A GENERALIZED ADAPTIVE FINITE ELEMENT ANALYSIS OF LAMINATED COMPOSITE PLATES

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1. Introduction

Use of composites for critical structural applications has led to the methodologies for accurate analysis of these structures. A class of plate models are available in literature for the analysis of laminated composite plate like structures. Generally, the shear deformable models [1] are very popular. Another class of plate models are zig-zag theories. Some of these theories have been shown close to three dimensional elasticity solution in energy norm (see [2]). These theories have attractive feature that the cost of computation is independent of number of layers in a laminate. However, they are not capable of predicting transverse stresses accurately. In case of unsymmetric laminates, interfacial damage, cutouts, ply drop off, etc. these dimensionally reduced models are ineffective. Layerwise models are used to alleviate these issues. In layerwise models standard models are applied layer by layer (see [3]). In some cases continuity of displacements and in some cases transverse shear stresses are imposed at interlaminar interfaces. This leads to accurate prediction of stress state. However, the cost of computation multiplies with number of layers.

In general, these plate models are implemented using popular finite element method for analysis. However, the solution obtained by this method for a particular plate model is an approximation to the true solution of the plate model considered. The error occurred is known as discretization or approximation error in finite element analysis. There is a class methods are available for estimation of this error (for example see [4]). Further, the particular plate model itself is an approximation to the three dimensional elasticity solution. The error occurred due to this is called modeling error.

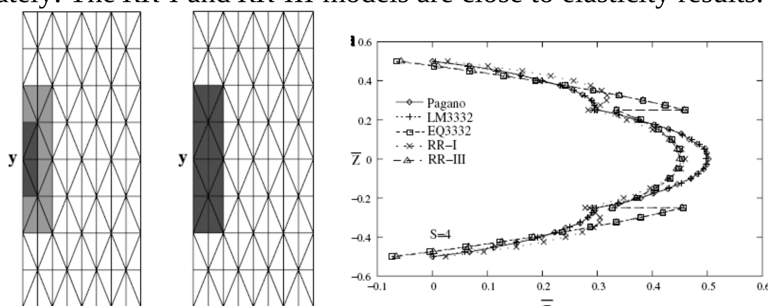
Thus, for accurate analysis of laminated plates, finite element demands for a proper choice of plate model and approximation (discretization) of domain. Further, the cost of computation should be economical.

The aim of the present paper is to give a generalized method for the analysis of laminated plate which is accurate as well as computationally economical. In the present paper a hierarchy of layerwise plate models, equivalent (all layers clubbed together) and intermediate (some of the layers clubbed together) plate models as degenerate cases are developed and tested for accuracy [3]. Further, a novel region by region plate model [5] developed by authors has been presented. In this model, in the regions where the state of stress is expected to be highly three dimensional a suitable layerwise model is implemented and away from this region a suitable equivalent and/or intermediate model is implemented. This model harnesses the advantage of accuracy of layerwise models and economy of computational cost of equivalent and intermediate models.

2. Layerwise and region by region plate modeling

A general layerwise model based on displacement continuity across the interlaminar interfaces is developed. The through thickness approximation of the displacement field is given by transverse functions. The displacement field is expressed as series of product of planar and transverse functions. Following [6], the order of these transverse functions for individual displacement component is chosen based on bending or membrane dominated actions. Thus, first order or higher order shear deformable model can be implemented layerwise. In intermediate model, some of the layers are grouped together and for equivalent model all layers are grouped together in thickness direction and transverse functions are expressed over these groups. Triangular wedge shaped elements with hierarchic shape functions for planar and transverse approximations are used.

A novel region by region plate model is developed wherein in desired regions fully layerwise, intermediate or equivalent model is applied. At the interface of two regions the continuity of displacements is imposed using constrained approximation (see [5] for details). Fig. 1 shows a [150/-150/150] laminate under cylindrical bending [5]. In Fig. 1(a) the darker region shows a layerwise model with transverse cubic approximation for *u* and *v* and quadratic for *w* displacement, for grey region is linear for *u*, *v* and quadratic for *w* and remaining region uses equivalent model with linear for *u*, *v* and quadratic transverse approximation for *w* (model is denoted as *RR-I*). In Fig. 1(b) the darker and unshaded region uses same model as in Fig. 1(a) (model is denoted as *RR-III*). The transverse shear stress at *x*=0 and middle of the length along *y* axis is shown in Fig. 1(c). It is seen that the equivalent model is not able to capture the elasticity result. The layerwise model captures results accurately. The *RR-I* and *RR-III* models are close to elasticity results.



(a) (b) (c)

Fig. 1 (a), (b) region by region modeling schemes *RR-I*, *RR-III*, (c) transverse shear stress through thickness variation

3. Focussed adaptivity and explicit modeling error estimator

The details of the pollution error based discretization error estimation in the quantity of interest (like stress state, displacement, etc) and control with focussed adaptivity [7] and explicit modeling error (see [6]) based on the interelement jumps will be presented in full paper. Further, in full paper some more results with mesh and model adaptivity, that is automatic selection of regions with layerwise, intermediate and equivalent models with different orders of transverse functions for displacement field in presence of damage in one of the layer in a laminate will be demonstrated in detail.

4. References

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