A New Three-Valued Paraconsistent Logic

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Paraconsistent logic and three-valued semantics: The term *Paraconsistent* was first used by the Peruvian philosopher Francisco Miró Quesada in the Third Latin America Conference on Mathematical Logic in 1976. A logic is called paraconsistent if there are formulas ϕ and ψ such that $\{\phi, \neg\phi\} \nvdash \psi$. Besides other semantics, the three-valued semantics of (paraconsistent) logics have always received special attentions from logicians like J. Łukasiewicz, S. C. Kleene and others and paraconsistentists like S. Jaśkowski, N.C.A. da Costa, G. Priest, R. Brady, C. Mortensen, D'Ottaviano, W. A. Carnielli, João Marcos etc. Parainconsistency axioms have been introduced in [5] in a way similar to classical two-valued logic.

Introduction of the three-valued matrix PS_3**:** Here we shall introduce a new three-valued matrix, $\mathrm{PS}_3 := \langle\{1, \frac{1}{2}, 0\}, \wedge, \vee, \Rightarrow, ^* \rangle$ where $\langle\{1, \frac{1}{2}, 0\}, \wedge, \vee\rangle$ is a distributive lattice and the *designated set* of PS_3 has been fixed as, $\{1, \frac{1}{2}\}$. From the truth tables of PS_3 it can easily be verified that $(\frac{1}{2}) \wedge (\frac{1}{2})^* \Rightarrow 0 = 0$ and hence PS_3 might be a three-valued semantics of some paraconsistent logic.

Proof theory for PS_3**:** The main part of this work is to develop a propositional logic LPS_3 so that PS_3 becomes the three-valued semantics of it. We have proved that LPS_3 is sound and complete with respect to PS_3. It will then be discussed how does LPS_3 satisfy Jaskowski's criterion (cf. [6]) of being a paraconsistent logic.

Comparison with other existing three-valued paraconsistent logics: A comparison between LPS_3 and some other paraconsistent logics having three-valued semantics, such as LP (*Priest's Logic of Paradox*) [8], LFI1 (*Logic of Formal Inconsistency 1*) and LFI2 (*Logic of Formal Inconsistency*

2) [3], J_3 (*D'Ottaviano's logic*) [4], RM_3 [2], P1 (*Sette's three-valued logic*) [9], $C_{0,2}$ (*Mortensen's paraconsistent logic*) [7] will be made. Particularly PS_3 has close connections with the three-valued models of the paraconsistent logics P1 (or $C_{0,1}$) and $C_{0,2}$. It is worthwhile to show, how do these logics differ pair wise. It is proved that LPS_3 is *maximal* relative to the *classical propositional logic*.

Paraconsistent set theory: The motivation of finding the algebra PS_3 is to build a model of some paraconsistent set theory. The paraconsistent logic LPS_3 can be used in some algebra-valued set theory construction similar to the Boolean-valued construction (cf. [1]) to obtain a model of a (weak) paraconsistent set theory.

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