On Categorical Relationship among various Fuzzy Topological Systems, Fuzzy Topological Spaces and related Algebraic Structures

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October 17, 2013

Abstract

Vickers introduced the notion of topological systems in [6]. Topological systems are closely related with the logic of finite observations or geometric logic. Vickers showed the interconnections among the categories of topological systems, topological spaces and frames. Syropoulos and Paiva [5] extended the notion to fuzzy topological systems and proposed a methodology of obtaining fuzzy topological spaces from systems. Maruyama studied the [4] interconnection between fuzzy topological spaces and Lukasiewicz logics from the viewpoint of duality. In this paper fuzzy topological systems are defined in a way different from Syropoulos et.al. The advantage of this definition is showed. The interconnections between the categories of fuzzy topological systems, fuzzy topological spaces and algebras are studied in this work. It is shown that categorical duality between fuzzy topological systems and algebraic model of Lukasiewicz logic holds. In a further generalization, based on fuzzy topology on fuzzy sets [2] another kind of fuzzy topological systems is defined. An interconnection among the category of this kind of fuzzy topological spaces and the category of corresponding topological systems and frames has been studied. Our goal is towards developing fuzzy geometric logic.

References

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