TDMA:

Q1: a) Overall capacity of 50 kbits/sec is ~~split into~~ divided among the 10 sessions.

Each session thus gets 5 kbits/sec.

Modelling each stream as an M/G/1 system

Arrival rate $\lambda = 2.5$ packets/sec

Avg. Service time $\overline{X} = \frac{1}{\mu} = (0.1)(0.02) + (0.9)(0.3)$

$\overline{X} = 0.272.$ $\quad (0.02 \Leftarrow \frac{100 \text{ bits}}{5 \times 10^3 \text{ bps}}, \quad 0.3 \Leftarrow \frac{1500 \text{ bits}}{5 \times 10^3 \text{ bps}})$

$\overline{X^2} = (0.1)(0.02)^2 + (0.9)(0.3)^2$

$= 0.081$

Utilization $= \rho = \frac{\lambda}{\mu} = (2.5)(0.272) = 0.68$

From PK formula, $W_q = \frac{1}{2} \cdot \frac{\lambda \overline{X^2}}{(1-\rho)}$

$= \frac{1}{2} \frac{(2.5)(0.081)}{(1-0.68)} = 0.3164$

$W = 0.3164 + 0.273 = 0.558 \quad (W = W_q + \overline{X})$

$L_q = \lambda W_q = (2.5)(0.3164) = 0.791$

$L = \lambda W = (2.5)(0.558) = 1.47$

b) Statistical Multiplexing:

Model as an M/G/1 system

where $\lambda = 10 \times 2.5$; $\overline{X} = (0.1)(0.002) + (0.9)(0.03)$

$= 0.0272$ $\quad (0.002 \Leftarrow \frac{100}{50 \times 10^3}, \quad 0.03 \Leftarrow \frac{1500}{50 \times 10^3})$

$\overline{X^2} = 0.00081$

In other words, λ increases by a factor of 10, $\overline{X}$ decreases by a factor of 10; $\overline{X^2}$ decreases by a factor of 100. ρ however remains constant

$\therefore$ All the terms W_q, W, L_q & L decrease by a factor of 10 (for each stream)

$\therefore$ $W_q = 0.03164$, $W = 0.0558$, $L_q = 0.0791$

$L = 0.147$

Q2: Open Jackson Network:

a) $\lambda_1 = \vartheta$

$\lambda_2 = 0.3\lambda_1$

$\lambda_3 = 0.7\lambda_1 + \lambda_2$

$\lambda_4 = 0.2\lambda_3$

Solving the equations, $\lambda_1 = \vartheta$, $\lambda_2 = 0.3\vartheta$

$\lambda_3 = \vartheta$, $\lambda_4 = 0.2\vartheta$.

utilization factor of each server

$\rho_1 = (\vartheta)(1) = \vartheta$

$\rho_2 = (0.3\vartheta)(9) = 2.7\vartheta$

$\rho_3 = (\vartheta)(2) = 2\vartheta$

$\rho_4 = (0.2\vartheta)(4) = 0.8\vartheta$

$\therefore$ Server 2 is the bottleneck since it has the highest utilization

b) server 2 has the highest utilization. It will become saturated when $\rho_2 = 1$

$$\Rightarrow \nu = \frac{1}{2.7} = 0.37.$$

c) Server 3 has the second highest utilization. To make server 3 the bottleneck, we need

$$\rho_2 < \rho_3 \Rightarrow (0.3\nu)\,\overline{x}_2 < 2\nu \quad \text{or} \quad \overline{x}_2 < \frac{2}{0.3} = \underline{\underline{6.666}}$$

d)

$$W = \frac{1}{\nu}\sum_{i=1}^{4} \frac{\rho_i}{1-\rho_i} \;;\quad \nu = 0.3$$

$$\rho_1 = 0.3,\ \rho_2 = 0.81;\ \rho_3 = 0.6,\ \rho_4 = 0.24$$

$$\therefore\ W = \underline{\underline{21.69}}$$

Q3:

a) Let $\overline{\lambda}_1 = \mu_1$ & $\overline{\lambda}_2 = p_2\mu_1$ be a solution to the equations $\lambda_j = \sum_{i=1}^{2} \lambda_i p_{ij}$, $j = 1, 2$

we have $\rho_1 = 1$, $\rho_2 = \dfrac{p_2\mu_1}{\mu_2}$

Let n be the # of jobs in the I/O, then

$$P(M-n, n) = \frac{\rho_2^{\,n}}{G(M)}$$

where

$$G(M) = \sum_{n=0}^{M} \rho_2^{\,n}$$

b) The CPU utilization is given by. Pr { the CPU is busy }

$$= 1 - P(0, M) = 1 - \frac{\rho_2^M}{G(M)}$$

$$= \frac{G(M) - \rho_2^M}{G(M)}$$

$$G(M-1) = G(M) - \rho_2^M \quad \left(\text{since } G(M) = \sum_{n=0}^{M-1} \rho_2^n + \rho_2^M\right)$$

$$\therefore \text{ CPU utilization} = \frac{G(M-1)}{G(M)}.$$

c) From Little's theorem,

$$\text{CPU utilization} = \frac{\lambda_1(M)}{\mu_1}$$

$$\Rightarrow \lambda_1(M) = \frac{\mu_1 \cdot G(M-1)}{G(M)}.$$