Comparing Semantics of Strategic Ability

Nils Bulling[1]
(joint work with Wojtek Jamroga[2]**)**

[1]TU Clausthal [2]University of Luxembourg

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1. Introduction

Motivation

- We study: Relationship between standard variants of the alternating-time temporal logics.
  - perfect recall / no memory
  - perfect / imperfect information
  - objective / subjective ability
- Focus is on the logics; i.e., on the level of valid sentences.
- Validities capture general properties of games.
- Same logics induce same kind of ability in games.

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- Validities capture general properties of games.
- Same logics induce same kind of ability in games.
- First step towards devising (practical) algorithms for satisfiability checking.

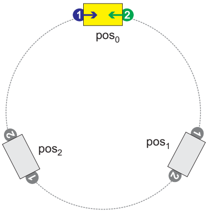
2. Reasoning about Strategic Ability

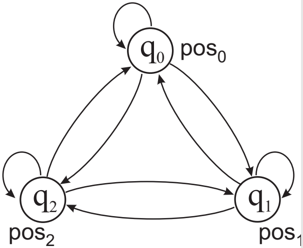
2.1 Temporal Logic

Temporal Logic and Reactive Systems

LTL: Modelling linear time

- computation: $q_0 q_1 q_2^\omega$
- "Some property p holds in some future state"





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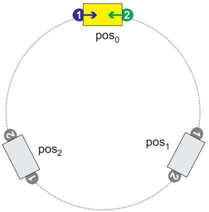
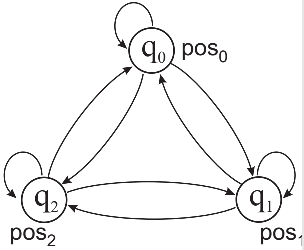


Example properties:

- Always
- Eventual
- Infinitely often



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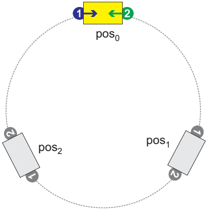

Example properties:

- Always $\rightsquigarrow$ $\Box pos_0$
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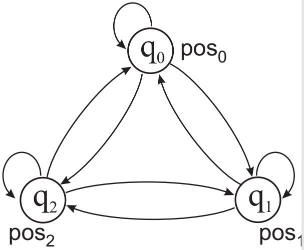

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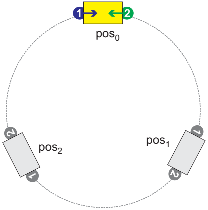

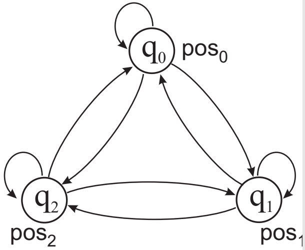

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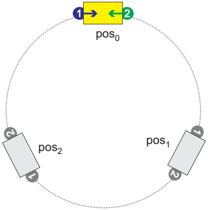

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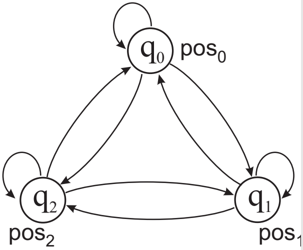

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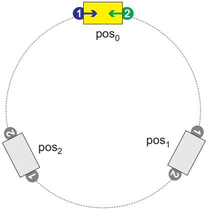

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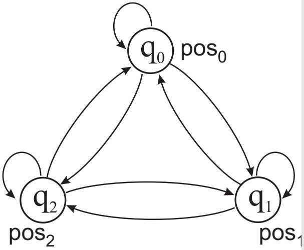

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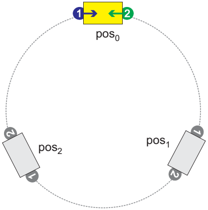

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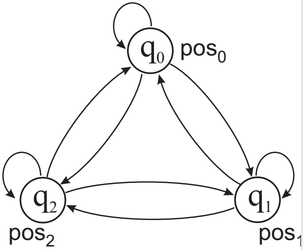

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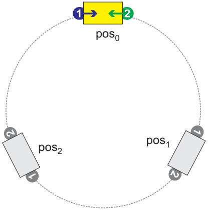

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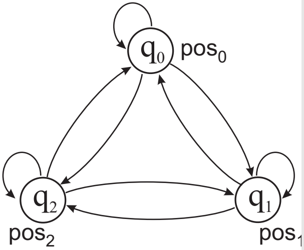
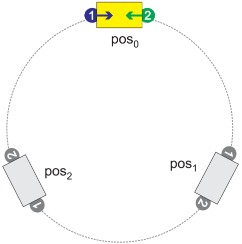


2.2 Strategic Logic

Strategic Logic and Multi-Agent Systems



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- execute actions
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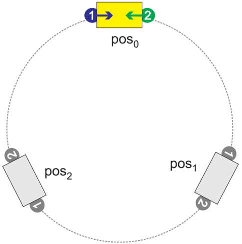

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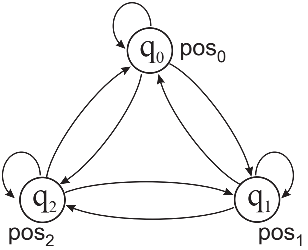

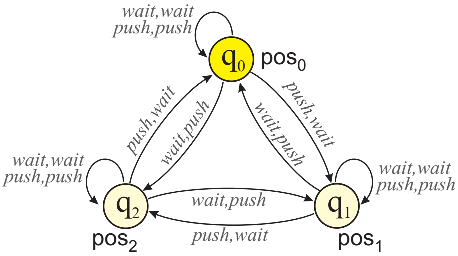

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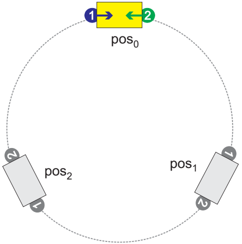

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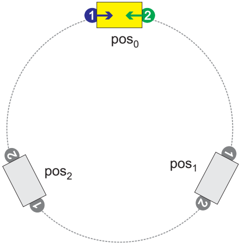

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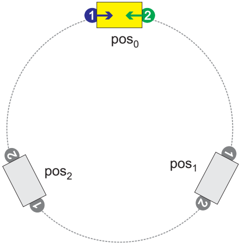

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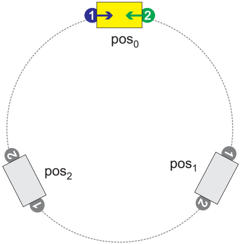

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- $\mathfrak{M}, q_0 \models \langle\langle 1\rangle\rangle \Box \neg \mathsf{pos}_1$
- $\mathfrak{M}, q_0 \not\models \langle\langle 1\rangle\rangle \Diamond \mathsf{pos}_1$

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Definition 1 (Language $\mathcal{L}_{ATL^*}$ [?])

The language $\mathcal{L}_{ATL^*}$ is given by all formulae generated by the following grammar:

$$\varphi ::= \mathsf{p} \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\langle A \rangle\rangle \gamma \quad \text{where}$$
$$\gamma ::= \varphi \mid \neg\gamma \mid \gamma \wedge \gamma \mid \gamma \mathcal{U} \gamma \mid \bigcirc \gamma,$$

$A \subseteq \mathbb{A}\mathrm{gt}$, and $\mathsf{p} \in \Pi$. Formulae φ (resp. γ) are called state (resp. path) formulae.

Definition 2 (Language $\mathcal{L}_{ATL}$[?])

The language $\mathcal{L}_{ATL}$ is given by all formulae generated by the following grammar:

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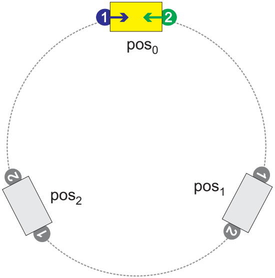
where $A \subseteq \mathbb{A}\text{gt}$ and $\mathsf{p} \in \Pi$.

Note: Every $\mathcal{L}_{ATL}$-formula is also a $\mathcal{L}_{ATL^*}$-formula!

ATL Models: Concurrent Game Structures

- **Agents**, **actions**, **transitions**, atomic **propositions**
- Atomic propositions + **interpretation**
- **Actions are abstract**



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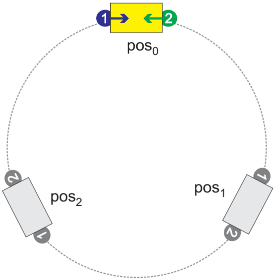

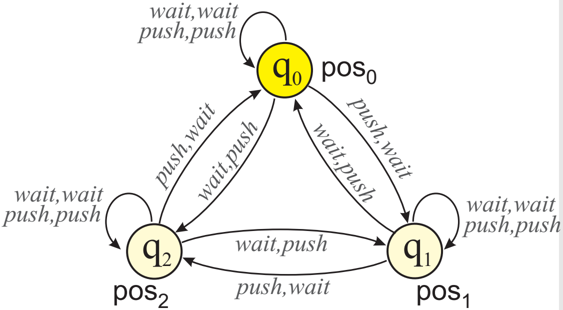

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- A perfect information perfect recall strategy for agent a (*IR*-strategy for short) is a function

$$s_a : Q^+ \to Act \text{ such that } s_a(q_0 q_1 \dots q_n) \in d_a(q_n).$$

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- A perfect information memoryless strategy for agent a (*Ir*-strategy for short) is a function

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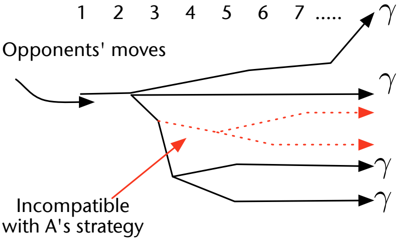
Outcome $out(q, s_A)$: set of all paths/executions possible if A follow s_A.

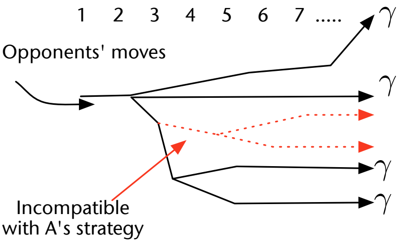


Outcome $out(q, s_A)$: set of all **paths/executions** possible if A **follow** s_A.

Semantics of ATL*

$\mathfrak{M}, q \models \langle\langle A \rangle\rangle \gamma$ iff

- there is a collective strategy s_A such that,
- for every path $\lambda \in out(q, s_A)$,
- we have that $\mathfrak{M}, \lambda \models \gamma$.



Definition 4 (Perfect information semantics)

$\mathfrak{M}, q \models_{\mathsf{Ix}} \langle\langle A \rangle\rangle \Phi$ iff there is a collective Ix-strategy s_A such that, for each path $\lambda \in out(q, s_A)$, we have $\mathfrak{M}, \lambda \models_{\mathsf{Ix}} \Phi$.

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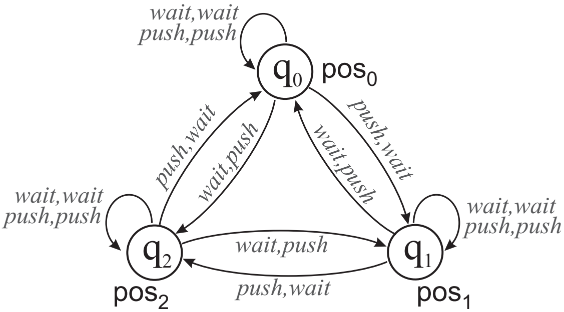
Note that temporal formulae and the Boolean connectives are handled as before.

Example: Robots and Carriage



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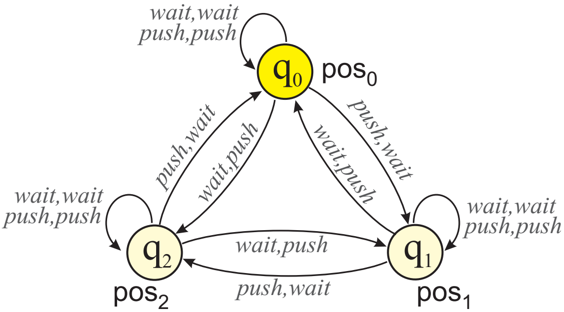

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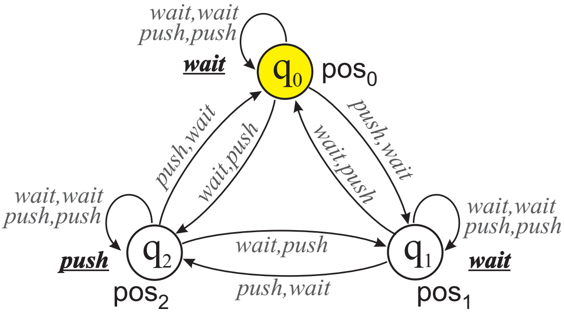

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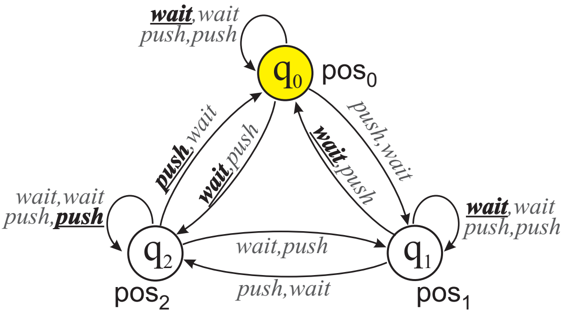

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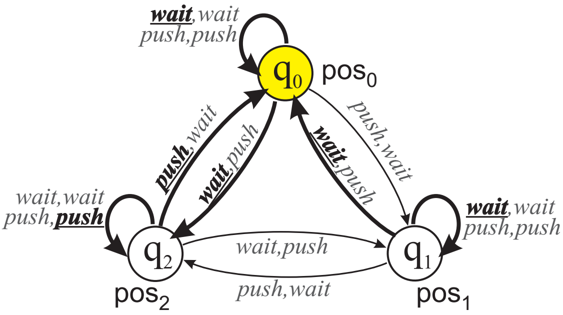

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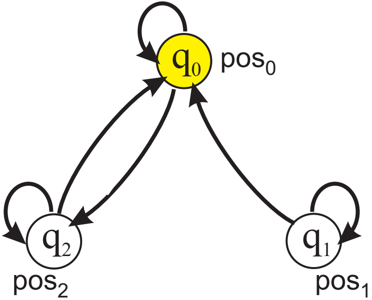

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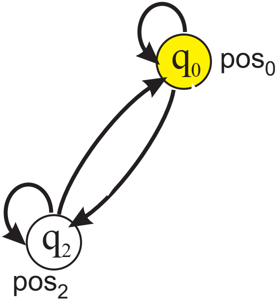

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Definition 5 ($\mathbf{ATL}_{Ix}$, $\mathbf{ATL}^*_{Ix}$, ATL, $\mathbf{ATL}^*$)

We define the following logics:

- $\mathbf{ATL}_{Ix}$ is the set of valid sentences over $(\mathcal{L}_{\mathsf{ATL}}, \models_{Ix})$
- $\mathbf{ATL}^*_{Ix}$ is the set of valid sentences over $(\mathcal{L}_{\mathsf{ATL}^*}, \models_{Ix})$

where $x \in \{r, R\}$, respectively.

Theorem 6

For $\mathcal{L}_{ATL}$, the perfect recall *semantics is* equivalent *to the* memoryless semantics *under perfect information , i.e.,*

$$\mathfrak{M}, q \models_{IR} \varphi \text{ iff } \mathfrak{M}, q \models_{Ir} \varphi.$$

That is

$$\mathbf{ATL} = \mathbf{ATL_{Ir}} = \mathbf{ATL_{IR}}.$$

Both semantics are different for $\mathcal{L}_{ATL^*}$; that is, $\mathbf{ATL^*_{Ir}} \neq \mathbf{ATL^*_{IR}}$.

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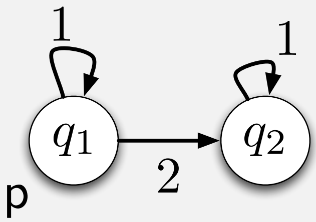
The property has been first observed in [?] but it follows from [?] in a straightforward way.

Example 7 ($ATL^*_{IR} \neq ATL^*_{Ir}$)



$$\varphi = \langle\langle a \rangle\rangle (\bigcirc p \wedge \bigcirc \bigcirc \neg p)$$

2.3 Imperfect Information

Imperfect information

How can we reason about agents/extensive games with imperfect information?

Imperfect information

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We combine **ATL*** and epistemic logic.

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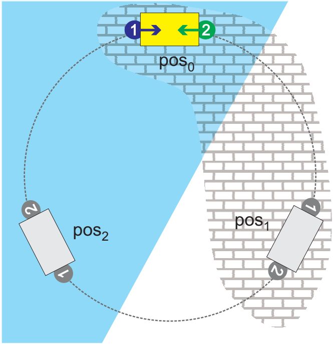
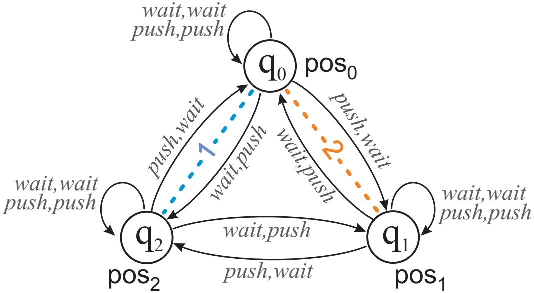
- A concurrent epistemic game structure (CEGS) is a CGS enriched with indistinguishability relations.
- We interpret $\langle\langle A \rangle\rangle\gamma$ epistemically ($\rightsquigarrow \models_{iR}$ and $\models_{ir}$): Group A knowsthat they can enforce γ.

Example: Robots and Carriage





What about $\langle\!\langle \mathbb{A}\mathrm{gt} \rangle\!\rangle \bigcirc \mathsf{pos}_1$ in q_0?

$\mathfrak{M}, q_0 \quad {}_{\mathtt{Ir}}\langle\!\langle \mathbb{A}\mathrm{gt} \rangle\!\rangle \bigcirc \mathsf{pos}_1$

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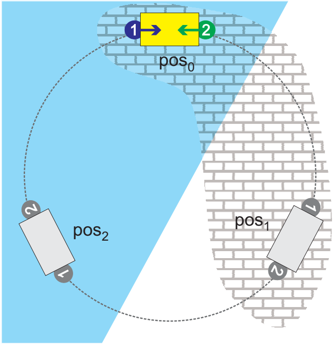

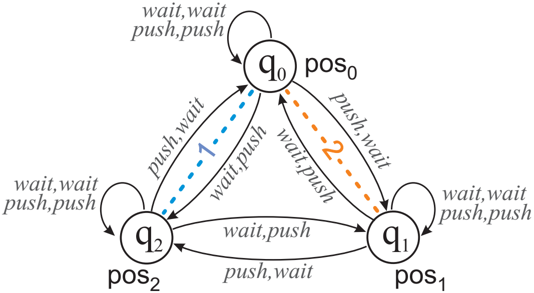

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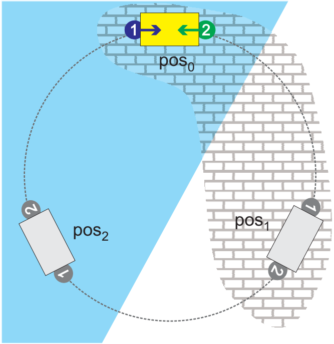

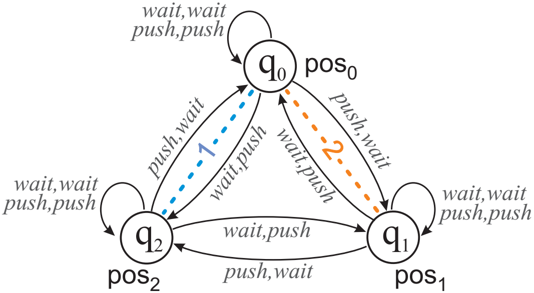

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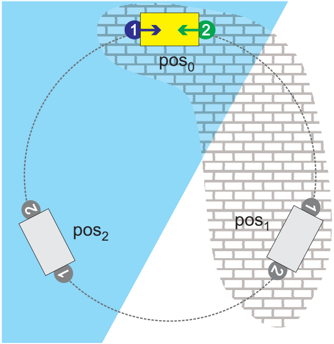

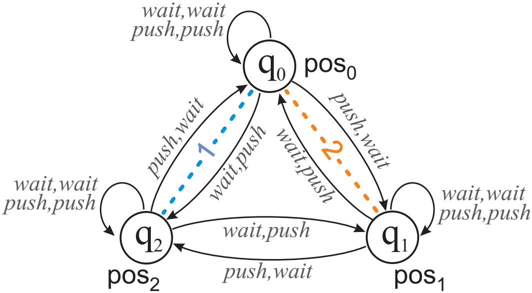

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Strategies should be executable $\leadsto$ uniform strategies

Definition 8 (Uniform strategy)

Strategy s_a is uniform iff it specifies the same choices for indistinguishable situations :

- Memoryless strategies:

$$\text{if } q \sim_a q' \text{ then } s_a(q) = s_a(q').$$

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A collective uniform strategy for A contains a uniform strategy for each agent in A.

Imperfect Information Strategies

Definition 9 (*IR*- and *Ir*-strategies)

- Imperfect information perfect recall strategy (*iR*-strategy):

= uniform IR-strategy.

Finally, we introduce two variants of ability under incomplete information.

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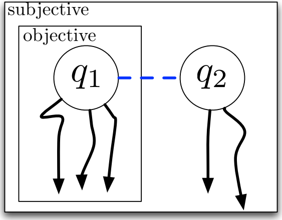
Objective vs. Subjective Semantics

There are two more characteristics of ability under imperfect infromation:

- Objective ability ($\mathrm{i_o}$): Only paths from the (real) current state are considered:

$$out^{\mathrm{i_o}\mathsf{y}}(q, s_A) = out(q, s_A) \text{ for } y \in \{\mathrm{r}, \mathrm{R}\}$$



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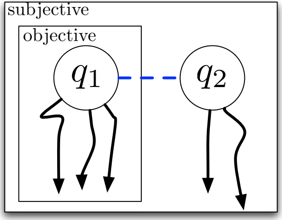

Imperfect Information Semantics

Definition 10 (Imperfect information semantics)

$\mathfrak{M}, q \models_{xy} \langle\langle A \rangle\rangle \varphi$ *iff*

- *there is a collective* xy-strategy s_A

where $x \in \{\mathrm{i_o}, \mathrm{i_s}\}$, $y \in \{r, R\}$ and $\sim_A := \cup_{a \in A} \sim_a$.

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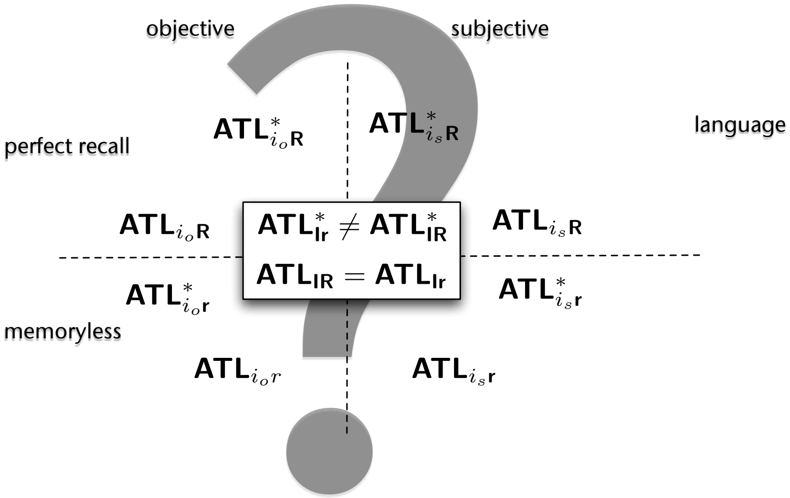
Remark 11

*This definition models that "*everybody *in* A *knows that* φ*".*

3. Comparing Semantics

How does the picture look?



Comparing Validities

Recall our motivation:

- Relationship between standard variants of ATL* on the level of valid sentences
- Logic = set of validities
- Validities capture *general properties of games* under consideration
- If two logics over $\mathcal{L}_{ATL^*}$ generate the same valid sentences then the underlying notions of ability induce the same kind of games

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- Validities capture general properties of games under consideration
- If two logics over $\mathcal{L}_{ATL^*}$ generate the same valid sentences then the underlying notions of ability induce the same kind of games
- First step towards devising algorithms for satisfiability checking

Semantic Variants of ATL

Memory of agents:

- Perfect recall (R) vs. imperfect recall strategies (r)

Available information:

- Perfect information (I) vs. imperfect information strategies (i)

Success of strategies:

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Important Validities and Invalidities

- $\langle\langle a\rangle\rangle \Diamond \mathsf{p} \leftrightarrow \mathsf{p} \vee \langle\langle a\rangle\rangle \bigcirc \langle\langle a\rangle\rangle \Diamond \mathsf{p}$

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- $\langle\langle a \rangle\rangle \Diamond \mathsf{p} \leftrightarrow \mathsf{p} \vee \langle\langle a \rangle\rangle \bigcirc \langle\langle a \rangle\rangle \Diamond \mathsf{p}$
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Important Validities and Invalidities

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- $\langle\langle a\rangle\rangle (\Diamond p_1 \land \Diamond p_2) \leftrightarrow \langle\langle a\rangle\rangle \Diamond ((p_1 \land \langle\langle a\rangle\rangle \Diamond p_2) \lor (p_2 \land \langle\langle a\rangle\rangle \Diamond p_1))$
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- $\neg\langle\langle \emptyset\rangle\rangle \Diamond \neg p \leftrightarrow \langle\langle \mathbb{A}\mathrm{gt}\rangle\rangle \Box p$
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3.1 Perfect vs. Imperfect Information

Comparing ATL_{ir} vs. ATL_{Ir}

Subjective incomplete information vs. perfect information.

Proposition 12

$Val(\mathsf{ATL}_{i_s r}) \subsetneq Val(\mathsf{ATL}_{Ir})$

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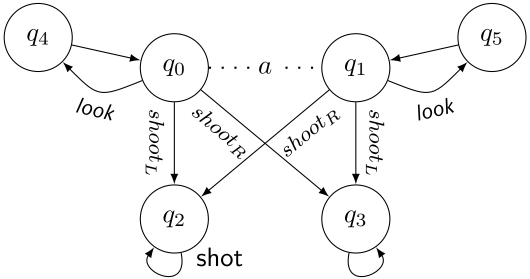

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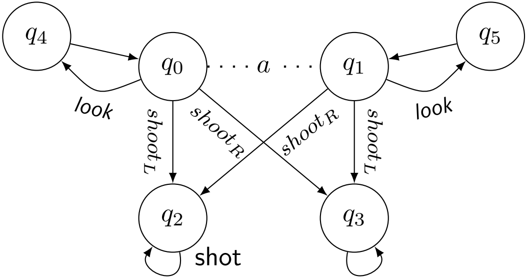
And with perfect recall?



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Objective incomplete information vs. perfect information.

Proposition 13

$Val(\mathsf{ATL}_{i_or}) \subsetneq Val(\mathsf{ATL}_{Ir})$

$$\mathfrak{M}, q_0' \not\models_{i_or} (\mathsf{shot} \vee \langle\langle a \rangle\rangle \bigcirc \langle\langle a \rangle\rangle \Diamond \mathsf{shot}) \rightarrow \langle\langle a \rangle\rangle \Diamond \mathsf{shot}$$

Objective incomplete information vs. perfect information.

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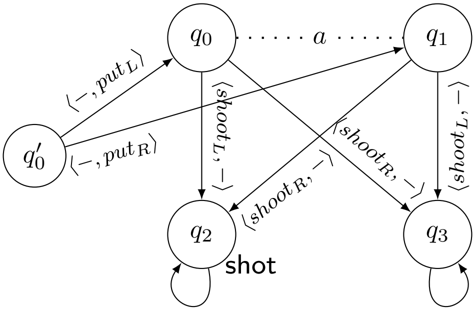

Comparing ATL_{iR} vs. ATL_{IR}

Objective incomplete information vs. perfect information under perfect recall.

By the same reasoning as above:

Corollary 14

$Val(ATL_{i_oR}) \subsetneq Val(ATL_{IR})$

Subjective ability and incomplete information vs. perfect information.

Proposition 15

$Val(\mathsf{ATL}_{\mathrm{i_s}\mathsf{R}}) \subsetneq Val(\mathsf{ATL}_{\mathsf{IR}})$

Subjective ability and incomplete information vs. perfect information.

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$Val(\mathsf{ATL}_{i_sR}) \subsetneq Val(\mathsf{ATL}_{IR})$

$$\mathfrak{M}, q_4 \not\models_{i_sR} \langle\langle a\rangle\rangle \Diamond \mathsf{shot} \rightarrow (\mathsf{shot} \vee \langle\langle a\rangle\rangle \bigcirc \langle\langle a\rangle\rangle \Diamond \mathsf{shot})$$

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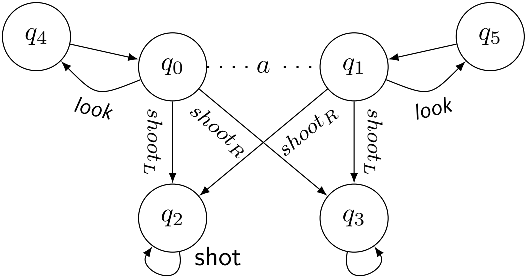

3.2 Perfect vs. Imperfect Recall

Now we compare memoryless and perfect recall strategies.

Is one class of strategies more powerful than the other?

Definition 16 (Tree-like CGS)

Let $\mathfrak{M}$ be a CGS and q be a state in it. M is called tree-like iff there is a state q (the root) such that for every q' there is a unique finite sequence of states leading from q to q'.

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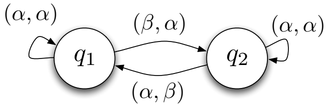
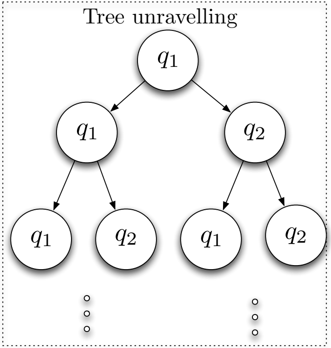
Lemma 17

For every tree-like CGS $\mathfrak{M}$, state q in $\mathfrak{M}$, and $\mathcal{L}_{ATL^*}$-formula φ, we have: $\mathfrak{M}, q \models_{Ir} \varphi$ iff $\mathfrak{M}, q \models_{IR} \varphi$.

Proof idea: The path to a state is unique. No state is visited a second time.

Idea: Fix a state and unravel the model to an infinite tree.





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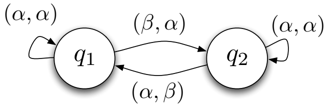

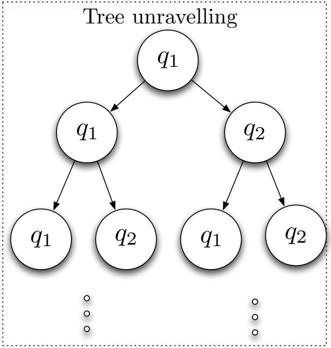

Idea: Fix a state and unravel the model to an infinite tree.

Definition 18 (Tree unfolding)

Let $\mathfrak{M} = (Agt, Q, \Pi, \pi, Act, d, o)$ be a CGS and q be a state in it. The tree-unfolding of $\mathfrak{M}$ starting from state q denoted $T(\mathfrak{M}, q)$ is defined as $(\mathbb{A}\mathrm{gt}, Q', \Pi, \pi', Act, d', o')$ where

- $Q' := \Lambda^{fin}_{\mathfrak{M}}(q)$, (i.e. states correspond to finite histories)
- $d'(a, h) := d(a, last(h))$,
- $o'(h, \vec{\alpha}) := h \circ o(last(h), \vec{\alpha})$, and
- $\pi'(h) := \pi(last(h))$.

We now compare perfect vs. imperfect memory.

Proposition 19

$Val(\mathsf{ATL}^*_{\mathsf{Ir}}) \subsetneq Val(\mathsf{ATL}^*_{\mathsf{IR}})$ (Even: $Val(\mathbf{ATL}^+_{\mathrm{Ir}}) \subsetneq Val(\mathbf{ATL}^+_{\mathrm{IR}})$)

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Membership: If $\models_{Ir} \varphi$ then $Treemodels \models_{Ir} \varphi$ then $Treemodels \models_{IR} \varphi$ then $\models_{IR} \varphi$

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$Val(\mathsf{ATL}^*_{\mathsf{Ir}}) \subsetneq Val(\mathsf{ATL}^*_{\mathsf{IR}})$ *(Even:* $Val(\mathbf{ATL}^+_{\mathsf{Ir}}) \subsetneq Val(\mathbf{ATL}^+_{\mathsf{IR}})$*)*

Membership: If $\models_{Ir} \varphi$ then $Treemodels \models_{Ir} \varphi$ then $Treemodels \models_{IR} \varphi$ then $\models_{IR} \varphi$

Strict inclusion:

$$\mathfrak{M}, q_0 \not\models_{Ir} \langle\langle a \rangle\rangle(\Diamond \mathsf{p}_1 \wedge \Diamond \mathsf{p}_2) \leftrightarrow \langle\langle a \rangle\rangle \Diamond((\mathsf{p}_1 \wedge \langle\langle a \rangle\rangle \Diamond \mathsf{p}_2) \vee (\mathsf{p}_2 \wedge \langle\langle a \rangle\rangle \Diamond \mathsf{p}_1)).$$

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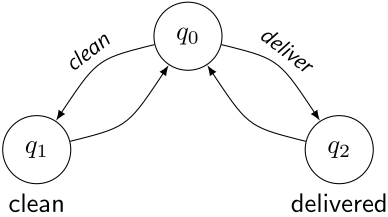
$\mathsf{p}_1 = \text{clean}$
$\mathsf{p}_2 = \text{delivered}$



Comparing $\text{ATL}_{i_o r}$ vs. $\text{ATL}_{i_o R}$

The case of objective ability under incomplete information is similar we only have to take into account epistemic relations in the tree:

$$h \sim_a^{T_{i_o R}(\mathfrak{M},q)} h' \text{ iff } h \approx_a^{\mathfrak{M}} h'$$

Again, memory does not matter:

Lemma 20

For every tree-like CEGS $\mathfrak{M}$, state q in $\mathfrak{M}$, and $\mathcal{L}_{ATL^*}$-formula φ, we have that $\mathfrak{M}, q \models_{\mathrm{i_o}r} \varphi$ iff $\mathfrak{M}, q \models_{\mathrm{i_o}R} \varphi$.

Again, memory does not matter:

Lemma 20

For every tree-like CEGS $\mathfrak{M}$, state q in $\mathfrak{M}$, and $\mathcal{L}_{ATL^*}$-formula φ, we have that $\mathfrak{M}, q \models_{\mathrm{i_o}r} \varphi$ **iff** $\mathfrak{M}, q \models_{\mathrm{i_o}R} \varphi$.

The tree unraveling preserves truth.

Lemma 21

For every node h in $T_{\mathrm{i_o}R}(\mathfrak{M}, q_0)$ it holds that

$$T_{\mathrm{i_o}R}(\mathfrak{M}, q_0), h \models_{\mathrm{i_o}R} \varphi \text{ iff } \mathfrak{M}, last(h) \models_{\mathrm{i_o}R} \varphi.$$

Objective ability: no memory vs. perfect recall.

Proposition 22

$Val(\mathsf{ATL}_{\mathrm{io}\mathsf{r}}) \subsetneq Val(\mathsf{ATL}_{\mathrm{io}\mathsf{R}})$.

Objective ability: no memory vs. perfect recall.

Proposition 22

$Val(\mathsf{ATL}_{i_o r}) \subsetneq Val(\mathsf{ATL}_{i_o R})$.

Recall: $\neg\langle\langle\emptyset\rangle\rangle\Diamond\neg p \leftrightarrow \langle\langle Agt\rangle\rangle\Box p$ for perfect recall.

Objective ability: no memory vs. perfect recall.

Proposition 22

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Recall: $\neg\langle\langle\emptyset\rangle\rangle\Diamond\neg\mathsf{p} \leftrightarrow \langle\langle Agt\rangle\rangle\Box\mathsf{p}$ for perfect recall.

$$\mathfrak{M}, q_0 \not\models_{\mathrm{i_o r}} \neg\langle\langle\emptyset\rangle\rangle\Diamond\neg(\neg\mathsf{suspicious}\vee\neg\mathsf{angry}) \rightarrow \langle\langle a\rangle\rangle\Box(\neg\mathsf{suspicious}\vee\neg\mathsf{angry})$$

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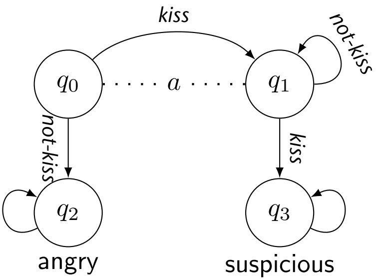
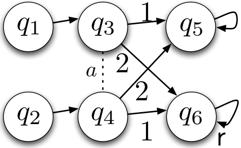


Comparing $ATL_{i_s r}$ vs. $ATL_{i_s R}$

In the case of subjective ability under incomplete information we need a more elaborated tree unraveling. Consider: $\langle\langle a\rangle\rangle \bigcirc \langle\langle a\rangle\rangle \Diamond r$



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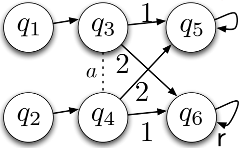

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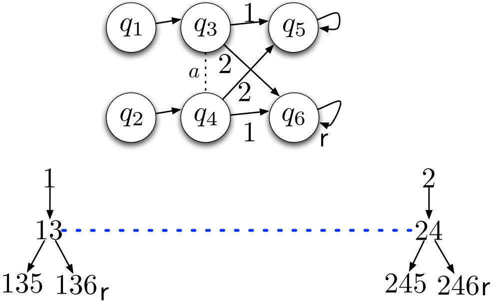

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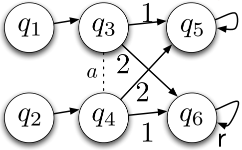

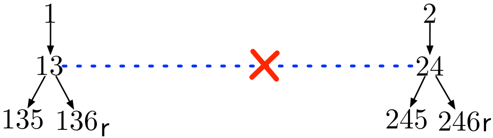

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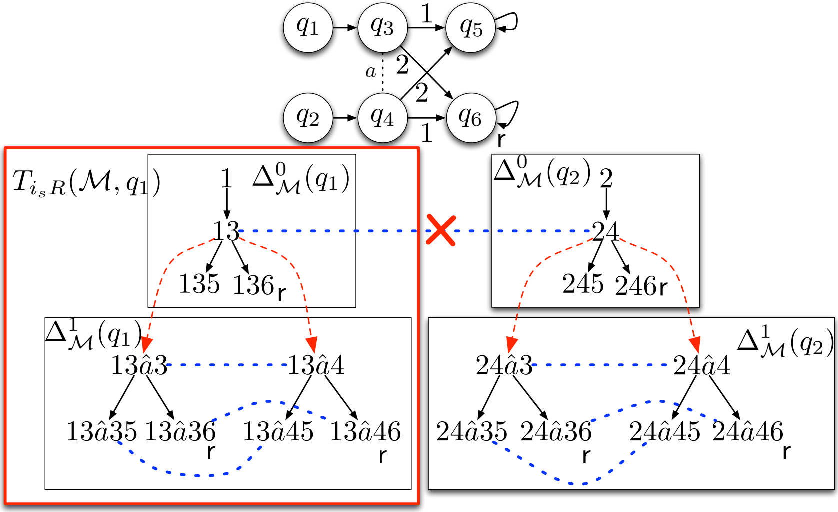

We have the same results as before.

Memory does not matter in trees:

Lemma 23

*For every CEGS $\mathfrak{M}$, state q in $\mathfrak{M}$, and **ATL*** formula φ, it holds that*

$$T_{i_sR}(\mathfrak{M}, q), h \models_{\mathrm{i_s}\boldsymbol{r}} \varphi \text{ iff } T_{\mathrm{i_s}\mathsf{R}}(\mathfrak{M}, q), h \models_{\mathrm{i_s}\boldsymbol{R}} \varphi.$$

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The $\mathrm{i_s}$-tree unraveling preserves truth:

Lemma 24

For every node h in $T_{i_s R}(\mathfrak{M}, q_0)$ it holds that

$$T_{i_s R}(\mathfrak{M}, q_0), h \models_{\mathrm{i_s R}} \varphi \text{ iff } \mathfrak{M}, last(h) \models_{\mathrm{i_s R}} \varphi.$$

Proposition 25

$Val(\mathsf{ATL}_{\mathrm{i}_\mathrm{s}\mathsf{r}}) \subsetneq Val(\mathsf{ATL}_{\mathrm{i}_\mathrm{s}\mathsf{R}})$

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$Val(\mathsf{ATL}_{i_s r}) \subsetneq Val(\mathsf{ATL}_{i_s R})$

Inclusion: $\models_{i_s r} \varphi$ then $Treemodels \models_{i_s r} \varphi$ then $Treemodels \models_{i_s R} \varphi$ then $\models_{i_s R} \varphi$

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Strict inclusion:

$$\mathfrak{M}, q_0 \not\models_{i_s r} \langle\langle a \rangle\rangle \bigcirc \langle\langle a \rangle\rangle \Diamond \mathsf{p} \rightarrow \langle\langle a \rangle\rangle \Diamond \mathsf{p}.$$

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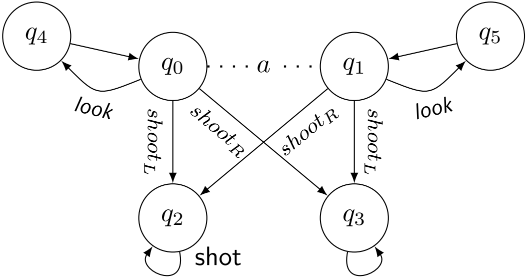

3.3 Subjective vs. Objective Ability

Proposition 26

$Val(\mathsf{ATL}_{i_o x}) \not\subseteq Val(\mathsf{ATL}_{i_s y})$ *for* $x, y \in \{r, R\}$.

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Formula $\Phi_2 \equiv \langle\langle a\rangle\rangle \Diamond \mathsf{p} \rightarrow \mathsf{p} \vee \langle\langle a\rangle\rangle \bigcirc \langle\langle a\rangle\rangle \Diamond \mathsf{p}$ is valid in $Val(\mathbf{ATL}_{i_o x})$

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$$\mathfrak{M}, q_4 \not\models_{i_s R} \langle\langle a \rangle\rangle \Diamond \mathsf{shot} \rightarrow \mathsf{shot} \vee \langle\langle a \rangle\rangle \bigcirc \langle\langle a \rangle\rangle \Diamond \mathsf{shot}$$

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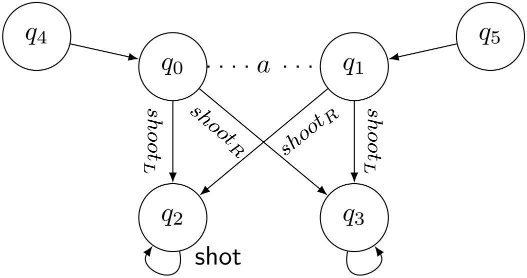
$$\mathfrak{M}, q_4 \not\models_{\mathrm{i_s R}} \langle\langle a\rangle\rangle \Diamond \mathsf{shot} \rightarrow \mathsf{shot} \vee \langle\langle a\rangle\rangle \bigcirc \langle\langle a\rangle\rangle \Diamond \mathsf{shot}$$



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$Val(\mathsf{ATL}_{i_s\mathsf{x}}) \not\subseteq Val(\mathsf{ATL}_{i_o\mathsf{y}})$ for $x, y \in \{r, R\}$.

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$\Phi_6 \equiv \langle\langle a\rangle\rangle \mathsf{N} \langle\langle c\rangle\rangle \bigcirc \langle\langle a\rangle\rangle \bigcirc \mathsf{p} \rightarrow \langle\langle a, c\rangle\rangle \Diamond \mathsf{p}$ is valid in $\mathbf{ATL}_{i_s\mathsf{x}}$

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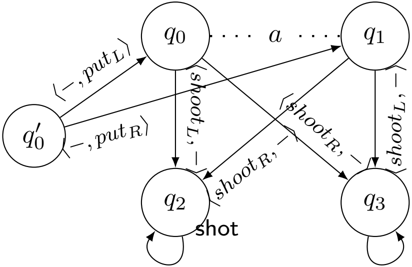
$$\mathfrak{M}, q_0' \not\models_{\mathrm{i_o R}} \langle\langle a\rangle\rangle \mathsf{N} \langle\langle c\rangle\rangle \bigcirc \langle\langle a\rangle\rangle \bigcirc \mathsf{p} \rightarrow \langle\langle a, c\rangle\rangle \Diamond \mathsf{p}$$



(Plus an agent c with no choices.)

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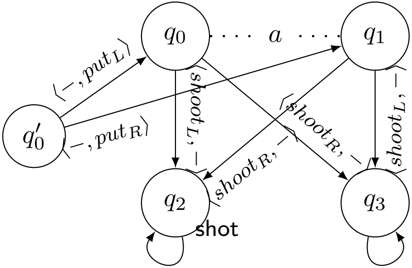


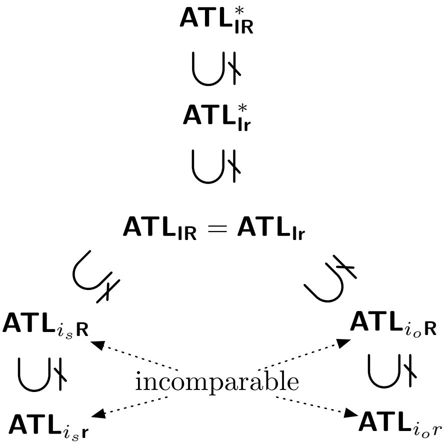
(Plus an agent c with no choices.)

So, we have: $Val(\mathsf{ATL}_{\mathrm{i_s}\mathsf{y}})$ and $Val(\mathsf{ATL}_{\mathrm{i_o}\mathsf{z}})$ are incomparable for every $\mathrm{y}, \mathrm{z} \in \{\mathrm{R}, \mathrm{r}\}$.

4. Conclusions

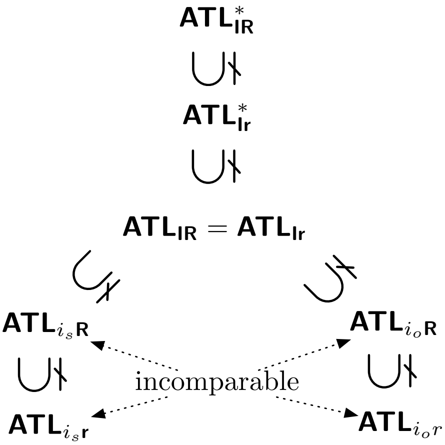
Overview of the Results

- "All" semantic variants are different on the level of general properties; before our study, it was by no means obvious.



Overview of the Results

- "All" semantic variants are different on the level of general properties; before our study, it was by no means obvious.
- Strong pattern of subsumption (memory and information)



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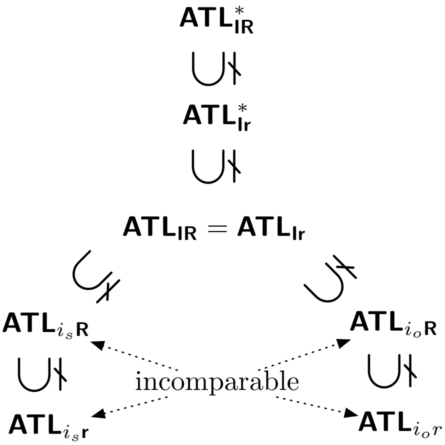
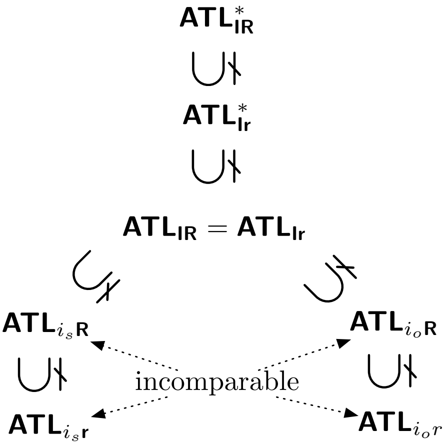
- "All" semantic variants are different on the level of general properties; before our study, it was by no means obvious.
- Strong pattern of subsumption (memory and information)
- Very natural when you see it (not obvious before).
- Some proofs are nontrivial



Overview of the Results

- "All" semantic variants are different on the level of general properties; before our study, it was by no means obvious.
- Strong pattern of subsumption (memory and information)
- Very natural when you see it (not obvious before).
- Some proofs are nontrivial
- In particular: non-validities are interesting.



Thank you for your attention!

Questions?