MSO 201a: Probability & Statistics
Quiz-I
2019-2020-II Semester

Time Allowed: 45 Minutes **Maximum Marks: 25**

Model Solutions

Problem No. 1:

In a probability space $(\Omega, \mathcal{F}, P)$, let A, B and C be events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{4}$, $P(A \cap B) = \frac{1}{4}$, $P(A \cap C) = \frac{1}{5}$, $P(B \cap C) = \frac{1}{5}$ and $P(A \cap B \cap C) = \frac{1}{6}$. Let D be the event that exactly one the event among A, B and C occurs. Derive the value of $P(D)$.

8 Marks

Solution

$$P(D) = P(A) + P(B) + P(C) - 2\left[P(A \cap B) + P(A \cap C) + P(B \cap C)\right] + 3P(A \cap B \cap C) \quad \ldots \quad \boxed{5 \text{ MARKS}}$$

$$= \frac{17}{60}$$

$$\boxed{P(D) = \frac{17}{60}} \quad \ldots \quad \boxed{3 \text{ MARKS}}$$

Problem No. 2:

Let X be a random variable with distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{4}, & 0 \le x < 1 \\ \frac{x}{3}, & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}.$$

Derive the values of $P(X \in \{0, 1, 2\})$, $P(1 \le X < 2)$, $P(1 \le X \le 2)$ and $P(1 < X < 2)$.

3+2+2+2 = 9 Marks

Solution

$P(X \in \{0,1,2\}) = P(X=0) + P(X=1) + P(X=2)$

$= [F(0) - F(0-)] + [F(1) - F(1-)] + [F(2) - F(2-)]$

$= 0 + \frac{1}{12} + \frac{1}{3} = \frac{5}{12}$ ... 3 MARKS

$P(1 \le X < 2) = F(2-) - F(1-) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$ ... 2 MARKS

$P(1 \le X \le 2) = F(2) - F(1-) = 1 - \frac{1}{4} = \frac{3}{4}$ ... 2 MARKS

$P(1 < X < 2) = F(2-) - F(1) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ ... 2 MARKS

Problem No. 3:

Let $F: \mathbb{R} \to \mathbb{R}$ be defined by

$$F(x) = \begin{cases} 0, & x < 0 \\ \dfrac{x+4}{16}, & 0 \le x < 2 \\ \dfrac{x^2+4b}{32}, & 2 \le x < 4 \\ \dfrac{x+c}{8}, & 4 \le x \le 6 \\ 1, & x > 6 \end{cases}$$

where b and c are real constants. Find the values of b and c so that F is a distribution function of some random variable.

4+4 = 8 Marks

Solution

$F(6) = F(6+) \Rightarrow \frac{6+c}{8} = 1 \Rightarrow c = 2$ ... 4MARKS

$F(2-) \le F(2) \Rightarrow \frac{6}{16} \le \frac{1+b}{8} \Rightarrow b \geq 2$

$F(4-) \le F(4) \Rightarrow \frac{4+b}{8} \le \frac{4+c}{8} \Rightarrow b \le 2$

$\Rightarrow b = 2$

... 4MARKS