On Axiomatizations of Dynamic Epistemic Logic

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(based on joint work with Guillaume Aucher)
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Background

Two modal logic approaches handling knowledge and actions:

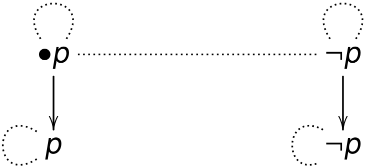
- *Epistemic* Temporal Logic (**ETL**): knowledge in distributed systems based on **temporal logic**. [Fagin et al., 1995, Parikh and Ramanujam, 1985]
- *Dynamic* Epistemic Logic (**DEL**): knowledge in multi-agent interactions based on **epistemic logic**. [Plaza, 1989, Gerbrandy and Groeneveld, 1997, Baltag et al., 1998]

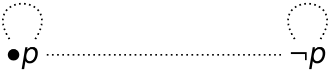
Background

They are both semantics-driven two-dimensional modal logics:

	language	model	semantics
ETL	time+K	temporal+epistemic	Kripke-like
DEL	K+events	epistemic	Kripke+*dynamic*

$\neg Kp \wedge EF\ Kp$ $\neg Kp \wedge [!p]Kp$





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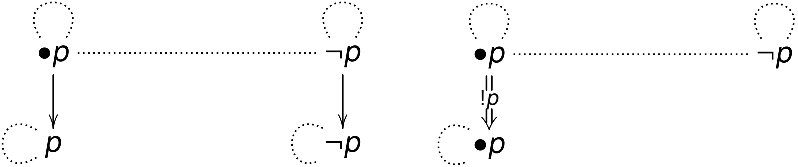


Dynamic semantics:
The **meaning** of an event is the **change** it brings to the knowledge states (dates back to [Stalnaker, 1978]).

Bridging the two

An earlier insight: Iterated updating epistemic structures generates special ETL-style "super models" [van Benthem et al., 2009].

Our approach: relate the two via **axioms**.

Previous work: [Wang and Cao, 2013] on PAL and [Wang and Li, 2012] on Dynamic Navigation Logic

In this work:

- New axiomatization of DEL using ETL-style axioms
- ETL-style completeness proof method for DEL-style logics
- Characterization results of product update and DEL-generatable ETL models.

Dynamic Epistemic Language (**LDEL**)

$$\phi \quad ::= \quad \top \mid p \mid \neg\phi \mid \phi\wedge\phi \mid \Box\phi \mid [e]\phi$$

where $p \in \mathbf{P}$ and $e \in \mathbf{\Sigma}$.

An event model $\mathcal{U}$ is a tuple $(\mathbf{\Sigma}, \rightarrowtail, Pre)$ where:

- $\mathbf{\Sigma}$ is a non-empty (countable) set of events.
- $\rightarrowtail \subseteq \mathbf{\Sigma}\times\mathbf{\Sigma}$ is a binary relation on $\mathbf{\Sigma}$ (image finite).
- $Pre : \mathbf{\Sigma} \to$ **LDEL** is a function assigning each event a precondition (an **LDEL** formula).

Given an (epistemic) model $\mathcal{M} = (S, \to, V)$ and a **fixed** event model $\mathcal{U}$, the semantics is as follows ([Baltag et al., 1998]):

$$\mathcal{M}, s \vDash \Box\psi \quad \Leftrightarrow \quad \forall t : s \to t \text{ implies } \mathcal{M}, t \vDash \psi$$
$$\mathcal{M}, s \vDash [e]\phi \quad \Leftrightarrow \quad \mathcal{M}, s \vDash Pre(e) \text{ implies } \mathcal{M}\otimes\mathcal{U}, (s, e) \vDash \phi$$

Reduction-to-static-based axiomatization

System $\mathcal{DE}$ (without Uni. sub.)

TAUT	all the instances of tautologies	MP	$\dfrac{\phi, \phi \to \psi}{\psi}$
DISTK	$\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$	NECK	$\dfrac{\phi}{\Box\phi}$
UATOM	$[e]p \leftrightarrow (Pre(e) \to p)$	RE	$\dfrac{\phi \leftrightarrow \chi}{\psi \leftrightarrow \psi[\chi/\phi]}$
UNEG	$[e]\neg\phi \leftrightarrow (Pre(e) \to \neg[e]\phi)$		
UCON	$[e](\phi \wedge \chi) \leftrightarrow ([e]\phi \wedge [e]\chi)$		
UK	$[e]\Box\phi \leftrightarrow (Pre(e) \to \bigwedge_{f:e\rightarrowtail f} \Box[f]\phi)$		

Proof of completeness via reduction to basic modal logic K:

$$\vDash \phi \iff \vDash t(\phi) \Longrightarrow \vdash_K t(\phi) \Longrightarrow \vdash_{\mathcal{DE}} t(\phi) \Longrightarrow \vdash_{\mathcal{DE}} \phi.$$

It does not come free. Be careful! [Wang and Cao, 2013]

New axiomatization

System $\mathcal{DEN}$

Axiom schemata		Rules	
TAUT	all the instances of tautologies	MP	$\dfrac{\phi, \phi \to \psi}{\psi}$
DISTK	$\Box(\phi \to \chi) \to (\Box\phi \to \Box\chi)$	NECK	$\dfrac{\phi}{\Box\phi}$
DISTU	$[e](\phi \to \chi) \to ([e]\phi \to [e]\chi)$	**NECU**	$\dfrac{\phi}{[e]\phi}$
INV	$(p \to [e]p) \wedge (\neg p \to [e]\neg p)$		
PRE	$\langle e\rangle\top \leftrightarrow Pre(e)$		
NM	$\Diamond\langle f\rangle\phi \to [e]\Diamond\phi$ (if $e \rightarrowtail f$ in $\mathcal{U}$)		
PR	$\langle e\rangle\Diamond\phi \to \bigvee_{f:e\rightarrowtail f} \Diamond\langle f\rangle\phi$		

New axiomatization

$\mathtt{PR}$ is in the shape of $\langle a\rangle\Diamond\phi \rightarrow \Diamond\langle a\rangle\phi$ (or $\Box[a]\phi \rightarrow [a]\Box\phi$).

$\mathtt{NM}$ is in the shape of $\Diamond\langle a\rangle\phi \rightarrow [a]\Diamond\phi$ (or $\langle a\rangle\Box\phi \rightarrow \Box[a]\phi$).

No Learning ($\mathtt{NL}$) in ETL: $\Diamond\langle a\rangle\phi \rightarrow \langle a\rangle\Diamond\phi$ (or $[a]\Box\phi \rightarrow \Box[a]\phi$).
Note the **difference** between $\mathtt{NM}$ and $\mathtt{NL}$:

$$\Diamond\langle a\rangle\phi \rightarrow [a]\Diamond\phi \ (\mathtt{NM}) \text{ vs. } (\mathtt{NL}) \ \Diamond\langle a\rangle\phi \rightarrow \langle a\rangle\Diamond\phi$$

$\mathtt{NL}$ is too strong: if you consider possible that an event is executable then it must be executable (take ϕ to be $\top$).

New proof method

Basic idea [WC12]: treat $[e]$ as a **standard** modality interpreted on the following ETL models:

$$(S, \to, \{\xrightarrow{e} \mid e \in \mathbf{\Sigma}\}, V)$$

$$\boxed{\mathcal{M}, s \Vdash [e]\phi \quad \Leftrightarrow \quad \forall t : s \xrightarrow{e} t \text{ implies } \mathcal{M}, t \Vdash \phi}$$

Proof strategy: find a class of ETL-style models $\mathbb{C}$ and show the following:

$$\vDash \phi \implies \mathbb{C} \Vdash \phi \implies \vdash_{\mathcal{DEN}} \phi.$$

It works for other DEL-like logics.

The class $\mathbb{C}$: **normal** ETL models

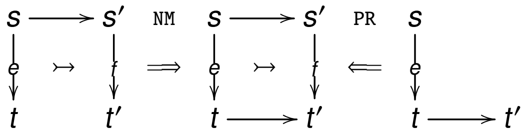
PRE: $\langle e\rangle\top \leftrightarrow Pre(e)$, INV: $(p \to [e]p) \wedge (\neg p \to [e]\neg p)$,
PR: $\langle e\rangle\Diamond\phi \to \bigvee_{f:e\rightarrowtail f}\Diamond\langle f\rangle\phi$, NM: $\Diamond\langle f\rangle\phi \to [e]\Diamond\phi$ (if $e \rightarrowtail f$).

Pre s has e-successors iff $\mathcal{N}, s \Vdash Pre(e)$.

Inv if $s \xrightarrow{e} t$ then for all $p \in \mathbf{P} : t \in V(p) \iff s \in V(p)$.

Nm if $s \to s'$ and $s' \xrightarrow{f} t'$ then for all e and t such that $s \xrightarrow{e} t$ and $e \rightarrowtail f$, we have $t \to t'$.

Pr if $s \xrightarrow{e} t$ and $t \to t'$ then there exists an s' such that $s \to s'$ and $s' \xrightarrow{f} t'$ for some f such that $e \rightarrowtail f$ in $\mathcal{U}$.



Same language, two logics: $\langle \mathbf{LDEL}, \mathbb{M}, \vDash \rangle$ and $\langle \mathbf{LDEL}, \mathbb{C}, \Vdash \rangle$.

Step 1 (Flatten the dynamics):

If $w \xrightarrow{e} v$ in a normal model $\mathcal{N}$, then $\mathcal{N}^- \otimes \mathcal{U}, (w, e) \leftrightarroweq \mathcal{N}^-, v$.

Step 2:

For any ϕ and any normal $\mathcal{N}, s$: $\mathcal{N}, s \Vdash \phi \iff \mathcal{N}^-, s \vDash \phi$.

Step 3: $\vDash \phi \implies \mathbb{C} \Vdash \phi$ (actually: $\vDash \phi \iff \mathbb{C} \Vdash \phi$).

Step 4: $\mathbb{C} \Vdash \phi \iff \vdash_{\mathcal{DEN}} \phi$.

Finally: $\vDash \phi \iff \mathbb{C} \Vdash \phi \iff \vdash_{\mathcal{DEN}} \phi$.

Application 1: Axiomatization of DEL with protocols

Enrich the epistemic model with sate-dependent protocols:

$$\mathcal{M}, \rho, s \Vdash [e]\phi \quad \Leftrightarrow \quad \mathcal{M}, \rho, s \Vdash Pre(e) \textbf{ and } e \in \rho(s) \text{ implies } (\mathcal{M}, \rho) \odot \mathcal{U}, (s, e) \vDash \phi$$

We can axiomatize it by $\mathcal{DEN}$ without $\mathtt{PRE}$ but the following:

$$\mathtt{PPRE} : \langle e \rangle \top \rightarrow Pre(e) \quad \text{and} \quad \mathtt{DET} : \langle e \rangle \langle h \rangle \top \rightarrow [e] \langle h \rangle \top$$

Reduction (to epistemic logic) is not possible.
The proof system is equivalent to the system of [Hoshi and Yap, 2009] based on an ETL semantics.

Application 2: Characterization theorems

Theorem

NM, PR, INV and PRE characterize the update product operation.

Similar result: [van Benthem, 2011, Ch 3.8] on PAL.

Theorem

Nm,**Pr**,**Inv** *and* **Pre** *characterize the product update generatable image-finite ETL models.*

Similar results:
[van Benthem and Liu, 2004, van Benthem et al., 2009]
(tree-like models and arbitrary event model)

Conclusions

- Event-model-based DEL can be viewed as a special case of step-wise ETL in terms of axioms.
- Our proof method does not rely on the reduction to static logic thus can be used for various DEL-like logics.
- Based on the axiomatization, various characterization results become transparent and simple.
- The new axioms are more meaningful (they even give new readings to red. axioms).

Reduction-to-static-based axiomatization

TAUT	all the instances of tautologies	MP	$\dfrac{\phi, \phi \to \psi}{\psi}$
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UK	$[e]\Box\phi \leftrightarrow (Pre(e) \to \bigwedge_{f: e \rightarrowtail f} \Box[f]\phi)$		

Future work

- DEL with common knowledge
- A Gentzen-style proof system
- DEL without `INV` nor `PRE` [Wang and Li, 2012] but iterations

	language	**model**	**semantics**
ETL	time+K	temporal+epistemic	Kripke-like
DEL	K+action	epistemic	Kripke+dynamic
PDL	program	temporal/epistemic	Kripke
Ideal	program+K	temporal+epistemic	dynamic+Kripke

Searching for the right logic:
Desired ETL properties of agents
$\Longrightarrow$ the corresponding dynamics (if possible)
$\Longrightarrow$ search for logics with the right computational properties

break.jpg

The reduction axioms are boring, let's have a (systematic) *prison break*!

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