Homework-3:

Question 1: M/G/1

Consider the M/G/1 system with the difference that before the first customer in each busy period starts service, it takes some time for setup of service from idle state. (This may arise in data networks where the first packet in the sequence may need routes to be setup and hence takes additional time.) Let this setup time be represented by the random variable Δ. We assume that Δ has a given general distribution and is independent of all other random variables in the model. Let $\rho = \lambda\overline{X}$ be the utilization factor. Further, assume that while setup is in progress the customer waits in queue. Show using **Residual Lifetime** analysis that

1. $p_0 = \mathrm{P}\{\text{the server is idle}\} = (1-\rho)/(1+\lambda\overline{\Delta})$ (Note that setting up service (routes) still implies server is busy, though it is not serving customers yet)
2. $\mathrm{P}\{\text{the server is in setup phase}\} = (1-\rho)\lambda\overline{\Delta}/(1+\lambda\overline{\Delta})$
3. Average length of the busy period $= (\overline{X}+\overline{\Delta})/(1-\rho)$
4. The average waiting time in queue is $W_Q = \frac{\lambda\overline{X^2}}{2(1-\rho)} + \frac{2\overline{\Delta}+\lambda\overline{\Delta^2}}{2(1+\lambda\overline{\Delta})}$

Question 2: Open Jackson Network

1. Determine the average delay of a customer (total time spent in the system) for the network given in Fig. 1.
2. Are the below two statements correct? Justify your answers.
   - If the external arrival rates r_1 and r_2 are doubled but the service rates μ_1, μ_2, μ_3 remain unchanged, the total number of customers in the system is doubled.
   - If both the external arrival rates r_1 and r_2 and service rates μ_1, μ_2, μ_3 are halved, the average delay remains unchanged.

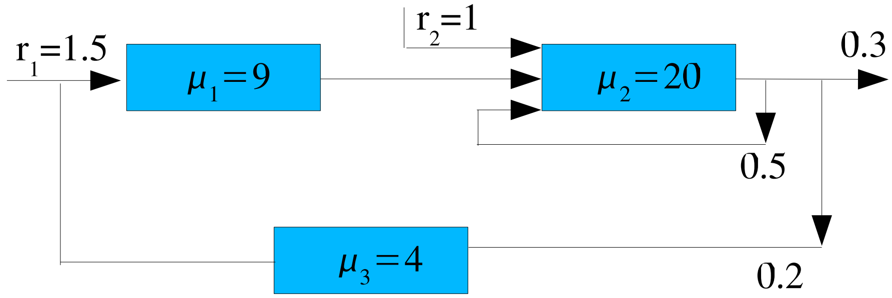

Figure 1:

Question 3: Closed Jackson Network

Consider the closed Jackson network given in the Fig. 2, with K=3 nodes and M customers.

1. Calculate the normalization constant $G(M)$ explicitly as a function of the number of customers M. [A very simple expression for G(M) exists. So, try to simplify it as much as possible]

2. Find the average arrival rate of each node i as a function of the number of customers M.

3. Use Buzen's algorithm, to calculate G(M) and $\lambda_i(M)$, for M = 1,2,3,4,5.

4. Consider the closed network of Fig. 3, with K=3 nodes and M customers. Show that this network has the same distribution as Fig. 2 network. [You can answer this question without calculating the stationary distributions explicitly.]

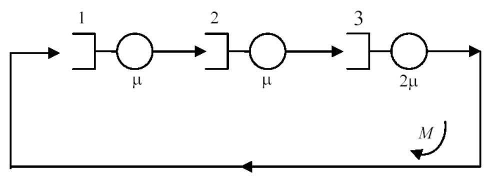

Figure 2:

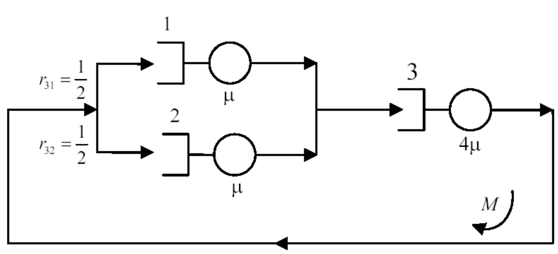

Figure 3: