Outline

Set theory is the standard foundational language for all of mathematics. A good knowledge of basic set theoretic techniques is important for every logician, independent of whether they work in mathematical logic, philosophical logic or applications of logic. The fundamental concepts that make set theory so important are the idea of transfinite counting (“counting beyond infinity") and that of distinguishing different infinite sets by their size (“cardinality"). Beyond the fundamental concepts, set theory is central for logic as its connection to topology and classification of sets by their descriptive complexity and the study of infinite games are genuinely set-theoretic in nature and use the above-mentioned two fundamental concepts.

In our 10-hour set theory course, we'll introduce the mentioned fundamental concepts and will see applications in descriptive set theory and the theory of infinite games. The course has three chapters.

Chapter 1. BASICS (taught by A. Walczak-Typke):

The motivation behind set theory. The first-order language of set theory. The axioms of set theory. Orders and Ordinals. Transfinite induction and transfinite recursion. Cardinal numbers. Basic cardinal arithmetic. Cofinalities.

Chapter 2. DESCRIPTIVE SET THEORY (taught by S.M. Srivastava):

The Borel hierarchy and universal sets. The projective hierarchy. Prewellordering property. Scale property.

Chapter 3. INFINITE GAMES (taught by B. Löwe):

Basics of infinite games. Existence of a non-determined game. Open determinacy theorem of Gale and Stewart. Large cardinals. Determinacy theorems from large cardinals.