A bird's eye view on provability logic

Provability logic is a modal logic that is used to investigate what arithmetical theories can express in a restricted language about their provability predicates. The logic has been inspired by developments in meta-mathematics such as Russell and Whitehead's *Principia Mathematica* (published a hundred years ago) and Gödel's incompleteness theorems of 1931. Gödel's second incompleteness theorem showed that consistent (and sufficiently strong) arithmetical theories like Peano Arithmetic can never prove their own consistency; this theorem gave a blow to Hilbert's Program. As a modal logic, provability logic has been studied since the early 1970s, and has had important applications in the foundations of mathematics. For instance, provability logic provides tools to study the notion of self-reference. In 1976, Solovay proved his landmark result that provability logic indeed proves everything you always wanted to prove about provability in Peano Arithmetic - and nothing more. Later developments in the 1990s have connected provability logic to weak systems of arithmetic, thereby associating questions about provability to open problems in complexity theory.

The lecture will introduce the axioms and semantics of propositional provability logic. Furthermore, some results will be sketched, such as a nice proof of Gödel's second incompleteness theorem that is much shorter than Gödel's original proof. The lecture will end with some open problems. As prerequisites for this lecture, acquaintance with propositional and predicate logic should be sufficient.