Generalizations of the Łoś-Tarski Preservation Theorem

Abhisekh Sankaran, Bharat Adsul, Supratik Chakraborty

Indian Institute of Technology (IIT), Bombay, India

November 1, 2013

Abstract

We present new preservation theorems that semantically characterize the $\exists^k \forall^*$ and $\forall^k \exists^*$ prefix classes of First Order Logic (FOL) formulae, for each natural number k. Unlike preservation theorems in the literature that characterize the $\exists^* \forall^*$ and $\forall^* \exists^*$ prefix classes as a whole, our theorems provide finer characterizations by relating the count of quantifiers in the leading block of the quantifier prefix to natural quantitative properties of models. As special cases of our results, we obtain the classical Łoś-Tarski preservation theorem for formulae, in both its forms, substructural and extensional. We extend these results to provide, for each natural number n, semantic characterizations of the subclasses of the Σ_n^0 and Π_n^0 prefix classes of FOL formulae, in which the number of quantifiers in the leading block of the quantifier prefix is fixed to a given natural number k. These extensions are new preservation theorems that give finer (in a similar sense as mentioned earlier) characterizations of the Σ_n^0 and Π_n^0 prefix classes than those in the literature.