Cultural Independence of Mathematical Thinking?

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Prelude…

- If there exist cognitive differences in ways people (across cultures) form their world-view, then will the development of mathematics (across cultures and even across social groups) be the same?
- If the initial development of mathematics is motivated from the needs and functions of a society, then can mathematics grow in isolation, without any impact from socio-cultural aspects of the community?

Alan Bishop, 'Western Mathematics: the secret of cultural imperialism', 1990, p. 53

"… it is now possible to put forward the thesis that all cultures have generated mathematical ideas, just as all cultures have generated language, religion, morals, customs and kinship systems. Mathematics is now starting to be understood as a pan-cultural phenomenon."

"The direction that mathematics takes in a culture is guided by the cultural needs or attributes – religious, philosophical, agricultural, navigational, industrial as well as mathematical – of the culture. The different directions taken by Greek and Chinese mathematics were determined by the primitive cultural conditions prevailing in these cultures during their prehistoric periods."

– R. L. Wilder, *Introduction to the Foundations of Mathematics*, p. 296

Cognitive development and its relation with culture:

- Cognitive development is a process of children learning to control the world and themselves by appropriating the cultural resources made available to them since birth by their families and community.
- Cultural designs for living are assumed to be apparent in the individual in the form of what is referred to as *cognitive styles* (not cognitive ability), that is, the preferred way in which a person processes information.
- A variety of terms characterise the cognitive styles associated with different cultural designs:
  - Field dependent / Field independent
  - Analytic / Holistic
  - Independent / Interdependent
  - Collectivistic / Individualistic

"Culture and cognition are not two separate phenomena: culture makes cognitive phenomena possible; humans cannot think except through culture. At the same time, culture is impossible without human thought and action."

- Michael Cole and Martin Packer
- "Culture and Cognition", 2011, p. 155

Cognitive Differences:

- Various empirical studies across different kinds of subject groups (inter-culture, intra-culture, inter-society) have revealed the presence of cognitive differences in terms of:
  i. (a) How they categorise the world

     (b) How and to what extent can they perform abstraction from the external world
  ii. Shape (geometric) Perception
  iii. Deduction and Inference
  iv. Reasoning and Problem Solving

CATEGORISATION AND ABSTRACTION

- Patterns of Categorisation in Illiterate Population (Alexander Luria)
- Patterns of Categorisation amongst Americans and East-Asians (Nisbett, et al)

Studies by Alexander Luria:

- Luria's general purpose was to show the socio-historical roots of all basic cognitive processes. For him, the structure of thought depends upon the structure of the dominant types of activity in different cultures.

- His studies with literate and semi-literate groups within a particular society indicated cognitive differences.

Patterns of Categorisation:

Rakmat (39, illiterate peasant):

Shown drawings of: **hammer - saw - log - hatchet**

Subject: "They are all alike. I think all of them have to be here. See, if you're going to saw, you need a saw, and if you have to split something you need a hatchet. So they are all needed here."

Experimenter: Look, here *you have three adults and one child*. Now clearly the child does not belong in this group.

Subject: "Oh, the boy must stay with the others! All three of them are working, you see, and if they have to keep running to fetch things, they'll never get a job done, but the boy can do the running for them… The boy will learn; that will be better, then they'll be able to work well together."

Experimenter: Look, here you have three wheels and a pair of pliers. Surely the pliers and the wheels aren't alike in any way, are they?

Subject: "No, they all fit together. I know that the pliers do not look like the wheels, but you will need them if you have to tighten something in the wheels."

- Studies revealed a tendency to categorise objects in a holistic manner, where situations are created in which different objects can co-exist and co-function as a group.

- However, this tendency was not observed among people from the same society exposed to some level of education.

Studies by Nisbett et al.

- Nisbett et al. studied cross-cultural groups (Americans and East-Asians comprising Chinese, Japanese and Koreans). They observed that the following differences between Greek and Chinese cultures (religion / philosophy/ forms of life) regarding:

(a) World-View:

G: Individualistic / Agent Centric / Discrete

C: Holistic / Harmony / Collective Agency

(b) Categorisation:

G: Essential Feature Abstraction necessary for categorising objects and events of the world

C: The concern with abstraction characteristic of ancient Greek philosophy has no counterpart in Chinese philosophy.

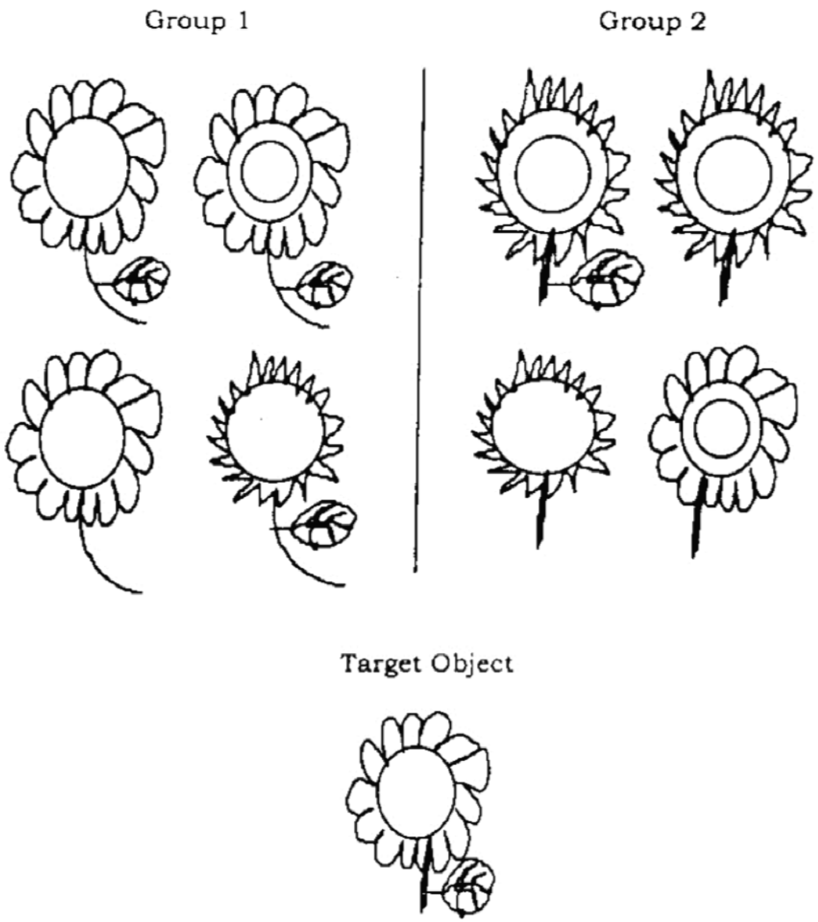
(c) Science and Mathematics:

G: Non-tolerance of Contradiction / Logic

C: Tolerance of Contradiction / Dialectism

Modes of Categorisation:

- Based on historical evidence for cognitive differences and Nisbett's theory about the social origins of them, Westerners would:
  - have a greater tendency to categorise objects than would Easterners;
  - find it easier to learn new categories by applying rules about properties to particular cases;
  - make more inductive use of categories, i.e. generalise from particular instances of a category to other instances or to the category as a whole.



Example of item measuring whether judgments of similarity are based on family resemblance or rules.

Koreans: The target object was similar to Group 1 (family resemblance).

European Americans: The target object was similar to Group 2.

(A rule that allows the object to belong to Group 2, i.e. the object "has a straight stem (as opposed to a curved one)".

DEDUCTION AND INFERENCE

- Of illiterate, peasant group of Central Asia (Alexander Luria)
- Of Kpelle farmers in West Africa (Cole et.al)

The questions…

- Are the logical schemas invariant at different stages of social and historical development?
- Do people have the same form in productive thinking processes in different cultures?
- What exactly is the structure of derivational and inferential processes among people whose life rests upon concrete practical activity?

Luria (1976)

- Luria was interested in the effect of literacy on the cognitive capacities of individuals. He experimented with syllogistic problems of the form:

  All A are B

  A certain *x* is an A

  Is this certain x a B? (or a WH-question, such as "What about this certain *x*?)

- The problem that was given to the subjects:

  In the Far North, where there is snow, all bears are white.

  Novaya Zemlya is in the Far North.

  What colours are bear there?

Response:

S: I don't know what colour the bears are there, I never saw them.

E: But what do you think?

S: Once I saw a bear in a museum, that's all.

E: But on the basis of what I said, what colour do you think the bears are there?

S: Either one-coloured or two-coloured … [ponders for a long time]. To judge from the place, they should be white. You say that there is a lot of snow there, but we have never been there!

Subject: Khamark, age: 40, miller from a remote village, illiterate

Cole at al:

All Kpelle men are rice farmers,

Mr. Smith is not a rice farmer,

Is he a Kpelle man?

S: I don't know the man in person. I have not laid eyes on the man himself.

E: Just think about the statement.

S: If I know him in person, I can answer that question, but since I do not know him in person I cannot answer that question.

Observations:

- There is a reluctance to draw any conclusion about things that are beyond personal experience.
- They do not think that the premises give information which can be used to answer the question.
- However, people with some level of education did not have the problem of answering the question based upon the information that is given in the premises.

COGNITIVE PROCESSES IN MATHEMATICAL THINKING

It has been reported that mathematical concepts and skills develop as children dynamically interact with a wide variety of sociocultural activities such as the exchange of goods, schooling and play activities in daycare settings.

- Noriyuki Inoue (2011)

Model of Mathematical Teaching:

US

1 Homework check

The teacher checks and reviews the students' answers to homework problems from the previous class.

2 Demonstration of examples

The teacher introduces a set of new mathematical problems from the textbook or worksheet and solves them, with explanations, in front of the students.

3 Assign problem solving tasks

The teacher assigns similar problems to the students from the textbook or worksheet.

4 Group work

The students work on the assigned problems individually or in groups.

5 Check answers

The teacher checks the students' answers.

6 Assign new homework

The teacher distributes new homework based on the new mathematical skill that was taught in class.

JAPAN

1 *Hatsumon*

After briefly reviewing prior knowledge, the teacher gives the students a rich mathematical problem that requires an understanding of key mathematical concepts.

2 *Kikanshido*

Students solve the problem individually or in groups as the teacher walks past their desks and guides their thinking.

3 *Neriage*

Students present their problem solving in front of the class, and the teacher facilitates a class discussion to evaluate and compare different strategies and build consensus on the mathematical concepts that are needed to solve the problem.

4 *Matome*

The teacher summarizes the lesson and introduces a new set of problems for the students to consider.

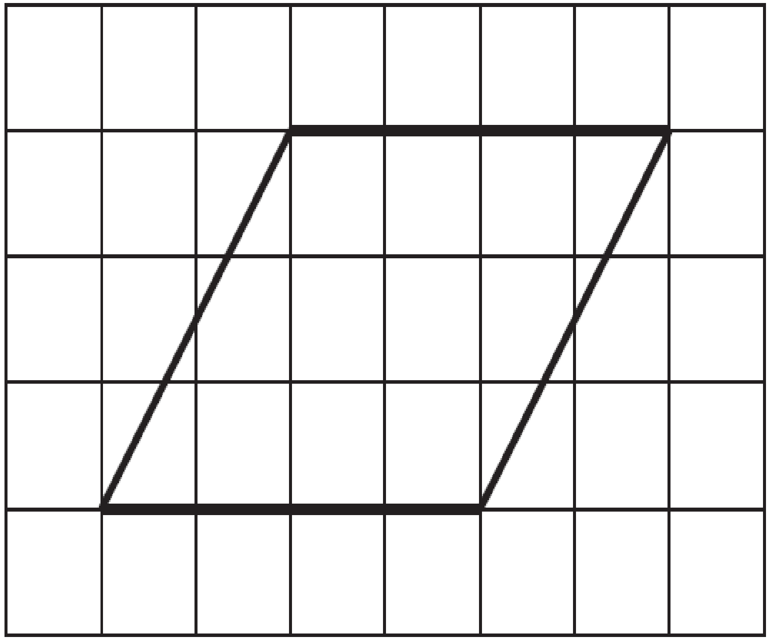


Figure 10.1 *Hastumon*: Find the area of the parallelogram.

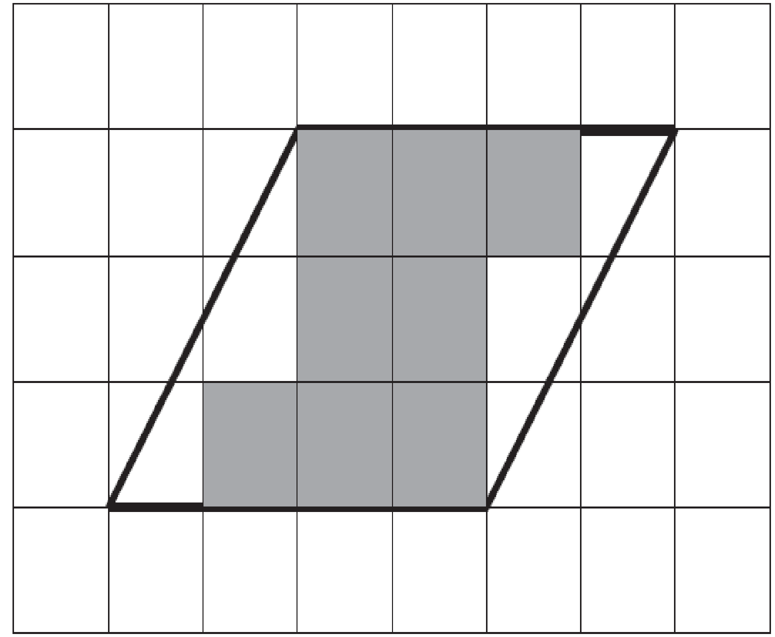


Figure 10.2 Counting the number of unit squares.

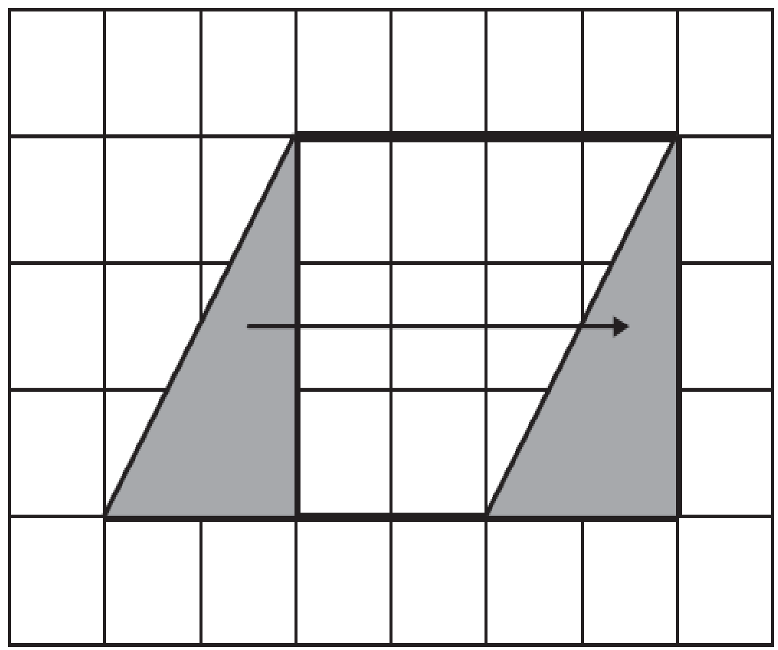


Figure 10.3 Transformation to a rectangle.

which a teacher could ask the students to think how to identify the area of the parallelogram in Figure 10.1 before learning the standard formula (area = base × height). In finding the area of the parallelogram, some students may count the number of unit squares within the parallelogram by ignoring the squares that are not fully included within the parallelogram, or estimating the areas of the squares that are not fully included (Figure 10.2). Other students may suggest cutting the triangular portion of the parallelogram on the left, moving it to the right side of the parallelogram, and creating a rectangle so that the area of the rectangle could be calculated instead (Figure 10.3).

At this point, the teacher would ask the students to think about the different strategies they have presented, and would build consensus on the best strategy. In the process, the teacher would ask the students to think about the strength and weakness of the presented ideas, discuss their rationales, and build consensus in the whole class discussion (e.g., "For the shapes that do not perfectly fit in the grids, moving the parts of the shape and making a rectangle can help us find the exact area of the original shape. Finding the number of unit squares is another way only if the area of the squares that do not fit in the grids can be accurately obtained."). Once the group has built consensus, the teacher would write the

Noriyuki Inoue, "A Cultural Approach to Deconstructing Cognitive Processes in the Mathematics Classroom: Japan and the United States", 2011

Traditional Mathematical Teaching in Pre-Colonial Bengal: Śubhaṅkarī

- Defined as a compendium of computational knowledge that was taught to children in the indigenous elementary schools in Bengal and also at home. It consisted mainly of learning of *Ārjyás* of two kinds:

  (i) Those which once memorised and understood facilitated computation when working with indigenous units of measure and currency; and

  (ii) Verses that posed simple mathematical problems

- The Subhankari manuscripts consisted such verses and other items such as multiplication tables.

(Indigenous) Units of Land Measurement:

Kuṛobā Kuṛobā Kuṛobā Lijje
Kāṭhāi Kuṛobā Kāṭhāi Lijje
Kāṭhāi Kāṭhāi Dhul Parimān
Biś Dhul Hai Kāthār Pramān
Dhul Bāki Thāke Jodi Kāṭhā Nile Par
Ṣolo Diye Pure Tāre Sārā Gaṇḍā Dhar
Chhaṭāk Dharite Habe Chhaṭāk Bighāe
Gaṇḍā Dhari Te Habe Chhaṭāk Kāṭhāi
Chhaṭāke Chhaṭāke hale Kāk Dhori Labe
Ekun Karile Pare Kāli Thik Pābe

1. A Bigha multiplied by a Bigha gives a Bigha
2. A Katha multiplied by a Bigha gives a Katha
3. A Katha multiplied by itself gives a Dhul
4. Twenty Dhuls make a Katha
5. If even after converting Dhuls to Kathas some Dhuls are left over
6. Multiply the remaining Dhuls by 16 converting them into Gandas
7. A Chhatak multiplied by a Bigha gives a Chhatak
8. A Chhatak multiplied by a Katha gives a Ganda
9. A Chhatak multiplied with itself gives a Kak
10. If you proceed thus your computation will be correct

Some Arithmetic Arjyas:

There was a temple very tall
The Wind God's son drowned it in rage
Half sank in mud, one-third in water
One-tenth rolled in the moss
Fifty-two gaj continued to stand
What is the total height, asks Subhankar…

There was a very large fish in the sea
A sea-monster consumed half of it
Kites consumed one-eight of it
Men found one-twelfth of it
Subhankar Das frames the problem
What's left is forty-nine hands of the carcass.

The problem is to find the full length of the whole carcass and also the lengths consumed by each of the parties.

Observations:

1. Can we actually look into finding out the best way of doing mathematical activities (problems)?
2. Are such efforts meaningful?
3. Pedagogical concerns.
   i. Whether differences in cognitive style affect efficiency of performance?
   ii. Whether adaptation of a uniform method of mathematical teaching is effective?
   iii. Whether such adaptation poses a challenge of isolation from one's indigenous form of life?
4. Issue of acceptance.
5. Shift from a prescriptive project to a descriptive one…

Thank you…