Concepts of Rough Naturals

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ABSTRACT

In this talk, I will try to give an idea about concepts of rough naturals introduced recently in some of my recent research papers. These have been used for a different evolution of rough semantics, new measures of vagueness and human reasoning contexts. Similar concepts have not been used earlier in the literature due to basic assumptions about the nature of the semantic domain.

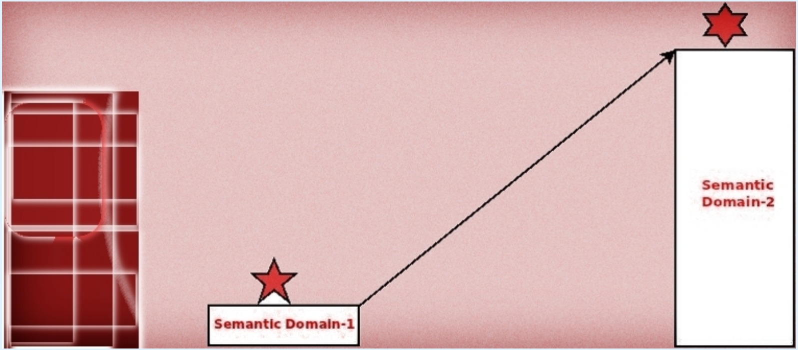
Outline

In RST, We May

- Deal with collections of relatively indiscernible objects
- Relative some entitities - Meta-R
- From a classicalist perspective - Meta-C
- Assumption*: Dynamics in Meta-R transforms to Meta-C.
- E.g. KDD, Vague Reasoning, Feature Selection, Multi-Agent Systems, Ambiguous Information, Human Reasoning, uncertainity...

Discernability

. . . depends on the domain of discourse or *semantic domains*.



Inductive Reasoning and ...

insufficient attributes.

Factors

- Simplification of the dynamics of cognition of human agents through such transforms can lead to serious errors.
- Mechanism of perception can be agent dependent.
- Linguistic Relativity - Objectivity of worlds is not an exact concept [Whorf'56 +]

To Count May Mean...

- The construction of a monic map $: S \longmapsto N$.
- The construction of all monic maps $: S \longmapsto N$.
- The existence of above mentioned maps.
- The existence of integral ratios.
- A process of discrete accumulation.

But to count a collection of objects, it is necessary to discern them first.

Why Rough Naturals?

- To Count a collection of objects is not the same as counting a set of objects.
- It may be possible to define *some* monic maps : $S \longmapsto N$, but not *all*.
- Dynamics in a rough semantic domains can be distorted by models that seek to model them with unjustified assumptions from a relatively exact semantic domain (higher meta level).
- Example: Measures introduced at initial stages are problematic - of the contamination problem.
- Psychological*: Acquired Number Senses are not applied uniformly.
- The Mathematics of Vagueness Programme.

Positively Totally Ordered Semigroups

- Beautiful part of Semigroup theory, 1950, ..., 1975, ...[SM'79]
- $(\forall a, b)\, a < a + b,\, b < a + b$ - 'Positive'
- $(a \leq b \longrightarrow (\exists x)\, a + x = b)$ - 'naturally ordered'
- PTOSG $\oplus$ Many nonequivalent Conditions $\equiv N$.
- Alimov'1950: Cancellative commutative PTOSG without AP - Example.
- An interpretation of 'counting strategies' implicit in PTOSGS - (in a forthcoming paper).

Fuzzy Cardinalities, Reals

- These are essentially real-valued weak measures that avoid 'counting'.
- Measures that are compatible with / dictated by t-norms and t-conorms.
- Simplest: scalar cardinality . . . controversial. . . controversial++ .

In Practical Contexts...

- the discernibility required for normal counting may not be feasible
- the number assigned to one may be forgotten while counting subsequent objects
- the concept of identification by way of attributes may not be stable
- the entire process of counting may be 'lazy' in some sense.
- the mereology necessary for counting may be insufficient.
- E.g. Counting fishes in a pond, crowd management procedures, Empirical Results on Number Conception.

Esoteric Rough Set Theory

- Introduced in [A Mani'08A]: assumes a form of transform from Meta-R to Meta-C.
- Partially Reflexive relations model 'indiscernibility'.
- Partially avoids the contamination problem.
- But in VPRS, VPRFS contexts, explicit measures are used.
- Uses usual concept of natural numbers.

Notation

- Which is Better - Lists, Multisets, Sets?
- Rough Y-Systems [AM'11A]
- R - some indiscernibility relation
- s- successor operation; f - the counting operation
- $Ran(f)$ is a partial algebra in general.
- $\{x_i\}_1^n$ - definable formally in Lists (without N).

General Rough Y-Systems (RYS) [AM'11A]

$$\left\langle S, \mathbf{P}, (l_i)_1^n, (u_i)_1^n, +, \cdot, \sim, 1 \right\rangle$$

- $(\forall x)\mathbf{P}xx$; $(\forall x, y)(\mathbf{P}xy, \mathbf{P}yx \longrightarrow x = y)$
- For each i, $(\forall x, y)(\mathbf{P}xy \longrightarrow \mathbf{P}(l_i x)(l_i y), \mathbf{P}(u_i x)(u_i y))$
- For each i, $(\forall x)\,\mathbf{P}(l_i x)x, \mathbf{P}(x)(u_i x))$
- For each i, $(\forall x)(\mathbf{P}(u_i x)(l_i x) \longrightarrow x = l_i x = u_i x)$
- ...

Desirable Granular Axioms

Absolute Crispness, ACG For each i, $(\forall y \in \mathscr{G})\, y^{l_i} = y^{u_i} = y$

Mereological Atomicity,MER $\forall i$,
$$(\forall y \in \mathscr{G})(\forall x \in S)(\mathbf{P}xy,\, x^{l_i} = x^{u_i} = x \longrightarrow x = y)$$

Lower Stability, LS $\forall i$, $(\forall y \in \mathscr{G})(\forall x \in S)\,(\mathbf{P}yx \longrightarrow \mathbf{P}(y)(x^{l_i}))$

Upper Stability, US $\forall i$, $(\forall y \in \mathscr{G})(\forall x \in S)\,(\mathbf{O}yx \longrightarrow \mathbf{P}(y)(x^{u_i}))$

Stability, ST Shall be the same as the satisfaction of LS and US.

Immediate Predecessor Based Counting (IPC) [AM'10B]

In this form of 'counting', the relation with the immediate preceding step of counting matters crucially.

1. Assign $f(x_1) = 1_1 = s^0(1_1)$
2. If $f(x_i) = s^r(1_j)$ and $(x_i, x_{i+1}) \in R$, then assign $f(x_{i+1}) = 1_{j+1}$
3. If $f(x_i) = s^r(1_j)$ and $(x_i, x_{i+1}) \notin R$, then assign $f(x_{i+1}) = s^{r+1}(1_j)$

- 2-type of the expression $s^{r+1}(1_j)$ is j.
- Alternative: $\{(\alpha(i), \beta(i))\}_i$: $i \in N$
- $\beta(i)$ - the 2-type and $\alpha(i)$ is a 0 or a successor

History Based Primitive Counting (HPC)

In HPC, the relation with all preceding steps of counting is taken into account.

1. Assign $f(x_1) = 1_1 = s^0(1_1)$
2. If $f(x_i) = s^r(1_j)$ and $(x_i, x_{i+1}) \in R$, then assign $f(x_{i+1}) = 1_{j+1}$
3. If $f(x_i) = s^r(1_j)$ and $\bigwedge_{k<i+1}(x_k, x_{i+1}) \notin R$, then assign $f(x_{i+1}) = s^{r+1}(1_j)$

Example

- $S = \{f, b, c, a, k, i, n, h, e, l, g, m\}$
- $R = cl\{(a, b), (b, c), (e, f), (i, k), (l, m), (m, n), (g, h)\}$.
- $Q = cl\{(a, b), (e, f), (i, k), (l, m), (m, n)\}$
- [IPC]: $\{1_1, 2_1, 1_2, 1_3, 2_3, 1_4, 2_4, 3_4, 1_5, 2_5, 1_6, 2_6\}$
- [HPC]: $\{1_1, 2_1, 1_2, 1_3, 2_3, 1_4, 1_5, 2_5, 1_6, 1_7, 1_8, 1_9\}$
- [HPPC]: $\{1_1, 2_1, *, *, 3_1, *, 4_1, 5_1, *, *, *, *\}$

Knowledge Interpretation of Rough Sets

- [ZP'91] In classical RST Sets of the form A^l and A^u represent clear and definite concepts.
- [ChS'08] basically accepts this in cover based RST contexts as well.
- [A Mani'09A, AM'11A] Aggregations of special types/disjoint sets of granules represent clear and definite concepts.
- Representation problem in axiomatic approach to granules [AM'11A, AM'11]
- Algebraic Semantics of the above different approaches can be found in [AM'11].
- Many concepts of 'knowledge consistency' are also considered in the same.

Measures of Knowledge Consistency

- If $Q \subseteq R$ on $\underline{S}$, then
- the state of the knowledge encoded by Q is a *refinement* of that of R.
- R-positive region of Q is $POS_R(Q) = \bigcup\{[y]_R; [y]_R \subseteq X\}$.
- Degree of dependence of knowledge Q on R:
  $\delta(Q,R) = \dfrac{Card(POS_R(Q))}{Card(S)}$.
- P depends on Q to a degree k, $P \multimap_k Q$ iff $k = \delta(P,Q)$.
- [CHP] *consistency degree* if $P \multimap_a Q$, $Q \multimap_b P$,
  $Cons(P,Q) = \dfrac{a+b+nab}{n+2}$ for a constant $n \in Z_+$

Granular Consistency Degree [AM'10B]

Definition

The *granular degree of dependence of knowledge Q on R*, $gk(Q,R)$ will be the tuple $(k_1, \ldots k_r)$, with k_is being the ratio of the number of granules of type i included in $POS_R(Q)$ to $card(S)$.

Definition

If $gk(Q,R) = (k_1, k_2, \ldots, k_r)$ and $gk(R,Q) = (l_1, l_2, \ldots, l_p)$ then the *granular consistency degree* $gCons(Q,R)$ of the knowledge represented by Q, R, will be

$$gCons(Q,R) = (k_1^*, \ldots, k_r^*, l_1^*, \ldots, l_p^*, nk_1^* l_1, \ldots nk_r^* l_p),$$

where $k_i^* = \frac{k_i}{n+2}$ for $i = 1, \ldots, r$ and $l_j^* = \frac{l_j}{n+2}$ for $j = 1, \ldots, p$.

Rough Inclusion Functions [AM'10B]

- [AM'11A] Rough inclusion functions that are not representable in terms of granules through term operations formed from the basic ones are not truly functions/degrees of the rough domain - *non-compliant for the rough context*.

$$k(X,Y) = \begin{cases} \dfrac{\#(X \cap Y)}{\#(X)}, & \text{if } X \neq \emptyset \\ 1, & \text{else} \end{cases}$$ with

$$k^*(X,Y) = \begin{cases} (y_1, y_2, \ldots, y_r) & \text{if } X^l \neq \emptyset, \\ \left(\dfrac{1}{r}, \ldots, \dfrac{1}{r}\right), & \text{else} \end{cases}$$

where $\dfrac{\#(G_i) \cdot \chi_i(X \cap Y)}{\#(X^l) = y_i}$, $i = 1, 2, \ldots r$.

Here it is assumed that $\{G_1, \ldots, G_r\} = \mathscr{G}$ (the collection of granules) and that the function χ_i is being defined via, $\chi_i(X) = \begin{cases} 1, & \text{if } G_i \subseteq X \\ 0, & \text{else} \end{cases}$

Example

- $S = \{f, b, c, a, k, i, n, h, e, l, g, m\}$
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Example (cont'd)

- $S|Q = \{\{a,b\},\{c\},\{e,f\},\{i,k\},\{l,m,n\},\{g\},\{h\}\}$
- $POS_R(Q) = \bigcup_{X \in S|Q} X^l = \{e,f,l,m,n\} = \{f,n,e,l,m\}$ (in the order)
- Induced HPC counts: $\{1_1, 2_5, 1_6, 1_7, 1_9\}$
- Induced HPC Count: $\{1_1, 2_3, 1_4, 1_5, 1_6\}$
- From this the measures can be constructed.

Mathematics of Vagueness

- Incorporate vagueness in more natural and amenable ways in the light of the contamination problem.
- IPC counts lead to two classes of partial algebras [AM'11].
- Rough Natural orders have been defined
- Algebraic Semantics of some RSTs have been deduced via these.
- Inverse Problems and Counts.
- Can the division operation be eliminated in the general measures proposed?

Contamination is not Linguistic Relativity

Four-year-old Jessica is standing at the bottom of a small rise in the preschool yard when she is asked by another four-year-old on the top of the rise to come up to her [PD'02].

- "No, you climb down here. It's much shorter for you."

The authors claim that "Jessica has adopted a developing concept of comparison of length to solve - at least for her - the physical dilemma of having to walk up the rise". But this is just one among the many concepts assignable.

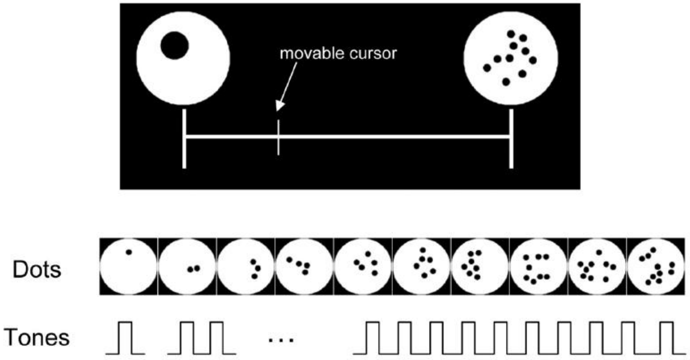
- The context leads to collections of concepts approximating a target.
- A concept of rough object ...
- if there is a rough evolution.

Number Line

- [AF] studies the poor performance of 'impact factors'.
- [SVEP]: Adults tend to map numbers (in nonsymbolic form) logarithmically...
- under conditions that discourage counting.
- [RB'08] -overview

Number Line+

Fig. 1. Number mapping task with numbers from 1 to 10.



Example: Goal Based Counting

- Dubious Goal: Reduce Number of Farmer Suicides [SP'10]
- Maharashtra (1997-2008): 41,404/ 2,000,000 (official, NCRB)
- 2006-08: 12,493 (official, NCRB).
- 40 Clauses,'Investigated Cases', 'Eligible Suicide'...
- Result: 90% reduction

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CHEERS !