rigorously tested for their ability to produce the oscillatory states usually associated with nonlinear effects. In addition, it can be argued, as by Deutsch (*10*), that the Navier-Stokes equations, although capable of simulating mean macroscopic characteristics, are inappropriate for determining sensitivities to initial conditions. Because all objects, including those in the climate system, really obey quantum theory not classical mechanics and quantum theory does not show such sensitivity to initial conditions, perhaps this modeling approach provides the wrong estimate of the real world sensitivity.

The uncertain importance of complexity in climate has implications for the resources needed to model the climate system and provide future forecasts. In practical terms, the questions become what spatial and temporal scales must be included in models, and how accurate must the depiction of the specific physical processes be? Depending on the perceived importance of the nonlinear effects, these questions may have very different answers.

From the point of view that focuses on the net radiation, detailed physics and fine scales are required only when necessary for modeling those processes that have the largest impact on the available energy. An appreciation of exactly what those scales must be awaits better understanding of some of the phenomena, for example, convection and cloud formation. Other scales and physical details are important primarily for localized impacts. For example, we probably need better understanding of how water moves through the soil, which includes both stochastic flow through a porous media and pipe flow through an irregular distribution of worm and root holes, if we truly want to be able to predict water availability in specific regions; a prime target would be forecasting the future recharge of the Ogallala aquifer, which provides much of the water for irrigation in the southwestern United States and is already being depleted (*11*).

If, on the other hand, there is a need to account for the various nonlinear effects and their up-scale potential, then the small scales acquire greater importance, as the key interactions that govern transitions from one state to the other may depend on local processes. Palmer (*12*, pp. 419–420) argues that "it may not be enough for climate models to have fluxes that are accurate to 4 W m^{-2} on global scales; they may also have to be accurate to 4 W m^{-2} in specific key sensitive regions, even if we are only interested in the hemispheric-mean response to imposed CO_2 doubling." Similarly, if the change in El Niño frequencies in the future is to be investigated, this imposes stringent requirements on modeling scales: None of the models used to simulate climate change seem to have sufficient resolution in the tropical oceans to induce realistic El Niños (*4*).

Conclusion: Limits to Forecasting?

Where does this leave us? Questions concerning the future climate in general will probably continue to be dominated by uncertainties in the radiative feedbacks. These feedbacks may be influenced by the system's nonlinearities and the future patterns of variability, but we do not know by how much. On the regional scale, the nonlinearities might play a larger role; they also might be extremely difficult to forecast. Climate, like weather, will likely always be complex: determinism in the midst of chaos, unpredictability in the midst of understanding.

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VIEWPOINT

Complexity and the Economy

W. Brian Arthur

After two centuries of studying equilibria—static patterns that call for no further behavioral adjustments—economists are beginning to study the general emergence of structures and the unfolding of patterns in the economy. When viewed in out-of-equilibrium formation, economic patterns sometimes simplify into the simple static equilibria of standard economics. More often they are ever changing, showing perpetually novel behavior and emergent phenomena. Complexity portrays the economy not as deterministic, predictable, and mechanistic, but as process dependent, organic, and always evolving.

Common to all studies on complexity are systems with multiple elements adapting or reacting to the pattern these elements create. The elements might be cells in a cellular automaton, ions in a spin glass, or cells in an immune system, and they may react to neighboring cells' states, or local magnetic moments, or concentrations of B and T cells. Elements and the patterns they respond to vary from one context to another. But the elements adapt to the world—the aggregate pattern—they co-create. Time enters naturally here via the processes of adjustment and change: As the elements react, the aggregate changes; as the aggregate changes, elements react anew. Barring the reaching of some asymptotic state or equilibrium, complex systems are systems in process that constantly evolve and unfold over time.

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Such systems arise naturally in the economy. Economic agents, be they banks, consumers, firms, or investors, continually adjust their market moves, buying decisions, prices, and forecasts to the situation these moves or decisions or prices or forecasts together create. But unlike ions in a spin glass, which always react in a simple way to their local magnetic field, economic elements (human agents) react with strategy and foresight by considering outcomes that might result as a consequence of behavior they might undertake. This adds a layer of complication to economics that is not experienced in the natural sciences.

Conventional economic theory chooses not to study the unfolding of the patterns its agents create but rather to simplify its questions in order to seek analytical solutions. Thus it asks what behavioral elements (actions, strategies, and expectations) are consistent with the aggregate patterns these behavioral elements co-create? For example, general equilibrium theory asks what prices and quantities of goods produced and consumed are consistent with (would pose no incentives for change to) the overall pattern of prices and quantities in the economy's markets. Game theory asks what moves or choices or allocations are consistent with (are optimal given) other agents' moves or choices or allocations in a strategic situation. Rational expectations economics asks what forecasts (or expectations) are consistent with (are on average validated by) the outcomes these forecasts and expectations together create. Conventional economics thus studies consistent patterns: patterns in behavioral equilibrium that would induce no further reaction. Economists at the Santa Fe Institute, Stanford, MIT, Chicago, and other institutions are now broadening this equilibrium approach by turning to the question of how actions, strategies, or expectations might react in general to (might endogenously change with) the aggregate patterns these create (*1*, *2*). The result—complexity economics—is not an adjunct to standard economic theory but theory at a more general, out-of-equilibrium level.

The type of systems I have described become especially interesting if they contain nonlinearities in the form of positive feedbacks. In economics, positive feedbacks arise from increasing returns (*3*, *4*). To ensure that a unique, predictable equilibrium is reached, standard economics usually assumes diminishing returns. If one firm gets too far ahead in the market, it runs into higher costs or some other negative feedback, and the market is shared at a predictable unique equilibrium. When we allow positive feedbacks, or increasing returns, a different outcome arises. Consider the market for online services of a few years back, in which three major companies competed: Prodigy, Compuserve, and America Online. As each gained in membership base, it could offer a wider menu of services as well as more members to share specialized hobby and chat room interests with—that is, there were increasing returns to expanding the membership base. Prodigy was first in the market, but by chance and strategy America Online got far enough ahead to gain an unassailable advantage. Today it dominates. Under different circumstances, one of its rivals might have taken the market. Notice the properties here: a multiplicity of potential solutions; the outcome actually reached is not predictable in advance; it tends to be locked in; it is not necessarily the most efficient economically; it is subject to the historical path taken; and although the companies may start out equal, the outcome is asymmetrical. These properties have counterparts in nonlinear physics where similar positive feedbacks are present. What economists call multiple equilibria, nonpredictability, lock-in, inefficiency, historical path dependence, and asymmetry, physicists call multiple metastable states, unpredictability, phase or mode locking, high-energy ground states, nonergodicity, and symmetry breaking (*5*).

Increasing returns problems have been discussed in economics for a long time. A hundred years ago, Alfred Marshall (*6*) noted that if firms gain advantage as their market share increases, "whatever firm first gets a good start will obtain a monopoly." But the conventional static equilibrium approach gets stymied by indeterminacy: If there is a multiplicity of equilibria, how might one be reached? The process-oriented complexity approach suggests a way to deal with this. In the actual economy, small random events happen; in the online services case, events such as random interface improvements, new offerings, and word-of-mouth recommendations. Over time, increasing returns magnify the cumulation of such events to select the outcome randomly. Thus, increasing returns problems in economics are best seen as dynamic processes with random events and natural positive feedbacks—as nonlinear stochastic processes. This shift from a static outlook into a process orientation is common to complexity studies. Increasing returns problems are being studied intensively in market allocation theory (*4*), international trade theory (*7*), the evolution of technology choice (*8*), economic geography (*9*), and the evolution of patterns of poverty and segregation (*10*). The common finding that economic structures can crystallize around small events and lock in is beginning to change policy in all of these areas toward an awareness that governments should avoid both extremes of coercing a desired outcome and keeping strict hands off, and instead seek to push the system gently toward favored structures that can grow and emerge naturally. Not a heavy hand, not an invisible hand, but a nudging hand.

Once we adopt the complexity outlook, with its emphasis on the formation of structures rather than their given existence, problems involving prediction in the economy look different. The conventional approach asks what forecasting model (or expectations) in a particular problem, if given and shared by all agents, would be consistent with (would be on average validated by) the actual time series this forecasting model would in part generate. This "rational expectations" approach is valid. But it assumes that agents can somehow deduce in advance what model will work and that everyone "knows" that everyone knows to use this model (the common knowledge assumption.) What happens when forecasting models are not obvious and must be formed individually by agents who are not privy to the expectations of others?

Consider as an example my El Farol Bar Problem (*11*). One hundred people must decide independently each week whether to show up at their favorite bar (El Farol in Santa Fe). The rule is that if a person predicts that more that 60 (say) will attend, he or she will avoid the crowds and stay home; if he predicts fewer than 60, he will go. Of interest are how the bar-goers each week might predict the numbers of people showing up, and the resulting dynamics of the numbers attending. Notice two features of this problem. Our agents will quickly realize that predictions of how many will attend depend on others' predictions of how many will attend (because that determines their attendance). But others' predictions in turn depend on their predictions of others' predictions. Deductively there is an infinite regress. No "correct" expectational model can be assumed to be common knowledge, and from the agents' viewpoint, the problem is ill defined. (This is true for most expectational problems, not just for this example.) Second, and diabolically, any commonalty of expectations gets broken up: If all use an expectational model that predicts few will go, all will go, invalidating that model. Similarly, if all believe most will go, nobody will go, invalidating that belief. Expectations will be forced to differ.

In 1993, I modeled this situation by assuming that as the agents visit the bar, they act inductively—they act as statisticians, each starting with a variety of subjectively chosen expectational models or forecasting

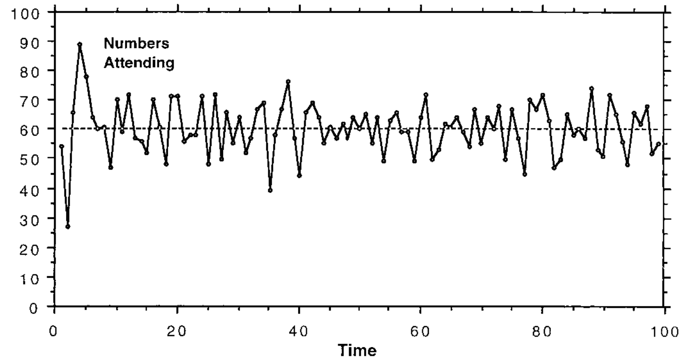

Fig. 1. Bar attendance in the first 100 weeks.

hypotheses. Each week they act on their currently most accurate model (call this their active predictor). Thus agents' beliefs or hypotheses compete for use in an "ecology" these beliefs create. Computer simulation (Fig. 1) showed that the mean attendance quickly converges to 60. In fact, the predictors self-organize into an equilibrium ecology in which, of the active predictors, 40% on average are forecasting above 60 and 60% below 60. This emergent ecology is organic in nature, because although the population of active predictors splits into this 60/40 average ratio, it keeps changing in membership forever. Why do the predictors self-organize so that 60 emerges as average attendance and forecasts split into a 60/40 ratio? Well, suppose 70% of predictors forecasted above 60 for a longish time, then on average only 30 people would show up. But this would validate predictors that forecasted close to 30, restoring the ecological balance among predictions. The 40%/60% "natural" combination becomes an emergent structure. The Bar Problem is a miniature expectational economy with complex dynamics (*12*).

One important application of these ideas is in financial markets. Standard theories of financial markets assume rational expectations—that agents adopt uniform forecasting models that are on average validated by the prices these forecast (*13*). The theory works well to first order. But it doesn't account for actual market anomalies such as unexpected price bubbles and crashes, random periods of high and low volatility (price variation), and the heavy use of technical trading (trades based on the recent history of price patterns). Holland, LeBaron, Palmer, Tayler, and I (*14*) have created a model that relaxes rational expectations by assuming, as in the Bar Problem, that investors cannot assume or deduce expectations but must discover them. Our agents continually create and use multiple market hypotheses—individual, subjective, expectational models—of future prices and dividends within an artificial stock market on the computer. These "investors" are individual, artificially intelligent computer programs that can generate and discard expectational hypotheses and make bids or offers based on their currently most accurate hypothesis. The stock price forms from their bids and offers and thus ultimately from agents' expectations. So this market-in-the-machine is its own self-contained, artificial financial world. Like the bar, it is a mini-ecology in which expectations compete in a world those expectations create.

Within this computerized market, we found two phases or regimes. If parameters are set so that our artificial agents update their hypotheses slowly, the diversity of expectations collapses quickly into homogeneous rational ones. The reason is that if a majority of investors believes something close to the rational expectations forecast, then resulting prices will validate it, and deviant or mutant predictions that arise in the population of expectational models will be rendered inaccurate. Standard finance theory, under these special circumstances, is upheld. But if the rate of updating of hypotheses is increased, the market undergoes a phase transition into a complex regime and displays several of the anomalies observed in real markets. It develops a rich psychology of divergent beliefs that don't converge over time. Expectational rules such as "if the market is trending up, predict a 1% price rise" that appear randomly in the population of hypotheses can become mutually reinforcing: If enough investors act on these, the price will indeed go up. Thus subpopulations of mutually reinforcing expectations arise, agents bet on these (therefore technical trading emerges), and this causes occasional bubbles and crashes. Our artificial market also shows periods of high volatility in prices, followed randomly by periods of low volatility. This is because if some investors discover new profitable hypotheses, they change the market slightly, causing other investors to also change their expectations. Changes in beliefs therefore ripple through the market in avalanches of all sizes, causing periods of high and low volatility. We conjecture that actual financial markets, which show exactly these phenomena, lie in this complex regime.

After two centuries of studying equilibria—static patterns that call for no further behavioral adjustments—economists are beginning to study the general emergence of structures and the unfolding of patterns in the economy. Complexity economics is not a temporary adjunct to static economic theory but theory at a more general, out-of-equilibrium level. The approach is making itself felt in every area of economics: game theory (*15*), the theory of money and finance (*16*), learning in the economy (*17*), economic history (*18*), the evolution of trading networks (*19*), the stability of the economy (*20*), and political economy (*21*). It is helping us understand phenomena such as market instability, the emergence of monopolies, and the persistence of poverty in ways that will help us deal with these. And it is bringing an awareness that policies succeed better by influencing the natural processes of formation of economic structures than by forcing static outcomes.

When viewed in out-of-equilibrium formation, economic patterns sometimes fall into the simple homogeneous equilibria of standard economics. More often they are ever changing, showing perpetually novel behavior and emergent phenomena. Complexity therefore portrays the economy not as deterministic, predictable, and mechanistic but as process dependent, organic, and always evolving.

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