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A new hybrid vibration control methodology using a combination of magnetostrictive and hard damping alloys

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Abstract. A new hybrid damping technique for vibration reduction in flexible structures, wherein a combination of layers of hard passive damping alloys and active (smart) magnetostrictive material are used to reduce vibrations, is proposed. While most conventional vibration control treatments are based exclusively on either passive or active based systems, this technique aims to combine the advantages of these systems and simultaneously, to overcome the inherent disadvantages in the individual systems. Two types of combined damping systems are idealized and studied here, *viz*., the Noninteractive system and the Interactive system. Frequency domain studies are carried out to investigate their performance. Finite element simulations using previously developed smart beam elements are carried out on typical metallic and laminated composite cantilever beams treated with hybrid damping. The influence of various parameters like excitation levels, frequency (mode) and control gain on the damping performance is investigated. It is shown that the proposed system could be used effectively to dampen the structural vibration over a wide frequency range. The interaction between the active and passive damping layers is brought out by a comparative study of the combined systems. Illustrative comparisons with 'only passive' and 'only active' damping schemes are also made. The influence and the mode dependence of control gain in a hybrid system is clearly illustrated. This study also demonstrates the significance and the exploitation of strain dependency of passive damping on the overall damping of the hybrid system. Further, the influence of the depthwise location of damping layers in laminated structures is also investigated.

Keywords: hybrid vibration control; magnetostrictive material; hard coatings; active control; smart structure.

1. Introduction

Reduction of undesirable vibrations in structures is a continual challenge to structural engineers. While elimination of vibrations is impossible in most operating environments, efforts to reduce these vibrations are necessaryto obtain better fatigue performance, noise free environment, position control

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etc.. Since most structures have low inherent damping, damping is enhanced by the incorporation of materials with high damping capability into the structure. Damping techniques are classified as one of passive (energy absorbing or dissipating), active (blocking force generating) or hybrid (combined active and passive) damping techniques. Though the term ‘damping’ is not strictly applicable to active vibration control, for the sake of convenience in discussing the active and passive systems, in this paper, active vibration control is referred to as active damping. Conventionally, passive damping, especially using soft viscoelastic (VE) or hard ceramics having high damping characteristics, has attracted extensive attention (Johnson 1995, Chung 2001). The strain energy dissipation in these materials occurs due to hysterisis behaviour, arising out of different irreversible motions like polymer chain movements in VE polymers and dislocation movements and/or magnetic domain movements in hard damping alloys. More recently, smart materials like piezoceramics, magnetostrictive materials and shape memory alloys are being investigated for active vibration suppression. Though they possess some inherent energy dissipation capability, significant vibration reduction is usually obtained through actuation or blocking force generation (Tani, *et al*. 1998).

One of the most widely used vibration reduction techniques is the layered damping treatment. In this, a layer(s) of passive and/or active damping materials is usually either bonded to the host surface or introduced as a subsurface layer in laminated constructions. Constrained Layer Damping (CLD) is one such popular technique (Nashif, *et al*. 1985), wherein a soft VE material is sandwiched between the host and a stiff constraining layer for energy dissipation. By a suitable choice of VE and constraining material, layer thickness etc., high damping can be achieved. Another layered damping treatment is in the form of coatings of hard damping materials like alloys of Aluminum, Zinc, Magnesium and ferromagnetic materials like Fe-Cr-Al (Cochardt 1953, Giauque, *et al*. 1998). Smart composites (Lim, *et al*. 1999) where smart materials are embedded in a suitable matrix to give a layered construction are also being developed. These are used as intelligent actuating layers over the host structure to introduce distributed control of vibration. One such smart magnetostrictive composite is Terfenol-D, which has shown good potential for active damping (Lim, *et al*. 1999, Clark 1980). Tani, *et al*. (1998) have reviewed extensively the uses of such materials for vibration control.

Table 1 provides a brief overview of the relative advantages and disadvantages of some of the

Table 1 Overview of salient merits and demerits in some passive and active damping techniques

Damping Material		Advantages	Disadvantages
Passive Energy Absorbing Materials	Viscoelastic	• High Damping • Low weight Penalty • No external effort needed	• Unsuitable for very low and high temperatures, high frequencies • No controllability
	Hard Damping alloys	• High temperature, high frequency operating range • No external effort needed	• Relatively low damping • High weight penalty • High strain dependency • No controllability
Active vibration suppression materials	Peizoelectric, Magnetostrictive alloys or composites	• Controlled blocking force generation • High or Moderate temperature, high frequency operating range	• Low inherent damping in most operating range • External energy supply • Instability issues in control
	Shape Memory alloys	• Large strain applications	• Lower controllability • Low frequency bandwidth • Needs either stress or temperature induced phase transformation

common passive and active damping techniques. In view of these relative merits and demerits, hybrid damping techniques involving a combination of both active and passive treatments are being explored. Hybrid CLD (also called as Active CLD) techniques using piezoelectric materials has received considerable attention (Clark 1980, Baz 1993, Lam, *et al*. 1993, Agnes and Napolitano 1993, Huang, *et al*. 1996, Sadri, *et al*. 1997, Liu and Wang 2000). In this technique, in addition the VE damping, a peizoelectric actuating layer(s) bonded to the stiff constraining layer is used to achieve both sensing and actuation. This has resulted in enhanced vibration suppression over a wider frequency and excitation range and has overcome some of the drawbacks of active or passive alone treatments.

A survey of the existing literature revealed that most hybrid damping treatments are based on ACLD involving a VE damping material. Thus, the constraints associated with VE materials still exist (Table 1). Further, for flexible structures with low frequency high amplitude vibrations, magnetostriction based vibration control are found to be efficient (Kazuhiko, *et al*. 2000). It is also reported that they perform better compared to currently available peizoelectric material based control (Reddy and Barbosa 2000). A hard damping coating can be suitable to augment the active system. In view of these, a novel hybrid damping technique is proposed here.

In the proposed hybrid damping treatment, a layer(s) of hard passive damping material and a layer(s) of active magnetostrictive material are bonded to the host structure. Vibration reduction is achieved from both these layers. The hard damping layer with strain dependent damping provides passive damping, while the blocking force generated in the magnetostrictive material by incorporating a suitable control system yields active damping. The principal motivating feature of the proposed hybrid system is as follows. For flexible structures either the ‘passive only’ or the ‘active only’ system, cannot offer desired vibration reduction. Hence both passive and active damping systems are used for enhanced performance. However, the nett damping is not always given by the addition of the contributions from the individual systems. Depending on the strains in the passive layer before and after the actuation, the passive damping contribution can either increase or decrease, suggesting strong interaction between the two systems. In principle, it is possible to favourably exploit this interaction. In this scenario, the passive layer, in addition to providing damping improves controllability and reduces control effort to achieve desired damping levels. Also, the active damping, in turn, can be tuned to obtain enhanced passive damping, leading to much higher levels of nett damping. Though it is possible to obtain higher nett damping by opting for higher active damping, it is not always desirable since it requires more control effort and hence higher energy supply to the actuator.

The performance of the proposed damping system is evaluated by studying both metallic and laminated composite structures with hybrid damping. The latter are finding increasing applications as flexible structures due to their high specific strength and stiffness. Since they have low inherent damping, damping layers are incorporated to enhance structural damping. Usually these can be considered during the design of the laminated structure itself. Due to the nature of layered construction, they offer an additional flexibility of locating the damping layers at any location through the thickness.

Two configurations of the combined active and passive system are considered:

- Noninteractive system, wherein the individual damping elements viz., the active and passive layers actions are decoupled
- Interactive system, coupling between the damping elements

The Noninteractive system, though not realistic, represents a scenario wherein the passive damping layer is not influenced by the active damping system. In the interactive (or the truly hybrid) mode, the strain induced in the passive layer and in-turn its damping contribution are reevaluated after considering the effect of the active damping induced through the magnetostrictive layer. Apart from being more realistic, this also facilitates a study of the influence of the active control gain on the nett damping of the structure. The comparison of the behaviour of these two combined systems brings out the extent and influence of coupling between the two damping layers.

The performance of these two systems is evaluated by finite element analysis of beams treated with the proposed hybrid damping systems. A general formulation for the analysis of laminated beam is presented here. A six degrees of freedom (DOF) beam finite element, developed in an earlier work on smart composite beams (Bhattacharya 1997), is adapted. The steady state performance of various damping configurations in a cantilever beam is evaluated for different structural and control parameters. It may be noted that in the combined system due to strain dependent material damping in the passive layer, the behaviour is nonlinear. However, in the present study, the nonlinear effects are ignored and the influence of active damping is considered only once to update the strains in the passive layer and obtain the material loss factors, which are subsequently used to recompute the structural loss factor. This facilitates a preliminary study to understand the damping behaviour in combined systems.

2. Background theory

A schematic figure (Fig. 1) of a typical laminated composite beam with the ply lay-up along the *XZ* plane is given below. In general the beam is made up of *n* layers or plies along with a layer each of magnetostrictive material and passive damping alloy. The complex stiffness representation is used to incorporate the material damping in each layer.

Closely packed coils enclose the magnetostrictive layer through out the beam span and by driving the required current, a distributed active strain is generated in the magnetostrictive layer. Closed loop velocity proportional feedback control and distributed sensing are assumed. The present formulation for laminated beam models is also applicable to conventional metallic beam, as a special single layer case.

Using an equivalent single layer theory (Reddy and Barbisa 2000) to model the laminated beam, the displacement field at any location is expressed as:

$$U(x, y, z, t) = u(x, t) - yv'(x, t) \tag{1}$$

$$V(x, y, z, t) = v(x, t) \tag{2}$$

where '/' denotes differentiation with respect to *x*; *y*, the distance from the neutral axis, *u* and *v* are, respectively, the midplane displacements along *X* and *Y* axes. Assuming plane-strain conditions, the

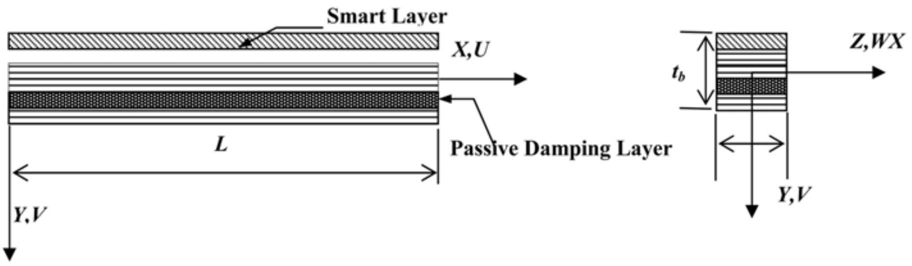


Fig. 1 A schematic of typical laminated beam with active and passive damping layers

total strain in any layer of the beam is given by:

$$\varepsilon_x^t = \varepsilon_x^e + \varepsilon_x^a \tag{3}$$

where, the subscripts *t*, *e* and *a* denote the total, elastic and active components, respectively. Following classical laminated beam theory, the total strain in terms of displacement becomes:

$$\varepsilon_x^t = u' - yv'' \tag{4a}$$

while the active strain can be expressed as (Liu and Wang 2000):

$$\varepsilon_x^a = d_M H(t) = d_M \frac{NI(t)}{\sqrt{l^2 + 4r_c^2}} \tag{4b}$$

where, d_M is the magneto-mechanical constant; N is the total number of turns in a specified actuation length l, and r_c is the equivalent coil-radius. Assuming a simple velocity proportional feedback with gain 'G', the coil current and the active strain may be farther expressed as

$$I(t) = G\dot{v}(t)$$

$$\varepsilon_x^a = C\dot{v}(t); \quad \text{where,} \quad C = Gd_M \frac{N}{\sqrt{l^2 + 4r_c^2}} \tag{5}$$

Substituting Eqs. (4a) and (5) into Eq. (3), the elastic strain in any layer of the composite laminate takes the form;

$$\varepsilon_x^{e_i} = u' - yv'' - \delta_i C\dot{v} \tag{6}$$

where, δ^i is 1 for the active layer and 0 for the others.

Following the complex modulus approach to model damping (Cochardt 1953), the constitutive relationship for different layers of the laminated beam is given by;

$$\sigma_x = E_m(1 + j\eta_m)\varepsilon_x \qquad \text{(magnetostrictive layer)} \tag{7}$$

$$\sigma_x = E_p(1 + j\eta_p)\varepsilon_x \qquad \text{(passive damping layer)} \tag{8}$$

$$\sigma_{x_i} = \overline{Q}(1 + j\eta_h)\varepsilon_x \qquad (i^{\text{th}} \text{ layer of the host laminate}) \tag{9}$$

where, η denotes the material loss-factor (Nashif, *et al.* 1985) and $\overline{Q}_{ij}$, the elastic constants of the layer.

3. Finite element formulation

The 6 DOF finite element model is shown in Fig. 2. The magnetostrictive layer in each element is assumed to be individually controllable and the cross-interference between the adjacent magnetic fields is neglected.

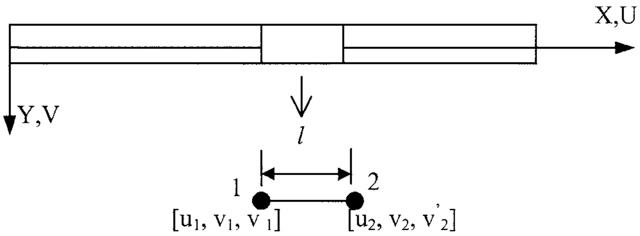


Fig. 2 Smart beam element

The displacement at any point inside the element may be expressed in terms of the nodal displacement as:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} F_1 & 0 & 0 & F_2 & 0 & 0 \\ 0 & H_1 & H_2 & 0 & H_3 & H_4 \end{bmatrix} \{q\} \tag{10}$$

or,

$$\{\tilde{u}\} = [A_q]\{q\} \tag{11}$$

where, the nodal displacement vector **q** is given by:

$$\{q\}^T = \{u_1 \ v_1 \ v_1' \ \ u_2 \ v_2 \ v_2'\}$$

and the polynomials,

$$F_1 = 1 - \xi; \ F_2 = \xi; \ \xi = x/l$$

$$H_1 = 1 - 3\xi^2 + 2\xi^2; \ H_2 = l(\xi - 2\xi^2 + \xi^3)$$

$$H_3 = 3\xi^2 - 2\xi^2; \ H_4 = l(-\xi^2 + \xi^3)$$

'l' denotes the element length.

The generalized displacement at any point on the cross-section of the beam may be expressed in terms of nodal DOF as

$$\bar{U} = \Gamma_1 \tilde{u} = \Gamma_1 A_q q = B_1 q \tag{12}$$

where the differential operator matrix Γ_1 is given by

$$\Gamma_1 = \begin{bmatrix} 1 & -y\dfrac{\partial}{\partial x} \\ 0 & 1 \end{bmatrix}$$

The total strain and the active strain in the element may also be represented as

$$\{\varepsilon_x^t\} = \Gamma_2 A_q q = B_2 q \qquad \Gamma_2 = \begin{bmatrix} \dfrac{\partial}{\partial x} & -y\dfrac{\partial}{\partial x^2} \end{bmatrix}$$

$$\{\varepsilon_x^a\} = CB_3\dot{q} \qquad B_3 = [0 \;\; 1] \tag{13}$$

Using the Hamilton's principle, the equation of motion is obtained as,

$$M\ddot{x} + C\dot{x} + Kx = F - R \tag{14}$$

$$M = A_b\int_0^1 B_1^T\rho B dx,\; C = \int B_2^T\overline{Q}_{11}CB_3 dV, K = \int B_2^T\overline{Q}_{11}B_2 dV,\; R = \int B_2^T\overline{Q}_{11}\varepsilon^i dV$$

where, '*M*' denotes the mass matrix, '*C*', the active damping matrix, '*K*', the stiffness matrix and '*F*' the force vector. Note that the stiffness matrix is complex and it has augmented hysteretic damping along with the structural stiffness produced by the ply and damping layers. '*R*' is the force vector generated corresponding to pre-strain in the elements. However, the present numerical analysis is carried out assuming zero pre-strain, and hence '*R*' is not considered. Eq. (14) corresponds to a generalised dynamic system, where the '*M*' is symmetric positive definite; '*C*' is asymmetric and '*K*' is complex and symmetric. Moreover, since the structural damping produced by the passive coating is a function of strain, stiffness matrix needs to be recomputed when there is a change in strain in the elements. For a harmonic forced excitation of magnitude *F* sinw *t*, Eq. (14) could also be expressed in frequency domain as:

$$-\omega^2\mathbf{M}x(s) + j\omega\mathbf{C}x(s) + \mathbf{K}x(s) = \mathbf{F}(s)$$

or,

$$x(s) = [K + j\omega C - \omega^2 M]^{-1}F(s) \tag{15}$$

A flow-chart for evaluating the damping performance using frequency domain analysis (FRF) is given in the Appendix.

4. Results and discussion

A cantilever beam ($L/t = 100$; $L/b = 10$) with the following material properties (Table 2) is chosen for the present study. A mesh with twenty elements is found to yield acceptable convergence for the first five modes. A forced harmonic excitation of known amplitude is applied at $x/L = 0.1$ from the fixed end. As a first step, the natural frequencies and the mode shapes are obtained for the first five bending modes. These results are used to excite the beam at natural frequencies and obtain the displacement and strains for the given excitation, as a function of various structural and control parameters. Also, a Frequency Response Function (FRF) to the harmonic excitation within the desired frequency range is obtained. The damping obtained in terms of the structural loss factor is computed from the Nyquist plots using the data from FRF analysis.

The performance of individual active and passive systems and the combined systems are first evaluated in the case of a metallic cantilever beam with the passive layer over it and the magnetostrictive layer as the outermost layer. The ratio of damping layer thickness to that of beam thickness (t_p/t) is set to 0.1 since the normally used range in practice is 0.05 to 0.3. The magnetomechanical constant ***d*** for Terfenol-D is taken to be 1.67e-08 m/A and the coil constant for the magnetizing coil, to be 10,000 turns/m. The

Table 2 Geometric and material properties of the beam

Material	Elastic Modulus (GPa)	Density (Kg/m^3)	Poisson's Raito, ν	Loss-Factor η
Aluminium	70	2700	0.33	0.001
Terfenol-D	35	9250	0.25	0.001
Fe-Cr-Al Alloy	180	7000	0.28	(Fig. 3)
CFRP Ply	E_{11} = 130; E_{22} = 10	1600	0.35	0.0025

parameters of interest are the excitation level (magnitude of F), control gain (G) and mode number or frequency. To get a better understanding of the performance of combined systems, cases with only passive or active contribution are also considered.

4.1. Passive damping system

For the current simulations, a typical hard damping alloy similar to Fe-Cr-Al alloy (Jones 1975) with strain dependent damping (Fig. 3) is considered. It is evident that there is a high strain dependency with a peak loss factor of about 0.1 in a narrow range of strain (between 70-100 micro-strain). This offers a scope to choose appropriate values of control gain to achieve maximum damping by tuning the strain induced in the passive layer. In order to find out the variation in damping as a function of strain at a given frequency, excitation levels of 1 N, 10 N and 100 N are chosen.

Fig. 4(a) illustrates the damping levels in the passive system. Typical Nyquist plot of the receptance FRF corresponding to the first mode of vibration and modal loss corresponding to a forced excitation of amplitude = 10N is presented in Fig. 4(b). For clarity, only the total damping for I, III and V bending modes are shown. Since $G = 0.$, the net damping is from the passive layer only. An increase in the excitation level from 1 N to 10 N has resulted in higher damping in all the three modes shown. However, at higher strains due to 100 N excitation there is a considerable decrease in damping. It may also be observed that at low excitation (1 N), the damping is more or less uniform in all the modes, whereas at higher excitation there are noticeable differences in the levels of damping across the modes. Because of its strain dependence, the material loss factor distribution in the passive layer varies along the length and with excitation levels. This is reflected clearly in the overall damping in the structure.

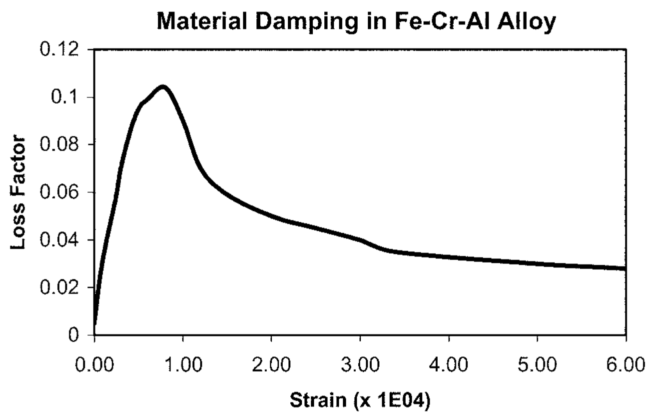


Fig. 3 Material loss factors in passive damping materials as a function of strain

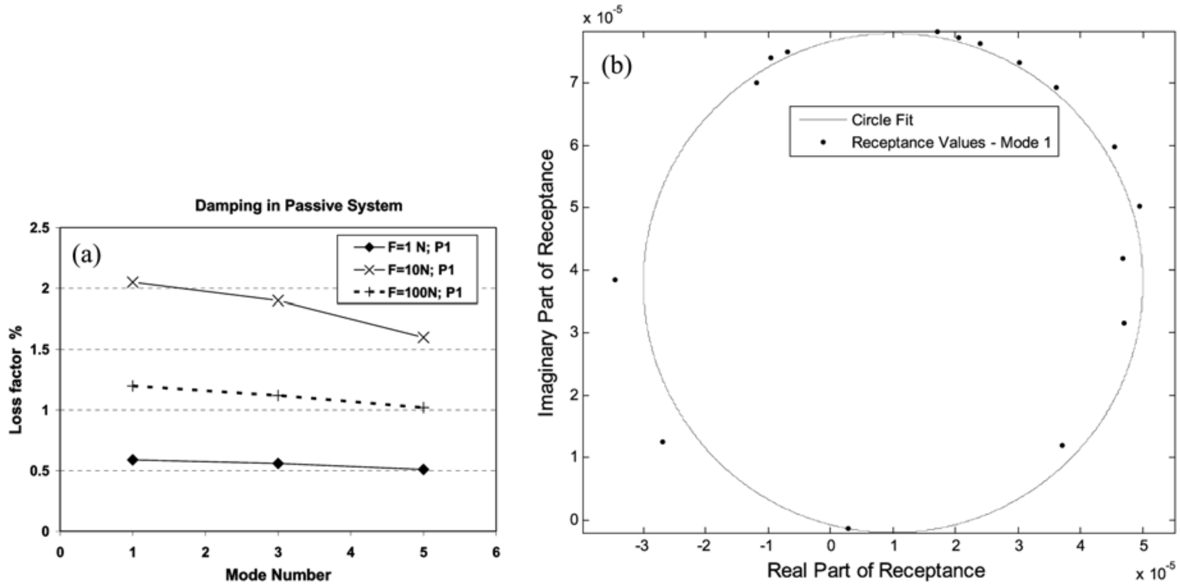


Fig. 4 (a) Passive damping at different excitation levels, (b) A typical Nyquist Plot for obtaining the first modal damping corresponding to a forced excitation of amplitude 10 N

4.2. Active damping system

By setting *a priori*, the damping in the passive layer to zero, the performance of the 'Active System' is evaluated. In this case the total damping from the system is from the active layer only. With a change in the control gain (G), the current passing through the magnetizing coils varies, changing the magnetic field around the Terfenol-D layer, thereby imparting different levels of actuation. Appropriate choice of the values of G leads to the reduction in levels of vibration. The damping from the 'Active system' for typical values of G from 1 to 10 is presented in Fig. 5. As expected, with the increase in control gain G, the damping values increase monotonically in all the modes. Also, at low gains, the damping levels in different modes are less mode sensitive. Also the amount increase in the damping is higher as compared to passively damped system. Though very high damping is noticed with $G = 10$, this corresponds to a high control effort which is not usually desirable.

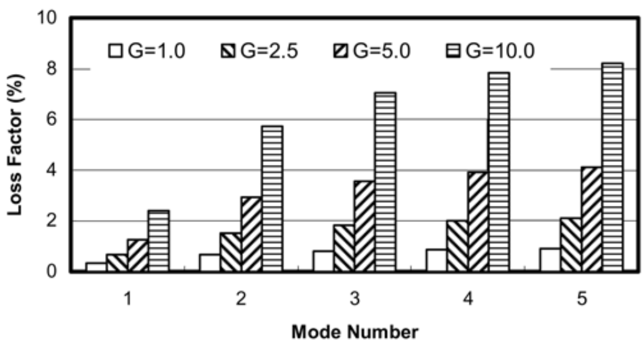


Fig. 5 Loss factors in active only system for different control gains

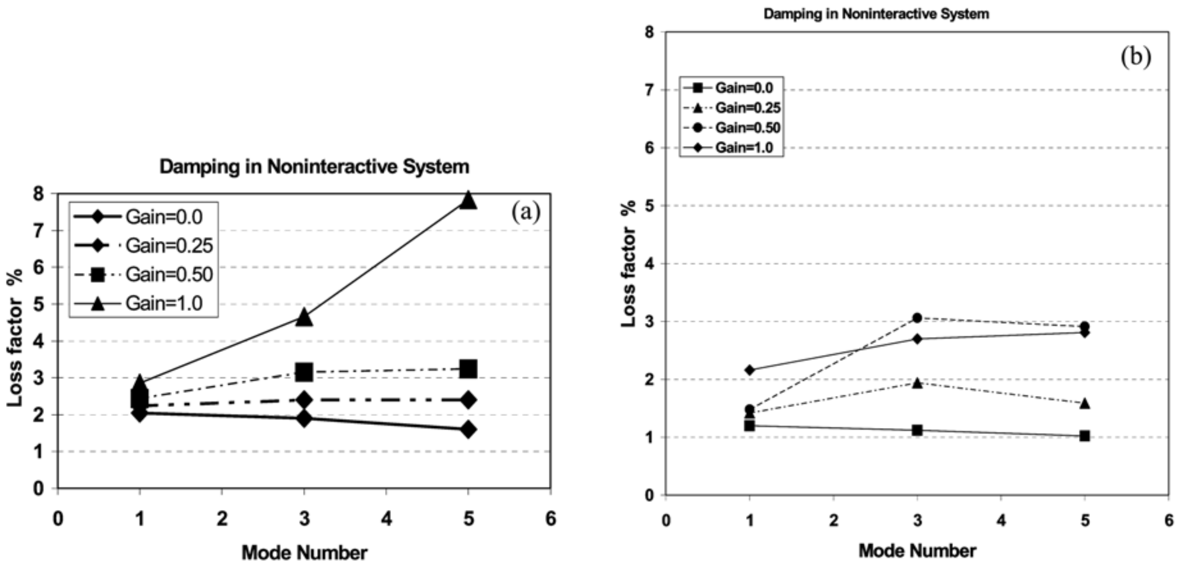


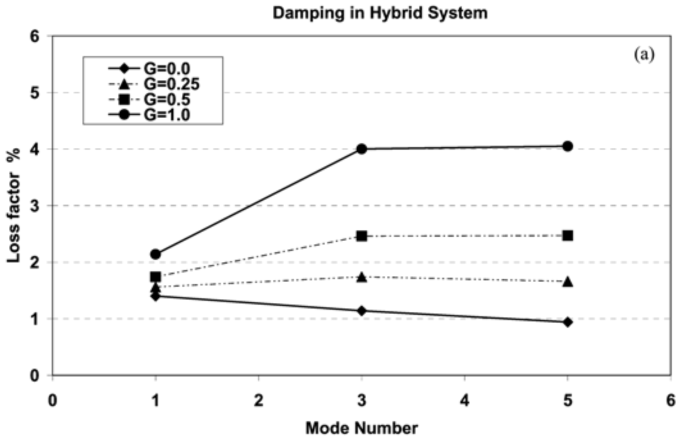
Fig. 6 (a) Loss Factors in a cantilever subjected to sinusoidal excitation of amplitude 10 N with noninteractive system, (b) Loss Factors in a cantilever subjected to sinusoidal excitation of amplitude 100 N with noninteractive system

4.3. Combined active and passive damping

In a further study, both active as well as passive damping is considered for the same beam model. In the Noninteractive system, the passive damping is computed based on the undamped strain level in the beam while the active damping is computed assuming a control gain. However, in the Interactive system, passive damping contribution is updated by re-computing the strains in the passive layer after considering the effect of active damping. The parameters of interest are the levels of excitation and control gain. The damping in the first five modes is computed corresponding to the harmonic excitations of magnitudes 10 N and 100 N. In order to limit the control effort, a low range of G ($0 \leq G \leq 1$) is used. Further, the efficacy of the hybrid system will be fully exploited only when comparable damping is achieved even with much lower gain values compared to an active system. It may be recalled that the case $G = 0$ correspond to 'passive only' system. The results for the two combined systems are shown in Figs. 6 and 7. η_P is the loss factor due to passive layer, and η_N and η_H are, respectively, total loss factor in Noninteractive and Interactive systems. Fig. 6 gives the net damping in noninteractive system (η_N) and Fig. 7 presents the net damping in interactive system (η_H).

4.3.1. Noninteractive system

As expected, the Noninteractive system performs better than the individual systems. From Fig. 6a, it can be seen that increase in gain results in increased damping in all the modes, with the amount of increase being mode dependent. For example, if the gain is doubled from 0.5 to 1.0, the corresponding increase in damping in modes III and V are significantly different. When the excitation level is increased to 100 N, compared to earlier case (10 N) there is a large reduction in damping, especially in higher modes (Fig. 6b). Another interesting observation is that at higher modes, there is much less increase in damping as the gain is increased. This is attributed to the significant reduction in passive damping contribution, as seen earlier.



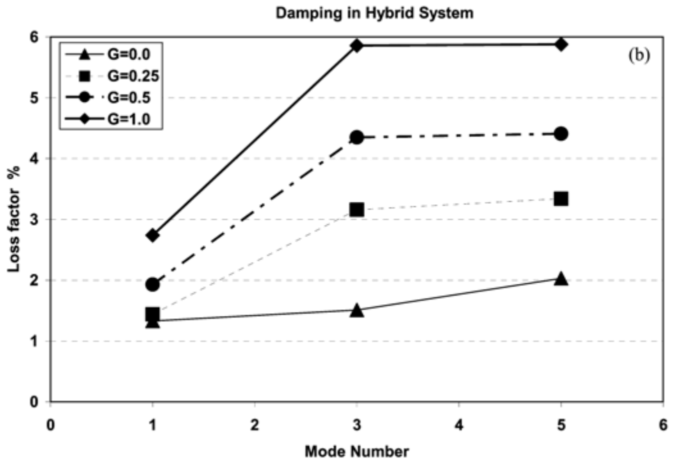


Fig. 7 (a) Damping in Interactive system with $F = 10$ N, (b) Damping in Interactive system with $F = 100$ N

4.3.2. Interactive system

The performance of the Interactive system is depicted Fig. 7. As expected, it is observed that for a given mode, increase in gain leads to higher overall damping. However, when contrasted with the behaviour of Noninteractive system (Fig. 6), following salient differences become evident. In the case of 10 N excitation amplitude, the damping in Interactive system is much less compared to Noninteractive system, with maximum decrease in damping in Mode-V (upto nearly 50%). In the specific instance of damping with Gain = 1.0, it is also apparent that while there is considerable increase in damping in Noninteractive system, Interactive system gives more or less same damping levels in Mode III and IV (Figs. 6a and 7a). This suggests that in this case, the Interactive system behaviour is dominated by the active layer (Fig. 5). However, at higher excitation level (100N), the Interactive system performs much better compared to the Noninteractive system (Figs. 6b and 7b). Only in the case of Mode I, these systems perform in a similar manner for all gains and excitation levels.

A better understanding of this contrasting behaviour is obtained by examining the passive damping contribution in the Interactive system. Fig. 8 shows the same in Interactive system for 10 N and 100 N excitation levels. Table 3. gives the strains in the passive layer with corresponding passive damping

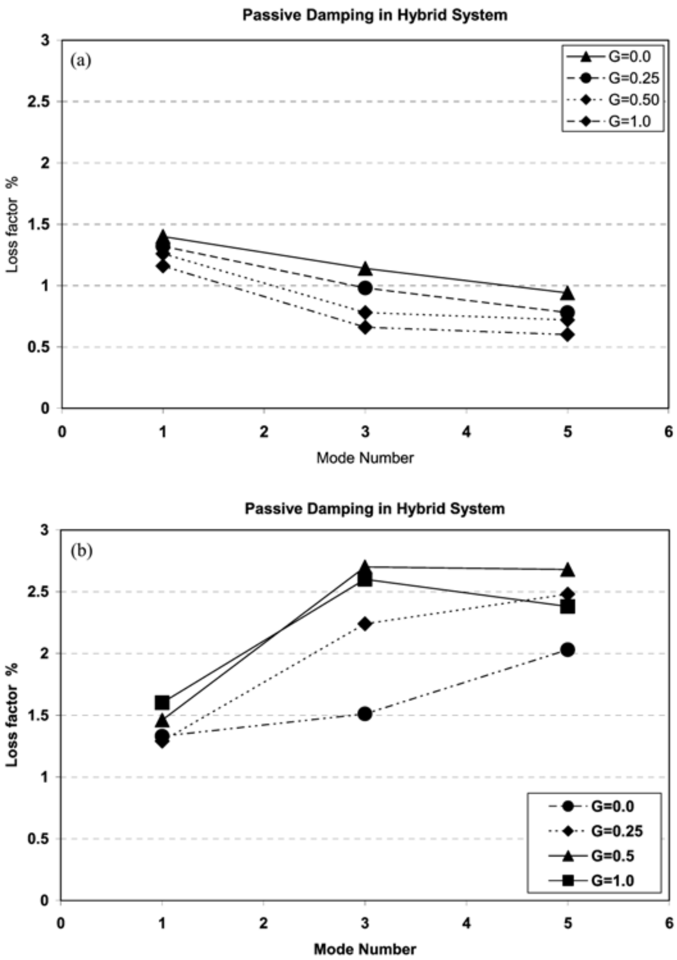


Fig. 8 (a) Passive damping contribution in Interactive system with F = 10 N, (b) Passive damping contribution in Interactive system with F = 100 N

Table 3 Strains in the passive layer for the cantilever with Interactive system; F = 100N

Control Gain	Mode-1		Mode-3		Mode-5	
	$\varepsilon_u \times 10^{4\ (\mathrm{Undamped})}$ 6.478		$\varepsilon_u \times 10^{4\ (\mathrm{Undamped})}$ 5.034		$\varepsilon_u \times 10^{4\ (\mathrm{Undamped})}$ 3.67	
G	$\varepsilon^o_p \times 10^4$	η_p (%)	$\varepsilon^o_p \times 10^4$	η_p (%)	$\varepsilon^o_p \times 10^4$	η_p (%)
0	4.159	1.33	2.689	1.51	1.538	2.03
0.25	3.927	1.30	2.289	2.24	1.395	2.48
0.5	3.504	1.46	1.753	2.70	1.07	2.68
1	2.854	1.60	1.184	2.60	0.729	2.38

values for F = 100 N. It is clearly seen that at lower excitation (10 N), passive layer contribution is reduced at all gains and modes. Further, an increase in gain leads to greater reduction in passive damping. At 100 N excitation level, this trend is, in general, reversed. Also, it is interesting to note that for Mode V, an increase of gain from 0.25 to 0.5 increases passive damping, while an increase from 0.5

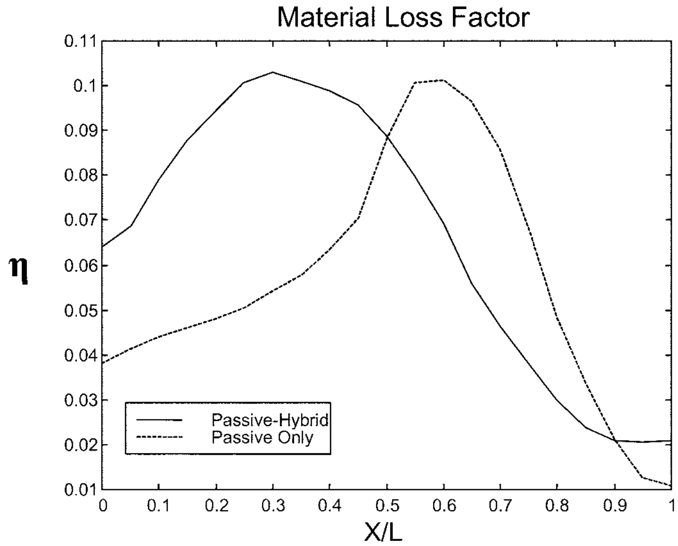


Fig. 9 Variation of the material loss factor η of the passive layer along the length of the beam

to 1 results in decreased performance (Table 3).

From these observations, it is obvious that strain dependency significantly alters the behaviour in the combined systems and that control gain influences the passive layer contribution significantly (Fig. 8). Hence, control gain can be an important parameter in the behaviour of the Interactive system. Fig. 9. illustrates the distribution of material loss factor in the passive layer in Mode-I before and after actuation. It may be noted that the passive damping contribution is a function of the area under this curve. This provides an added evidence of the amount of coupling between the two damping layers.

The efficacy of the proposed hybrid damping technique is clearly established by comparing the performance of the 'Passive only', 'Active only' and the Interactive system. It may be noted that the loss factors in 'Active only' system with G = 1.0 are less than 1%. Also, the loss factors in 'Passive only' system are less than 2.1% (Mode-I) for 10 N and less than 1.3 % (Mode-I) for 100 N excitation levels. However, the maximum loss factor in the proposed Interactive or hybrid system is around 4% (Mode III and V) for 10 N and around 6% (Mode III and V) for 100 N excitation levels. Hence the damping in proposed system is better than the cumulative damping in the individual systems. Fig. 10 illustrates the comparative performance of different systems in Mode-I. The top graph provides the mode shape and the bottom graph provides the strain in the passive layer in different systems. The change in strain in the passive layer would affect the net damping performance of the system.

4.4. Hybrid damping of laminated beam

A laminated cantilever beam with L/b = 10 and L/t = 100 is chosen to study the influence of hybrid control on laminated structures. The host structure is a carbon fiber reinforced epoxy laminate with 11 plies of 0° fibre orientation, with material properties as given in Table 2. A layer each of passive damping material (P) and active Terfenol-D (A) are included to obtain combined damping. One of the main advantages in laminated constructions is that the depthwise position of the damping layers could be varied. This alters the strain experienced by the passive layer and thereby its damping performance.

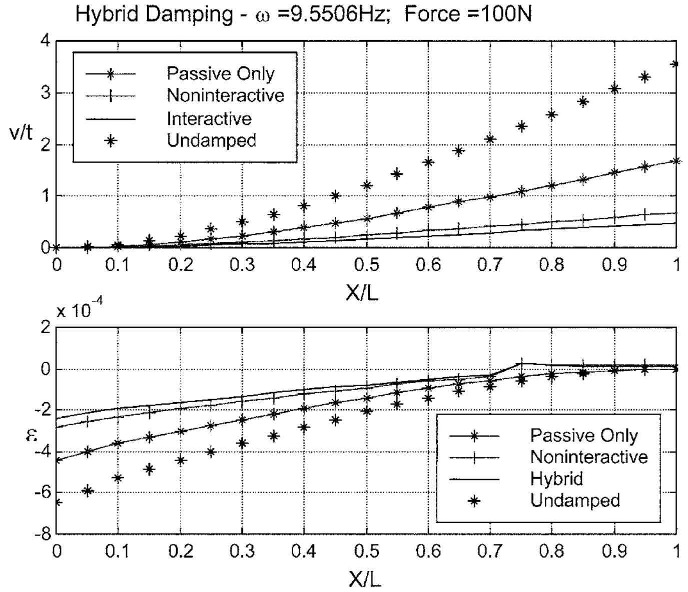


Fig. 10 Steady state response of the cantilever beam in Mode-I with different damping systems

Figs. 11a-c provides the FRF of a $[0_{10}/A/P]$ with Interactive system and $F = 100$ N and $G = 0.5$. Figs. 11a and b show the response in the first and third bending modes from different damping systems. It is clear that the relative contribution from the active and passive layers are different in these modes. Also there is significant damping in all five modes considered (Fig. 11c).

4.5. Ranking of stacking sequences for damping

The influence of placement of the damping layers on damping is investigated by considering different stacking sequences. Percentage loss factors in Mode-I for several configurations at a force level of 100 N and control gain of 0.5 are given in Table 4. From these results, it is evident that the level of damping differs significantly (upto 100%) in different arrangements. Also, with the present excitation level and mode, placing the passive damping layer as the outermost layer yields best results. Due to low active damping contribution in Mode-I, maximizing the passive contribution by appropriately locating 'P' assumes significance.

From Table 4, it appears that $[0_{10}/A/P]$ is the best for Mode-I. A similar study for Mode-III and Mode-V with two different gain levels is carried out. The stacking sequences are ranked based on net damping performance and the results are given in Table 5. The structural loss factors are given alongside the sequences. Several important observations can be made from these results (Tables 4,5). There is no single stacking sequence that is best in all the modes and is independent of gain. Though $[0_{10}/A/P]$ is the best for $G = 0.5$ in all the modes, an increase in G leads to a different combination, viz., $[0_{10}/P/A]$. This is due to increased active contribution and its influence on passive damping due to higher G. Also, lower ranked stacking sequences exhibit both mode and gain dependency. The choice of the placement of 'P' and 'A' depends on various factors, which could even give rise to conflicting requirements due to strong interaction between passive and active damping layers. Further, in practice, it might be desirable to separate 'P' and 'A' due to other design requirements like structural integrity etc..

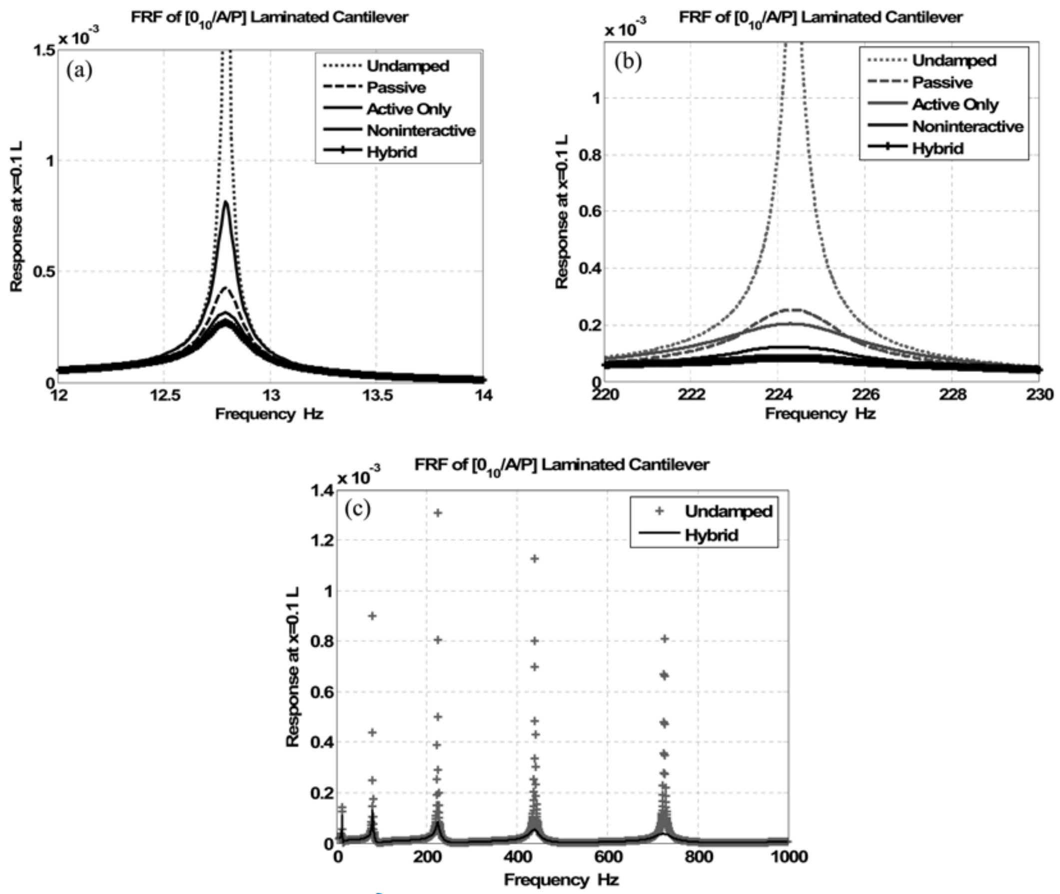


Fig. 11 (a) Frequency response in the [010/A/P] cantilever; *F* = 100 N, Gain = 0.5, showing different levels of damping from different systems in First bending mode, (b) Frequency response in the [010/A/P] cantilever; *F* = 100 N, Gain = 0.5, showing different levels of damping from different systems in third bending mode, (c) Frequency response in the [010/A/P] cantilever; *F* = 100 N, Gain = 0.5 showing different damping in hybrid systems for the first 5 modes

Table 4 Influence of damping layer location on damping in interactive system; $G = 0.5$; $F = 100$ N

Damping %	$[0_{10}/A/P]$	$[0_9/A/0/P]$	$[0_{10}/P/A]$	$[0_8/A/0_2/P]$	$[0_9/A/P/0]$	$[0_6/A/0_4/P]$	$[0_9/P/0/A]$	$[0_9/P/A/0]$
η_p	2.04	1.81	1.66	1.92	1.34	1.4	1.15	1.06
η_h	2.33	2.30	2.26	2.16	1.74	1.62	1.62	1.52

5. Conclusions

A new combined active and passive damping technique for hybrid vibration control was proposed. In the proposed system, the active damping is realized through a layer of magnetostrictive material (Terfenol-D) while strain dependent hard alloys were used as passive damping layers. Two types of combined damping systems, viz., Noninteractive and Interactive systems, are studied. Numerical simulations using a 6 DOF smart beam element were carried out to evaluate the proposed hybrid system *vis a vis* different damping scenarios. Structural loss factors for the first five modes of

Table 5 Rank of different stacking sequences based on damping performance

Rank	gain =0.5		gain = 1.0	
	Mode-III	Mode-V	Mode-III	Mode-V
1	$[0_{10}/A/P]$ 3.66	$[0_{10}/A/P]$ 3.46	$[0_{10}/P/A]$ 4.80	$[0_{10}/P/A]$ 4.90
2	$[0_{10}/P/A]$ 3.42	$[0_{10}/P/A]$ 3.32	$[0_{10}/A/P]$ 4.64	$[0_{10}/A/P]$ 4.56
3	$[0_9/A/0/P]$ 3.28	$[0_9/A/0/P]$ 3.06	$[0_8/A/0_2/P]$ 4.18	$[0_9/P/0/A]$ 4.52
4	$[0_8/A/0_2/P]$ 2.90	$[0_9/P/0/A]$ 2.82	$[0_9/A/0/P]$ 4.00	$[0_9/A/0/P]$ 3.90
5	$[0_9/P/0/A]$ 2.82	$[0_8/A/0_2/P]$ 2.70	$[0_9/P/A/0]$ 3.50	$[0_9/P/A/0]$ 3.72
6	$[0_9/A/0/P]$ 2.64	$[0_9/A/0/P]$ 2.52	$[0_8/A/0_2/P]$ 3.42	$[0_8/A/0_2/P]$ 3.42
7	$[0_9/P/A/0]$ 2.45	$[0_9/P/A/0]$ 2.41	$[0_9/A/0/P]$ 3.40	$[0_9/A/0/P]$ 3.38
8	$[0_6/A/0_4/P]$ 2.20	$[0_6/A/0_4/P]$ 2.15	$[0_6/A/0_4/P]$ 2.34	$[0_6/A/0_4/P]$ 2.24

vibration were obtained using a frequency response study on both metallic and laminated cantilever beams. Further, keeping in mind the strain dependent damping of the passive layer, the possibility of tuning the active control gain to extract high levels of damping from passive layer was investigated. Results clearly indicate that proper choice of gain depends on both the mode and the amplitude of vibration, which determine the strain in the passive layer. Also, in the case of laminated structures, the depth-wise location of the passive damping layer was shown to have significant influence on the damping. For control of vibrations in multiple modes with a variable control gain, it may be necessary to carry out an optimization in terms of the control gain and the placement of the damping layers. From the present study it can be concluded that the proposed hybrid damping technique performs better than the individual 'passive only' and 'active only' systems and provides high damping in all the five bending modes investigated.

It may be mentioned here that there are several challenges in the practical development of the hybrid system. Real time control of the strain in passive layer to tune its performance through the gain in the active layer is one of the prime challenges. Further, the embedding of the active and passive layers in appropriate depth-wise locations to optimise the damping performance is another important aspect while realizing such a system. The authors feel that with recent developments of miniature local active damping systems and good control algorithms for real time control, the hybrid system conceptualized above could become viable. A separate study could be carried out to investigate the influence of number of active elements on the damping behavior.

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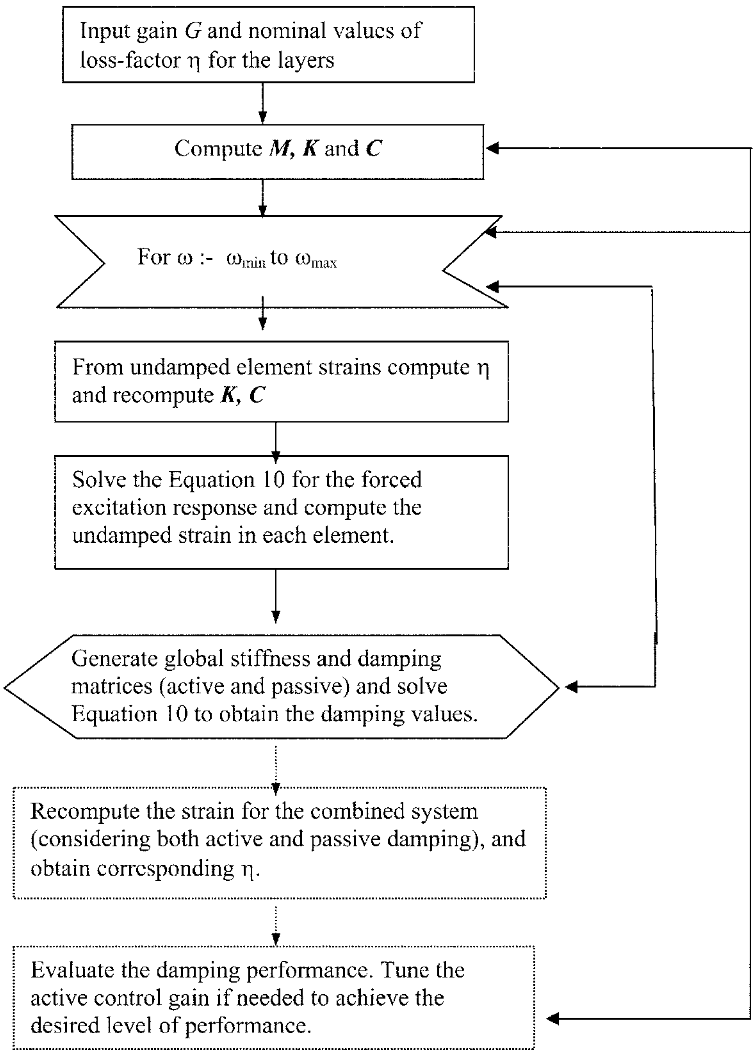
Notation

C	: a function of control gain and magnetomechanical constant
$\mathbf{C}$	: active damping matrix
$\mathbf{E_m}$	: modulus of elasticity of the magnetostrictive material
$\mathbf{E_p}$	: modulus of elasticity of the passive damping layer
F	: forced excitation signal
G	: control gain
$\boldsymbol{K}$	: complex stiffness matrix
L	: length of the beam
$\boldsymbol{M}$	: mass matrix
Q	: elastic constants for the laminated composite
R	: element force vector corresponding to the initial strain in the elements
U, V	: displacements along x and y directions respectively

t, b : thickness and width of the cantilever beam respectively
η : material loss-factor
η_{H} : total structural loss-factor in the beam for hybrid system
η_p : passive contribution to the structural loss-factor in the beam for hybrid system
σ : stress in any layer
ν : poisson's ratio
ε_x^t : total strain in the beam at any point along x direction
ε_x^a : active strain in the beam at any point along x direction
ε_x^e : elastic strain in the beam along x direction

Appendix



Flowchart of the procedure to estimate damping in the hybrid system