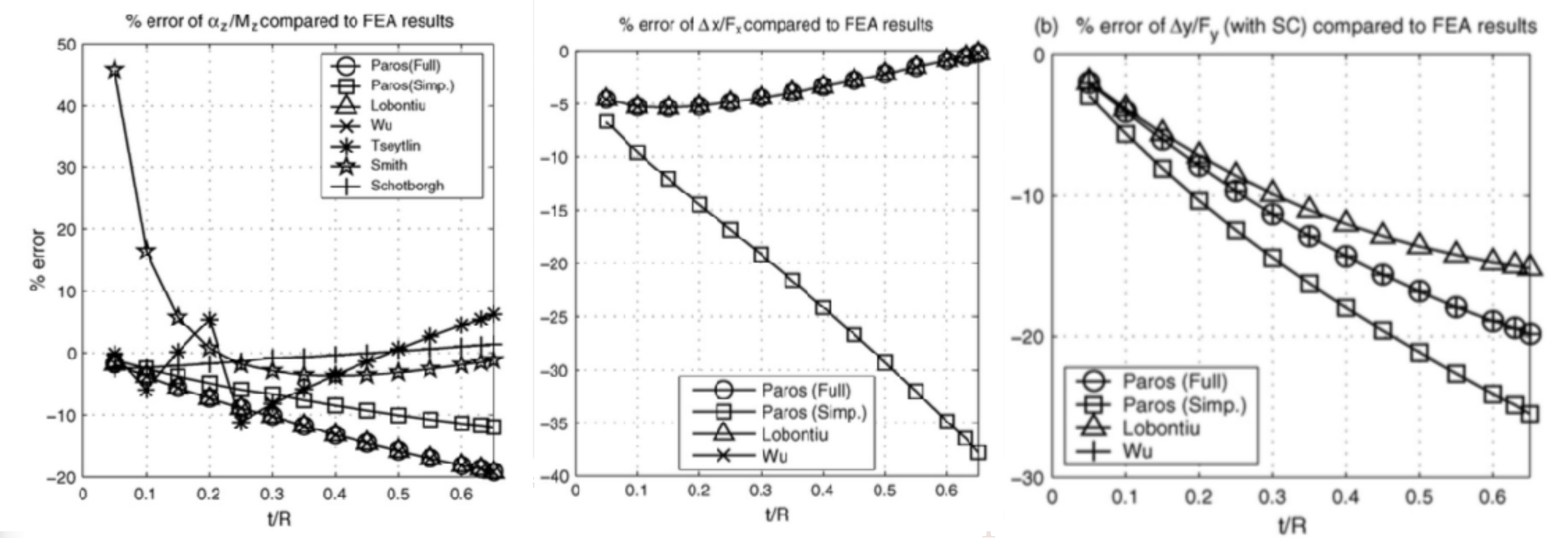

Errors could be quite HIGH in BOTH DIRECTIONS

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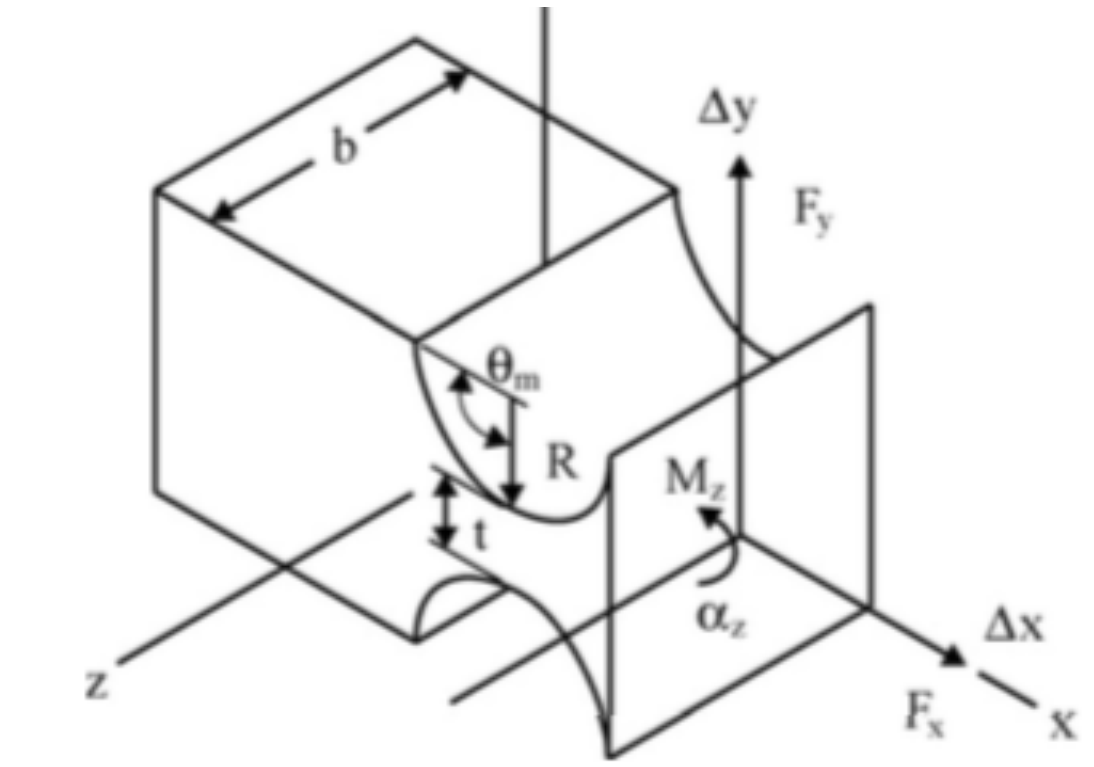

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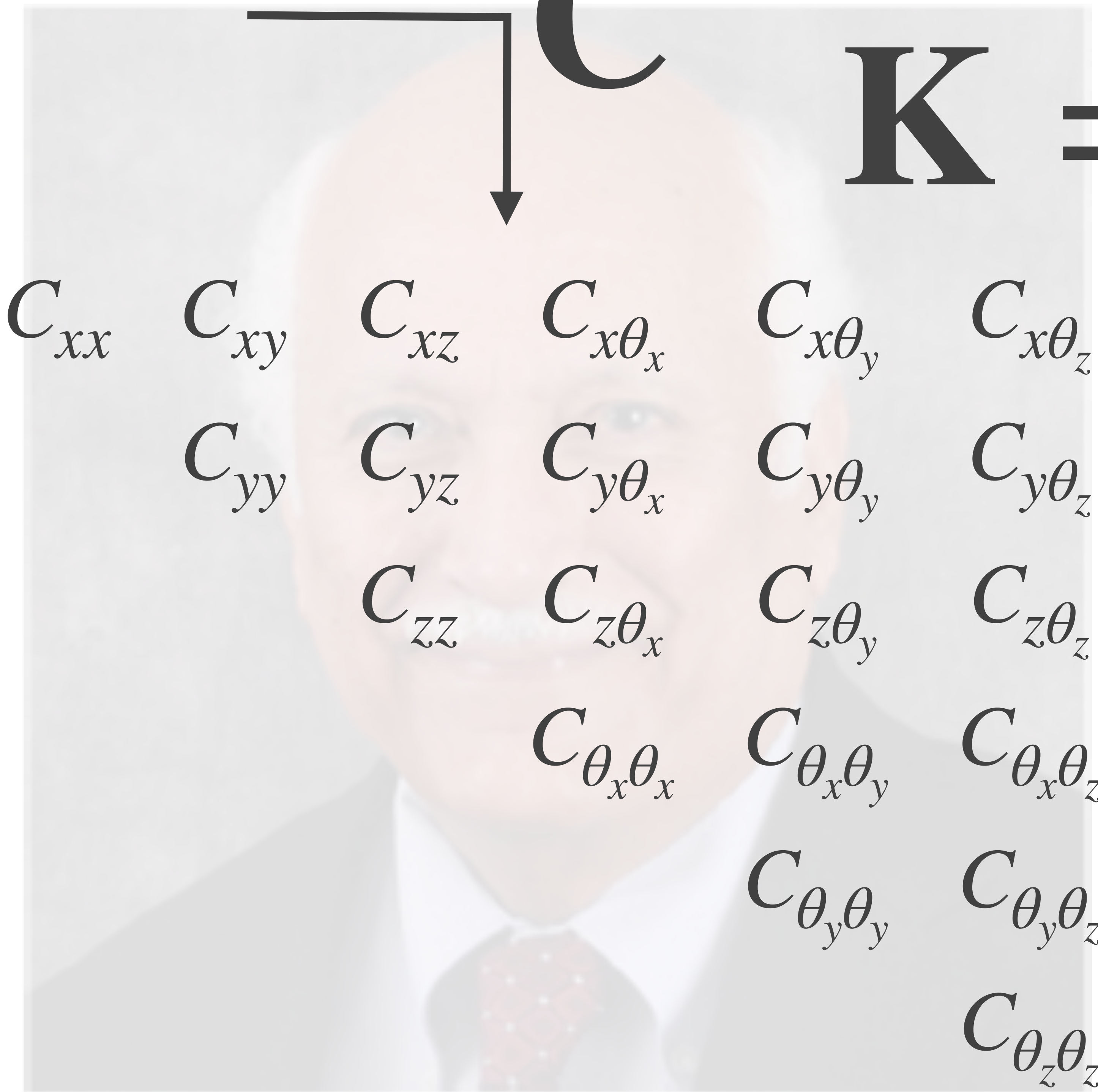
Typical process to design a notch flexure (?)

Compliance Matrix

$$\mathbf{C} \qquad \mathbf{K} = \mathbf{C}^{-1}$$



$$\begin{bmatrix} u_x \\ v_y \\ w_z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} C_{xx} & C_{xy} & C_{xz} & C_{x\theta_x} & C_{x\theta_y} & C_{x\theta_z} \\ & C_{yy} & C_{yz} & C_{y\theta_x} & C_{y\theta_y} & C_{y\theta_z} \\ & & C_{zz} & C_{z\theta_x} & C_{z\theta_y} & C_{z\theta_z} \\ & & & C_{\theta_x\theta_x} & C_{\theta_x\theta_y} & C_{\theta_x\theta_z} \\ & & & & C_{\theta_y\theta_y} & C_{\theta_y\theta_z} \\ & & & & & C_{\theta_z\theta_z} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix}$$

https://engineering.purdue.edu/ME/Seminars/2021/compliant-mechanisms-memory-lane-and-some-novel-and-exciting-applications/amidha.PNG

Numerical coefficients (small deformation) can be determined using FEA

Prof. Ashok Midha

Precision of Rotation   Stress Considerations

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