

# Problem of Non-existence: a Free Logic Approach

- An ancient problem concerning non-existence is – if a thing does not exist, how can one deny *its* existence?

- Of any fictitious character, of anything non-existent, anything impossible it is intuitively true to say that it does not exist.
- Examples:
  1. Vulcan does not exist.
  2. The golden mountain does not exist.
  3. The round square does not exist.

- The main point is to give an explanation of our understanding of assertions involving non-designating singular terms, that would respect our linguistic intuition.
- The problem lies in that the standard explanation of truth and falsity is not available here outright.

# Solution on the Russell-Quine Line

To consider expressions of the form ‘the so-and-so’, and also ordinary proper names as ultimately eliminable in terms of quantifiers, propositional functions, and identity.

‘Vulcan does not exist.’ is translated as –  
 $\sim(\exists x)[x \text{ Vulcanizes} \ \& \ (y)(y \text{ Vulcanizes} \ \rightarrow y=x)]$ .

This approach considers that -

- i) assertions involving non-designating terms **imply** existence of that which are purported to be designated by those singular terms.
- ii) Thus, simple singular assertions become false when singular terms involved in them fail to designate.
- iii) The so called singular terms, whether designating or non-designating, are not referring expressions.

# A Solution within a System of Free Logic with Supervaluational Model

- Unlike the formal language of a standard first order quantification logic (SQ), the formal language of a system of free quantification logic (FQ) contains -
- a monadic predicate 'E!' ('E!*t*' reads '*t* exists', '*t*' is an individual term), satisfying
- Axioms –1.  $(x)A \rightarrow (E!a \rightarrow A(a/x))$   
2.  $(x)E!x$

# Supervaluational Model in Three Stages:

## Stage-I

- ◇  $\mathcal{M} = \langle D, f \rangle$  is a model, where  $D$  is a possibly empty set of objects, and  $f$  is an interpretation function partially defined on the set of individual constants, and totally defined on predicates, such that,
  - i)  $f(a) \in D$ , if  $f(a)$  is defined, 'a' individual constant.

- ii)  $f(P) \in \mathcal{P}(D^n)$ ,  $P$   $n$ -adic predicate.
- iii) Every member of  $D$  has a name.

A statement is assigned truth value in  $\mathcal{M}$  by a valuation function  $V_{\mathcal{M}}$  :

- i) a) if  $f(a_i)$  is defined for each  $i = 1, \dots, n$ , then  $V_{\mathcal{M}}(Pa_1, \dots, a_n) = T$  if  $\langle f(a_1), \dots, f(a_n) \rangle \in f(P)$ ;
- i) Otherwise, false.



(b) if  $f(a_i)$  is undefined for some  $i = 1, \dots, n$ ,

$V_{\mathcal{M}}(Pa_1, \dots, a_n)$  is neither true, nor false.

ii)  $V_{\mathcal{M}}(E!a) = F$  iff  $f(a)$  is undefined in  $\mathcal{M}$ .

## Stage – II

- A completion  $\mathcal{M}^*$  of  $\mathcal{M} = \langle D, f \rangle$  is a model  $\langle D^*, f^* \rangle$  s.t. -

- i)  $D$  is a subset of  $D^*$ ,  $D^*$  non empty.
- ii)  $f^*(a) \in D^*$  is defined for all individual constants 'a'.
- iii)  $f^*(a) = f(a)$  if  $f(a)$  is defined.
- iv)  $f(P)$  is a subset of  $f^*(P)$ ,  $P$  n-adic predicate.

- A statement is assigned truth value in  $\mathcal{M}^*$  vis-à-vis  $\mathcal{M}$  by a valuation function  $V_{\mathcal{M}^*}^*$  as:

- i) a)  $V_{\mathcal{M}^*}^*(Pa_1, \dots, a_n) = T/F$  in  $\mathcal{M}^*$   
 accordingly as  $V_{\mathcal{M}}(Pa_1, \dots, a_n) = T/F$  in  $\mathcal{M}$ .
- b) If 'Pa<sub>1, ..., a<sub>n</sub></sub>' has no truth value in  $\mathcal{M}$ ,  
 then  $V_{\mathcal{M}^*}^*(Pa_1, \dots, a_n)$  is to be evaluated  
 independently of  $\mathcal{M}$ .
- ii)  $V_{\mathcal{M}^*}^*(E!a) = T/F$  vis-à-vis  $\mathcal{M}$  accordingly as  
 $V_{\mathcal{M}}(E!a) = T/F$  in  $\mathcal{M}$ .

## Stage – III

- Supervaluation for a model  $\mathcal{M}$  is the function  $S_{\mathcal{M}}$  s.t. –
  - i)  $S_{\mathcal{M}}(A) = T$  if  $V_{\mathcal{M}^*}^*(A) = T$ , for all completion  $\mathcal{M}^*$  of  $\mathcal{M}$ .
  - ii)  $S_{\mathcal{M}}(A) = F$  if  $V_{\mathcal{M}^*}^*(A) = F$ , for all completion  $\mathcal{M}^*$  of  $\mathcal{M}$ .

iii)  $S_{\mathcal{M}}(A)$  is neither true, nor false iff  $A$  is true in  $\mathcal{M}^*$  **vis-à-vis**  $\mathcal{M}$  for some  $\mathcal{M}^*$  based on  $\mathcal{M}$ , and is false **vis-à-vis**  $\mathcal{M}$  in others.

# Conclusion:

- It is alleged that Russell-Quine solution to the problem of non-existence/non-designating singular terms is counter intuitive with respect to common understanding of ordinary language.
- On the other hand, Strawson's view is nearer to common linguistic understanding.
- However, the question remains how singular existential assertions or their negations are to be explained on Strawson's view.

- It appears that an appropriate system of free logic with supervaluational model structure can answer this question in harmony with our understanding of negative existential assertions as true, while keeping up the spirit of Strawson's approach.

- References:

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