

Gentzen's consistency proof for arithmetic

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Abstract

David Hilbert pioneered the foundational programme of 'formalism', whose aim can be roughly paraphrased as:

- I. Axiomatize the whole of mathematics
- II. Prove that the axioms obtained in step I are consistent.

Among other things, this led to the birth of the beautiful subject of 'proof theory'.

In 1931, Gödel showed, with his first incompleteness theorem, that step I was not achievable, and with his second incompleteness theorem, that the second was not, too – if by proofs one meant 'finitistic' proofs. In this context, Gerhard Gentzen, in 1936, proved one of the gems of proof theory, a result which states that the consistency of Peano arithmetic (including full induction over the natural numbers) was provable by 'transfinite' means (assuming induction over certain well orderings of the natural numbers, but for a restricted set of formulas). The significance of his consistency proof also lies in the fact that he developed several fundamental tools along the way, which revolutionized the study of proofs.

In this talk, we will have a bird's eye view of Gentzen's theorem.