Gravitational Collapse, Negative World and Complex Holism

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Abstract

Building on the engine-pump paradigm of ChaNoXity, this paper argues that complex holism — as the competitive homeostasis of dispersion and concentration — is the operating mode of Nature. Specifically, we show that the negative world $\mathfrak{W}$ is a gravitationally collapsed black hole that was formed at big-bang time $t = 0$ as the pair $(W, \mathfrak{W})$, with W a real world, and gravity the unique expression of the maximal multifunctional nonlinearity of the negative world $\mathfrak{W}$ in the functional reality of W. The temperature of a gravitationally collapsed system does enjoy the relationship $T \propto 1/r$ with its radius, but the entropy follows the usual volumetric alignment with microstates, reducing to the surface approximation only at small r. It is not clear if quantum non-locality is merely a linear manifestation of complex holism, with the interaction of quantum gates in quantum entanglements resulting in distinctive features from the self-evolved structures of complex holism remaining an open question for further investigation.

1 Introduction

In a recent two-part discourse [22], a rigorous, scientific, self-contained, and unified formulation of complex holism has been developed. Science of the last 400 years has essentially evolved by the reductionist tools of linear mathematics in which a composite whole is regarded as the sum of its component parts. Increasingly however, a realization has grown that most of the important manifestations of nature in such diverse fields as ecology, biology, social, economic and the management sciences, beside physics and cosmology, display a holistic behaviour which, simply put, is the philosophy that parts of any whole cannot exist and be understood except in their relation to the whole. These complex self-organizing systems evolve on emergent feedback mechanisms and processes that "interact with themselves and produce themselves from themselves": they are "more than the sum of their parts". Thus society is more than a collection of individuals, life is more than a mere conglomeration of organs as much as human interactions are rarely dispassionate.

Living organisms require both energy and matter to continue living, are composed of at least one cell, are homeostatic, and evolve; life organizes matter into increasingly complex forms in apparent violation of the Second Law of Thermodynamics that forbids order in favour of discord, instability and lawlessness; infact "a living organism continually increases its entropy and thus tends to approach the dangerous state of maximum entropy, which is *death*". However, "It can only keep aloof from it, i.e. stay *alive*, by continually drawing from its environment *"negative entropy"*. It thus maintains itself stationery at a fairly high level of orderliness (= fairly low level of entropy) (by) continually sucking orderliness from its environment" [19]. Holism entails "life (to be) a far-from-equilibrium dissipative structure that maintains its local level of self organization at the cost of increasing the entropy of the larger global system in which the structure is imbedded" [18], "a living individual is defined within the cybernetic paradigm as a system of inferior negative feedbacks subordinated to (being at the service of) a superior positive feedback" [10], "life is a balance between the imperatives of survival and energy degradation" [4], and "life is a special complex system of activating mind and restraining body" [21] identifiable respectively by an anti-thermodynamic backward and thermodynamic forward arrows.

The linear reductionist nature of present mainstream science raises many deep-rooted and fundamental questions that apparently defy logical interpretation within its own framework; as do questions involving socio-economic, collective (as opposed to individualistic), and biological relations. The issues raised by this dichotomy have been well known and appreciated for long leading often to bitter and acrimonious debate between protagonists of the reductionist and holistic camps: ChaNoXity [22] aims at integrating Chaos-Nonlinearity-compleXity into the unified structure of holism that has been able to shed fresh insight to these complex manifestations of Nature. The characteristic features of holism are self-organization and emergence: Self-organization involves the internal organization of an open system to increase from numerous nonlinear interactions among the lower-level hierarchical components *without being guided or managed from outside.* The rules specifying interactions among the system's components are executed using only local information, without reference to the global pattern. *Self-organization* relies on three basic ingredients: positive-negative feedbacks, exploitation-exploration, and multiple interactions. In *emergence*, global-level coherent structures, patterns and properties arise from nonlinearly interacting local-level processes. The structures and patterns cannot be understood or predicted from the behavior or properties of the components alone: the global patterns cannot be reduced to individual behaviour. Emergence involves multi-level systems that interact at both higher and lower level; these emergent systems in turn exert both upward and downward causal influences.

Complexity results from the interaction between parts of a system such that it manifests properties not carried by, or dictated by, individual components. Thus complexity resides in the interactive competitive collaboration[1] between the parts; the properties of a system with complexity are said to "emerge, without any guiding hand". A complex system is an assembly of many interdependent parts, interacting with each other through competitive nonlinear collaboration, leading to self-organized, emergent holistic behaviour.

What is chaos? Chaos theory describes the behavior of dynamical systems — systems whose states evolve with time — that are highly sensitive to initial conditions. This sensitivity, expressing itself as an exponential growth of perturbations in initial conditions, render the evolution of a chaotic system appear to be random, although these are fully deterministic systems with no random elements involved. Chaos responsible for complexity [20] is the eventual outcome of *non-reversible* iterations of one-dimensional *non-injective* maps; noninjectivity leads to irreversible nonlinearity and one-dimensionality constrains the dynamics to evolve with the minimum spatial latitude thereby inducing emergence of new features as required by complexity. In this sense chaos is the maximal ill-posed irreversibility of the maximal degeneracy of multifunctions; features that cannot appear through differential equations. The mathematics involve topological methods of convergence of nets and filters[2] with the multifunctional graphically converged adherent sets effectively enlarging the functional space in the outward manifestation of Nature. Chaos therefore is more than just an issue of whether or not it is possible to make accurate long-term predictions of the system: chaotic systems are necessarily sensitive to initial conditions and topologically mixing with dense periodic orbits; this, however, is not sufficient, and maximal ill-posedness of solutions is a prerequisite for the evolution of complex structures.

ChaNoXity involves a new perspective of the dynamical evolution of Nature based on the irreversible multifunctional multiplicities generated by the equivalence classes from iteration of noninvertible maps, eventually leading to chaos of maximal ill-posedness. The iterative evolution of difference equations is in sharp contrast to the smoothness, continuity, and reversible development of differential equations which cannot lead to the degenerate irreversibility inherent in the equivalence classes of maximal ill-posedness. Unlike evolution of differential equations, difference equations update their progress at each instant with reference to its immediate predecessor, thereby satisfying the crucial requirement of adaptability that constitutes the defining feature of complex systems. Rather than the smooth continuity of differential equations, Nature takes advantage of jumps, discontinuities, and singularities to choose from the vast multiplicity of possibilities that rejection of such regularizing constraints entail. Non-locality and holism, the natural consequences of this paradigm, are to be compared with the reductionist determinism of classical Newtonian reversibility suggesting striking formal correspondence with superpositions, qubits and entanglement of quantum theory. Complex holism is to be understood as complementing mainstream simple reductionism — linear science has after all stood the test of the last 400 years as quantum mechanics is acknowledgedly one of the most successful yet possibly among the most mysterious of scientific theories. Its success lies in the capacity to classify and predict the physical world — the mystery in what this physical world must be like for it to be as it is supposed to be. For an in-depth analysis of this line of reasoning, [22] may be consulted.

[1]Competitive collaboration — as opposed to reductionism — in the context of this characterization is to be understood as follows: *The interdependent parts retain their individual identities, with each contributing to the whole in its own characteristic fashion within a framework of dynamically emerging global properties of the whole.* Although the properties of the whole are generated by the parts, the individual units acting independently on their own cannot account for the global behaviour of the total.

[2]These are generalizations of the usual concept of sequences and, in what follows, may be read as such.

2 ChaNoXity: The New Science of Complex Holism

The mathematical structure of ChaNoXity is based on the discrete evolution of difference equations rather than on the smooth and continuous unfolding of differential equations. The fundamental goal of chanoxity is to suggest, justify and institute the existence of an anti-thermodynamic arrow that allows open systems the privilege of metaphorically "sucking orderliness from the environment" and thereby survive in the highly improbable state of being "alive". For an exhaustive account of the very brief overview recounted below, reference should be made to [20, 22].

2.1 Mathematics of ChaNoXity. [6, 7, 11, 12]

(A) Topologies. (i) If $\sim$ is an equivalence relation on a set X, the class of all saturated sets $[x]_\sim = \{y \in X : y \sim x\}$ is a topology on X; this *topology of saturated sets* constitutes the defining topology of chaotic systems. In this topology, the neighbourhood system at x consists of all supersets of the equivalence class $[x]_\sim$. (ii) For any subset A of the set X, the *A-inclusion topology on X* comprises $\emptyset$ and every superset of A, while the *A-exclusion topology on X* are all subsets of $X - A$. Thus A is open in the inclusion topology and closed in the exclusion, and in general *every open set of one is closed in the other.* For a $x \in X$, the *x-inclusion* neighbourhood $\mathcal{N}_x$ consists of all non-empty open sets of X which are the supersets of $\{x\}$, while for a point $y \neq x$, $\mathcal{N}_y$ are the supersets of $\{x, y\}$. In the *x-exclusion* topology, $\mathcal{N}_x$ are the non-empty open subsets of $\mathcal{P}(X - \{x\})$ that *exclude* x.

The possibility of generating different topologies on a set is of great practical significance in emergent, self-organizing systems because open sets define convergence properties of nets and continuity characteristics of functions that nature can play around with to its best possible advantage.

(B) Initial-and-Final Topology. The topological theory of convergence of nets and filters in terms of residual and cofinal subsets plays a defining role in the development of this formalism, one of the goals being understanding of the Second Law "dead" state of maximum entropy. We consider this problem as a manifestation of the change of the topologies in $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ induced by a non-injective-surjective map f to a state of *ininality* of *initial* and *final* topologies [12] of X and Y respectively. For a continuous f there may be open sets in X that are not inverse images of open sets of Y, just as it is possible for non-open subsets of Y to contribute to $\mathcal{U}$. When the triple $\{\mathcal{U}, f, \mathcal{V}\}$ is tuned in a manner such that neither is possible, the topologies so generated are the *initial* (smallest/coarsest) and *final* (largest/finest) topologies on X and Y for which $f : X \to Y$ is continuous.

For $e : X \to (Y, \mathcal{V})$, the *preimage or initial topology of X generated by e and $\mathcal{V}$* is[3]

$$\mathrm{IT}\{e; \mathcal{V}\} \triangleq \{U \subseteq X : U = e^{-}(V), V \in \mathcal{V}_{\text{comp}}\} \tag{1}$$

and for $q : (X, \mathcal{U}) \to Y$, the *image or final topology of Y generated by $\mathcal{U}$ and q* is

$$\mathrm{FT}\{\mathcal{U}; q\} \triangleq \{V \subseteq Y : q^{-}(V) = U, U \in \mathcal{U}_{\text{sat}}\}. \tag{2}$$

A bijective *ininal function* $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ is a *homeomorphism,* and ininality for functions that are neither $1 : 1$ nor onto generalizes homeomorphism; thus

$$U, V \in \mathrm{IFT}\{\mathcal{U}; f; \mathcal{V}\} \Leftrightarrow \{f(U)\} = \mathcal{V} \text{ and } U = f^{-}(V)$$

reduces to

$$U, V \in \mathrm{HOM}\{\mathcal{U}; f; \mathcal{V}\} \Leftrightarrow \mathcal{U} = \{f^{-1}(V)\} \text{ and } \{f(U)\} = \mathcal{V}$$

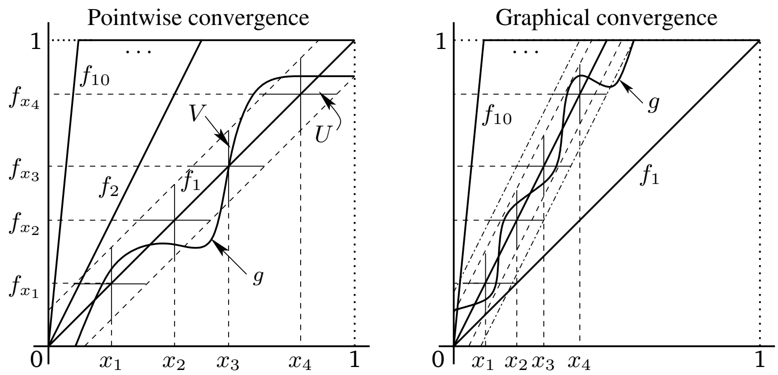
for a bijective, open-continuous function. A homeomorphism $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ renders the homeomorphic spaces $(X, \mathcal{U})$ and $(Y, \mathcal{V})$ topologically indistinguishable in as far as their geometrical properties are concerned. It is our hypothesis that the driving force behind the evolution of a system toward a state of dynamical homeostasis is the attainment of the ininal triple state (X, f, Y) for the system. The ininal interaction f between X and Y generates the smallest possible topology of f-saturated sets on X and the largest possible topology of images of these sets in Y constitutes the state of uniformity represented by the maximum entropy of the second law of thermodynamics. *Ininality of f is simply an instance of non-bijective homeomorphism.*

[3] For a non-bijective function $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$,

$$\mathcal{U}_{\text{sat}} \triangleq \{U \in \mathcal{U} : U = f^{-} f(U)\}$$
$$\mathcal{V}_{\text{comp}} \triangleq \{V \in \mathcal{V} : V = f f^{-}(V) = V \cap f(X)\}$$

Here the "inverse" f^{-} of f is defined by the projective conditions $f f^{-} f = f$ and $f^{-} f f^{-} = f^{-}$.

(C) Multifunctional Extension of Function Spaces is the smallest dense extension Multi(X) of the function space map(X). The main tool in obtaining the space Multi(X) from map(X) is a generalization of pointwise convergence of continuous functions to (discontinuous) functions [20] by a process of graphical convergence of a net of functions illustrated in the figure below. This defines neighbourhoods of $f \in$ map(X,Y) to consist of those functional relations in Multi(X,Y) whose images at any point $x \in X$ lies not only arbitrarily close to $f(x)$ (as in the usual case of topology of *pointwise convergence* $\mathcal{T}_Y$), but whose inverse images at $y = f(x) \in Y$ contain points arbitrarily close to x. Thus the graph of f must not only lie close enough to $f(x)$ at x in V, but must also be such that $f^{-}(y)$ has at least branch in U about x so that f is constrained to cling to f as the number of points on the graph of f increases. Unlike for simple pointwise convergence, *no gaps in the graph of the converged multi is permitted not only on the domain of f, but on its range too.*



(a) Limit: $([-1,0],0) \cup ((0,1],1)$ (b) Limit: $([-1,0],0) \cup (0,[0,1]) \cup ([0,1],1)$

Figure 1: Pointwise and graphical biconvergence. Local neighbourhoods of $f_n(x) = \begin{cases} 0 & -1 \le x \le 0 \\ nx, & 0 < x \le 1/n \\ 1, & 1/n < x \le 1 \end{cases}$ at $(x_i)_1^4$ with corresponding neighbourhoods (U_i) and (V_i) at $(x_i, f(x_i))$. The converged limit in (a) is a discontinuous function, in (b) it is a multifunction. It is this extension, from functional to general relations with its various ramifications, that constitutes the basis of chanoxity.

The usual topological treatment of pointwise convergence of functions is generalized to generate the boundary[4] Multi$_{||}(X,Y)$ between map(X,Y) and multi(X,Y)

$$\text{Multi}(X,Y) = \text{map}(X,Y) \bigcup \text{Multi}_{||}(X,Y) \bigcup \text{multi}(X,Y),$$

observe that the boundary of map(X,Y) in the topology of pointwise biconvergence is a "line parallel to the Y-axis".

Let $(f_\alpha \colon (X,\mathcal{U}) \to (Y,\mathcal{V}))_{\alpha\in\mathbb{D}}$ be the iterative evolutions of a function f. The existence of a maximal non-functional element in this evolutionary process, obtained as the set theoretic "limit" of the net of functions with increasing nonlinearity, does not imply that it belongs to the functional chain as a fixed point. The net defines a corresponding net of increasingly multivalued functions ordered inversely by the relation

$$f_\alpha \preceq f_\beta \Leftrightarrow f_\beta^- \preceq f_\alpha^- . \tag{4}$$

from which it follows that [20]

Chaotic map. Let A be a non-empty closed set of a compact Hausdorff space $(X,\mathcal{U})$. A function $f \in \mathrm{Multi}(X)$ is *maximally non-injective* or *chaotic* on $\mathcal{D}(f) = A$ w.r.t. to $\preceq$ if (a) for any f_i there exists an f_j satisfying $f_i \preceq f_j$ $\forall i < j \in \mathbb{N}$, (b) the dense set $\mathcal{D}_+ := \{x : (f_\nu(x))_{\nu\in\mathrm{Cof}(\mathbb{D})}\}$ of isolated singletons is countable.[5]

[4]The *boundary of A in X* is the set of points $x \in X$ such that every neighbourhood N of x intersects both A and is complement $X - A$:

$$\mathrm{Bdy}(A) \triangleq \{x \in X \colon (\forall N \in \mathcal{N}_x)((N \bigcap A \neq \emptyset) \wedge (N \bigcap (X - A) \neq \emptyset))\} \tag{3}$$

with $\mathcal{N}_x$ the neighbourhood system at x.

[5]The *residual* and *cofinal* subsets

$$\mathrm{Res}(\mathbb{D}) = \{\mathbb{R}_\alpha \in \mathcal{P}(\mathbb{D}) \colon \mathbb{R}_\alpha = \{\beta \in \mathbb{D} \text{ for all } \beta \succeq \alpha \in \mathbb{D}\}\}. \tag{5}$$

$$\mathrm{Cof}(\mathbb{D}) = \{\mathbb{C}_\alpha \in \mathcal{P}(\mathbb{D}) \colon \mathbb{C}_\alpha = \{\beta \in \mathbb{D} \text{ for some } \beta \succeq \alpha \in \mathbb{D}\}\} \tag{6}$$

of a directed set $\mathbb{D}$ are the basic ingredients of the topological theory of convergence of a net of functions.

The collective macroscopic cooperation between map(X) and its extension Multi(X) generates the equivalence classes through fixed points and periodic cycles of f. As all points in a class are equivalent under f, a net or sequence converging to any must necessarily converge to every other in the set. This implies that the cooperation between map(X) and Multi(X) conspires to alter the topology of X to large equivalence classes. This dispersion throughout the domain of f of initial localizations suggests increase in entropy/disorder with increasing chaoticity; complete chaos therefore corresponds to the second law state of maximum entropy enlarging the function space to multifunctions.

(D) The Negative World $\mathfrak{W}$. Motivation: Competitive Collaboration. Of the axioms defining a vector space V, that of the additive inverse which stipulates that for all $u \in V$ there exists an inverse $-u \in V$ such that $u + (-u) = 0$, comprises the crux of *competitive collaboration*. This *participatory* existence $\mathbb{R}_-$ of $\mathbb{R}_+$ inducing a *reverse arrow* in $\mathbb{R}_+$, *competing collaboratively* with the forward arrow in $\mathbb{R}_+$, serves to *complete* the structure of $\mathbb{R}$.

In a parallel vein, let W be a set such that for every $w \in W$ there exists a negative element $\mathfrak{w} \in \mathfrak{W}$ with the property that

$$\mathfrak{W} \triangleq \{\mathfrak{w} \colon \{w\} \bigoplus \{\mathfrak{w}\} = \emptyset\} \tag{7a}$$

defines the negative, or exclusion, set of W[6]. Hence for all $A \subseteq W$ there is a neg(ative) set $\mathfrak{A} \subseteq \mathfrak{W}$ associated with (generated by) A that satisfies

$$\begin{aligned} A \bigoplus \mathfrak{G} &\triangleq A - G, \quad G \leftrightarrow \mathfrak{G} \\ A \bigoplus \mathfrak{A} &= \emptyset. \end{aligned} \tag{7b}$$

The pair $(A, \mathfrak{A})$ act as relative discipliners of each other in the evolving dissipation and tension, "undoing", "controlling", or "stabilizing" the other. The exclusion topology of large equivalence classes in Multi(X) successfully competes with the normal inclusion topology of map(X) to generate a state of dynamic homeostasis in W *that permits out-of-equilibrium complex composites of a system and its environment to coexist despite the privileged omnipresence of the Second Law.* The evolutionary process ceases when the opposing influences in W and its moderator $\mathfrak{W}$ balance in dynamic equilibrium by the generation of the inimal triple.

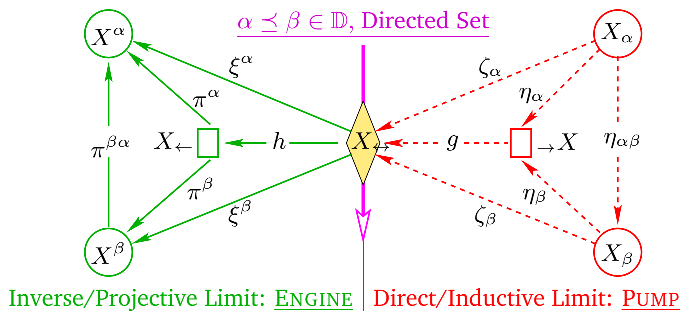


Figure 2a: Direct and inverse limits of direct and inverse systems $(X_\iota, \eta_{\iota\kappa})$, $(X^\iota, \pi^{\kappa\iota})$. Induced homeostasis is attained between the two adversaries by the respective arrows opposing each other as shown in the next figure where expansion to the atmosphere is indicated by decreasingly nested subsets.

(E) Inverse And Direct Limits This abstract conceptual foundation for the existence of a complimentary negative world $\mathfrak{W}$ for every real W permits participatory competitive collaboration between the two to generate self-organizing complex structures as summarized in Figs. 2a and 2b, see Refs. [6, 22] for the necessary details. Summarily, the mathematical goal of chanoxity of establishing the existence of an anti-thermodynamic arrow for every dispersive thermodynamic eventuality of large maximal equivalence classes of open sets through the attainment of the ininal topology, is additionally corroborated by the existence of these complimentary limits[7], [6], possessing the following salient features.

For a given direction $\mathbb{D}$, the connecting maps π and η between the family of subsets $\{X^\alpha\}$ and $\{X_\alpha\}$ are oriented in opposition, the respective inverse and direct limits of the systems being $X_\leftarrow$ and $_\rightarrow X$[8]. The mathematical existence of these opposing limits, applicable to the problem under consideration, validates

[6]Notice that this definition is meaningless if restricted to W or $\mathfrak{W}$ alone; it makes sense, in the manner defined here, only in relation to the pair $(W, \mathfrak{W})$.

[7]For a family of sets $(X_{\kappa\in\mathbb{D}})$ the disjoint union is the set $\coprod_{\alpha\in\mathbb{D}} X_\alpha \triangleq \bigcup_{\alpha\in\mathbb{D}}\{(x,\alpha) : x \in X_\alpha\}$ of ordered pairs, with each X_α being canonically embedded in the union as the pairwise disjoint $\{(x,\alpha) : x \in X_\alpha\}$, even when $X_\alpha \cap X_\beta \neq \emptyset$. If $\{X_k\}_{k\in\mathbb{Z}_+}$ is an increasing family of subsets of X, and $\eta_{mn} \colon X_m \to X_n$ is the inclusion map for $m \leq n$, then the direct limit is $\bigcup X_k$.
For $\{X^k\}_{k\in\mathbb{Z}_+}$ a decreasing family of subsets of X with $\pi^{nm} \colon X^n \to X^m$ the inclusion map, the inverse limit is $\bigcap X^k$.

[8]These limits are conventionally denoted $\varprojlim$ and $\varinjlim$ respectively

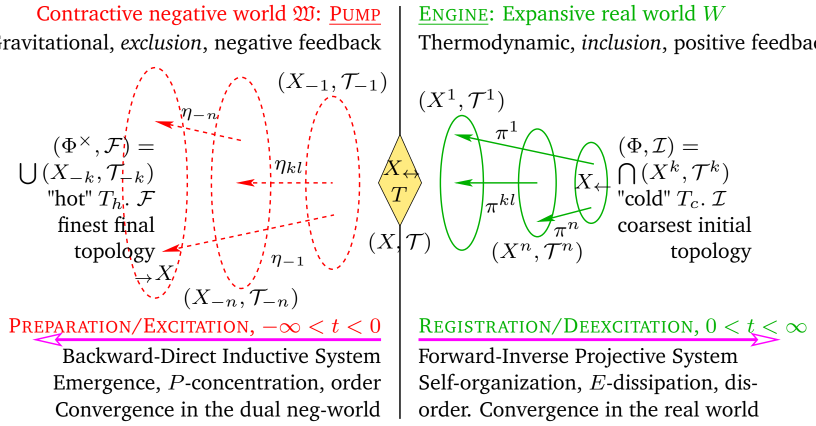


Figure 2b: Intrinsic arrows of time based on inverse-direct limits of inverse-direct systems. Intrinsic irreversibility follows since the thermodynamic forward-inverse arrow is the natural arrow in $\mathbb{R}_+$ equipped with the usual inclusion topology, while the backward-direct positive arrow of $\mathbb{R}_-$ manifests itself as a dual "negative" exclusion topology in $\mathbb{R}_+$. Notice that although E and P are born in $[T_h, T]$ and $[T, T_c]$ respectively, they operate in the domain of the other in the true spirit of competitive-collaboration. The entropy decreases on contraction since the position uncertainty decreases faster than the increase of momentum uncertainty.

the arguments above and bestows the anti-thermodynamic arrow with the sanction of analytic logic. Thus in Fig. 2b, reversal of the direction of $\mathbb{D}$ to generate the forward and backward arrows completes the picture; observe the significant interchange of the relative positions of the two diagrams defining the homeostatic equilibrium $X_{\leftrightarrow}(T)$. If either of the two were to be absent, the remaining would operate within the full gradient $T_h - T_c$; in the homeostatic competitive case, however, the condition T is generated and defined as will be seen below.

The inverse and direct limits are thus generated by opposing directional arrows whose existence follow from very general mathematical principles; thus for example existence of the union of a family of nested sets entails the existence of their intersection, and conversely. As a concrete example, Fig. 2b specializes to *rigged Hilbert spaces* $\Phi \subset \mathcal{H} \subset \Phi^{\times}$

$$\Phi^{\times} \triangleq \bigcup_k \mathcal{H}_{-k} \supset \cdots \supset \mathcal{H}_{-1} \supset \mathcal{H} \supset \mathcal{H}^1 \supset \cdots \supset \bigcap_k \mathcal{H}^k \triangleq \Phi$$

with Φ the space of physical states prepared in actual experiments, and $\Phi^{\times}$ are antilinear functionals on Φ that associates with each state a real number interpreted as the result of measurements on the state. Mathematically, the space of test functions Φ and the space of distributions $\Phi^{\times}$ represent definite and well-understood examples of the inverse and direct limits that enlarge the Hilbert space $\mathcal{H}$ to the *rigged Hilbert space* $(\Phi, \mathcal{H}, \Phi^{\times})$, with $\mathcal{H}$ the homeostatic condition.

2.2 Thermodynamics of ChaNoXity

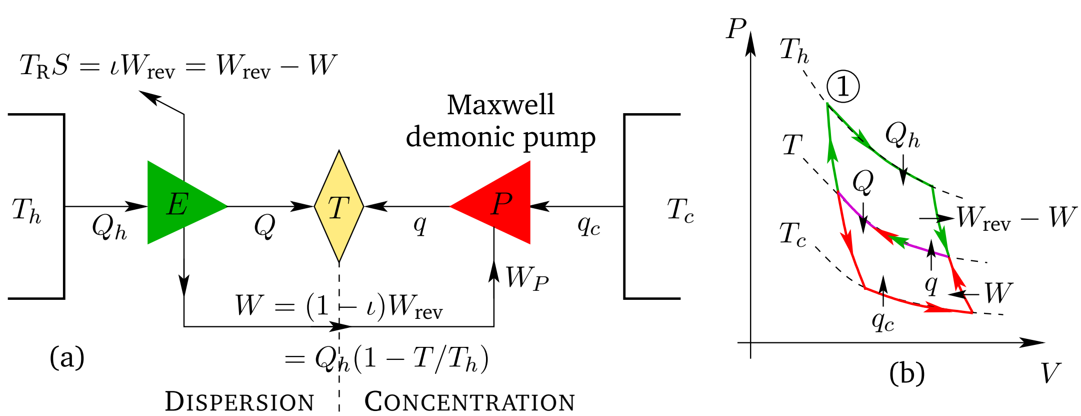


Figure 3: Reduction of the dynamics of opposites to an equivalent engine-pump thermodynamic system.

Assume that a complex adaptive system is distinguished by the complete utilization of a fraction $W := (1-\iota)W_{\text{rev}}$ of the work output of an imaginary reversible engine (T_h, E, T_c) to self-generate the pump P in competitive collaboration with E. The *irreversibility factor*

$$\iota \triangleq \frac{W_{\text{rev}} - W}{W_{\text{rev}}} \in [0,1] \tag{8a}$$

accounts for that part ιW_{rev} of available energy W_{rev} that cannot be gainfully utilized but must be degraded in increasing the entropy of the universe. Hence

$$\iota = \left(\frac{T_{\text{R}}}{W_{\text{rev}}}\right) S \tag{8b}$$

yields the effective entropy

$$S = \frac{W_{\text{rev}} - W}{T_{\text{R}}}. \tag{8c}$$

The self-induced pump decreases the temperature-gradient $T_h - T_c$ to $T_h - T$, $T_c \le T < T_h$, inducing dynamic stability to the system.

Let ι be obtained from

$$\begin{aligned} W_E := Q_h \left(1 - \frac{T}{T_h}\right) &\triangleq W_P \\ &= Q_h(1-\iota)\left(1 - \frac{T_c}{T_h}\right); \end{aligned} \tag{9}$$

hence

$$\iota(T) = \frac{T - T_c}{T_h - T_c} \tag{10a}$$

shows a remarkable formal similarity to the quality

$$x(v) = \frac{v - v_f}{v_g - v_f} \tag{10b}$$

of a two-phase mixture, where $T_h - T_c$ represents the internal energy that is divided into the non-entropic $T_h - T$ free energy A internally utilized to generate the pump P and a reduced $T - T_c$ entropic dissipation by E, with respect to the induced equilibrium temperature T.

The generated pump is a realization of the energy available for useful, non-entropic work arising from reduction of the original gradient $T_h - T_c$ to $T - T_c$. The irreversibility $\iota(T)$ is adapted by the engine-pump system such that the induced instability of P balances the imposed stabilizing effort of E to the best possible advantage of the system and the environment. Hence a measure of the energy in a system that cannot be utilized for work W but must necessarily be dumped to the environment is given by the *generalized entropy*

$$\begin{aligned} TS = \iota W_{\text{rev}} &= W_{\text{rev}} - W && (11a) \\ &= U - A && (11b) \end{aligned}$$

which the system attains by adapting itself *internally* to a state of optimal competitive collaboration.

Figure 3 represents the essence of competitive collaboration: the entropic dispersion of E is proportional to the domain $T - T_c$ of P, and the anti-entropic concentration of P depends on $T_h - T$ of E. Thus an increase in ι can occur only at the expense of P which opposes this tendency; reciprocally a decrease in ι is resisted by E. The induced pump P prevents the entire internal resource $T_h - T_c$ from dispersion at $\iota = 1$ by defining some $\iota < 1$ for a homeostatic temperature $T_c < T < T_h$, with E and P interacting with each other in the spirit of competitive collaboration at the induced interface T.

Defining the equilibrium steady-state representing $X_{\leftrightarrow}$ of homeostatic E-P adaptability $\alpha := \eta_E \zeta_P$, the *equation of state of the participatory universe*

$$\alpha(T) = \left(\frac{T_h - T}{T_h}\right)\left(\frac{T}{T - T_c}\right) \triangleq \frac{q}{Q_h} \tag{12}$$

in the form $Pv = f(T)$, where $P \equiv \zeta_P = 0$ at $T = 0$ and $v \equiv \eta_E$, be the product of the efficiency of a reversible engine and the coefficient of performance of a reversible pump. Fig. 4 for $T_h = 480°$K and

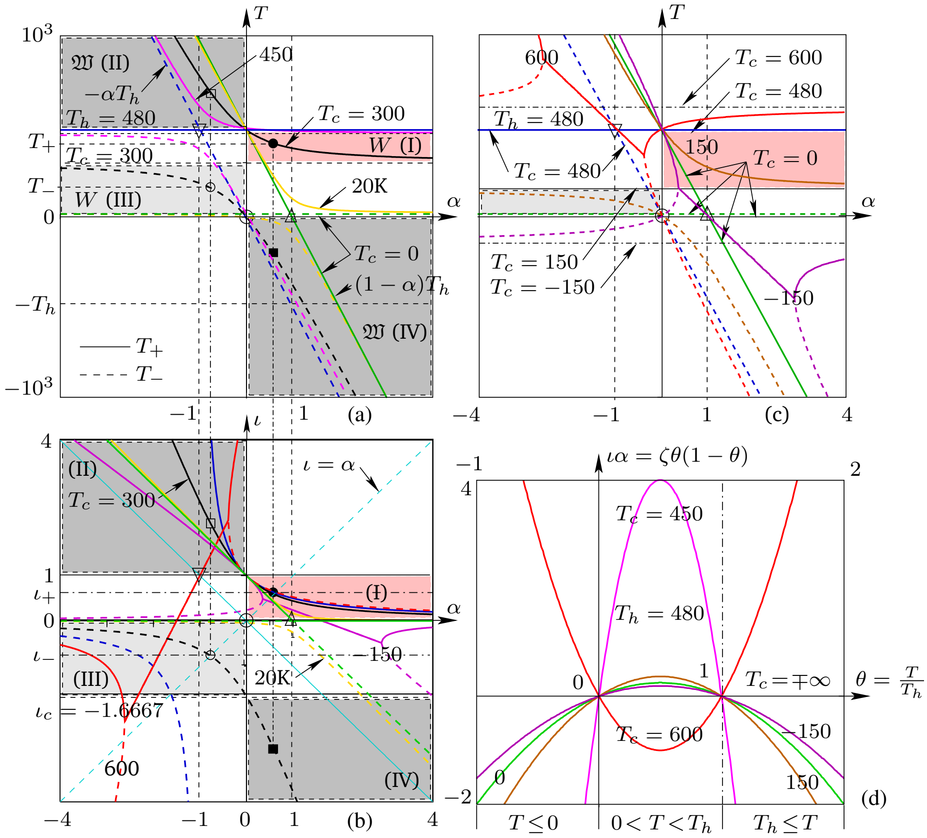


Figure 4: The interactive "participatory universe", $T_h = 480$K. The straight lines connecting the $T < T_c$ and $T > T_h$ segments in (b) and (c) correspond to complex roots.

$T_c = 300$°K shows that the engine-pump duality has the significant property of supporting two different states

$$T_{\pm}(\alpha) = \frac{1}{2}\left[(1-\alpha)T_h \pm \sqrt{(1-\alpha)^2 T_h^2 + 4\alpha T_c T_h}\right] \tag{13a}$$

$$= \begin{cases} ((1-\alpha)T_h,\, 0) = (0,0), & T_c = 0,\ \alpha = 1 \\ (T_h,\, -\alpha T_h) = (T_h, T_h), & T_c = T_h,\ \alpha = -1 \end{cases} \tag{13b}$$

for any given value of α.

Fig. 4(b) suggests that the balancing condition

$$\iota(T) = \alpha(T) \tag{14}$$

can be taken to define the most appropriate equilibrium criterion

$$T_{\pm} = \frac{T_h(T_h + T_c) \pm (T_h - T_c)\sqrt{T_h^2 + 4T_c T_h}}{2(2T_h - T_c)} \tag{15a}$$

$$= \begin{cases} (0.5T_h, 0), & T_c = 0 \\ (T_h, T_h), & T_c = T_h \end{cases} \tag{15b}$$

of the homeostatic complex state.

A complex system can hence be represented as

$$\underbrace{\begin{array}{r} \underline{\text{BACKWARD-DIRECT ARROW}} \\ P\text{-synthesis of concentration,} \\ \text{order, entropy decreasing,} \\ \textit{bottom-up emergence} \,{}_{\rightarrow}C \end{array}}_{\text{collaborative, } (\downarrow)} \bigoplus \underbrace{\begin{array}{l} \underline{\text{FORWARD-INVERSE ARROW}} \\ E\text{-analysis of dispersion,} \\ \text{disorder, entropy increasing,} \\ \textit{top-down self-organization } C_{\leftarrow} \end{array}}_{\text{competitive, } (\uparrow)} \tag{16}$$

$$\Longleftrightarrow$$

$$\text{Synthetic cohabitation of opposites } C_{\leftrightarrow},$$

where $\bigoplus$ denotes a non-reductionist sum of the components of a *top-down engine* and its complimentary *bottom-up pump* that behaves in an organized collective manner with properties that cannot be identified with any of the individual parts but arise from the structure as a whole: these systems cannot dismantle into their components without destroying themselves.[9] Analytic methods cannot simplify them as these techniques do not account for characteristics that belong to no single component but relate to the parts taken together, with all their interactions. Complexity is a dynamical, interactive and interdependent hierarchical homeostasis of P-emergent, ordering instability of collaborative positive feedback in cohabitation with the adaptive, E-organized, disordering stability of competitive negative feedback generating non-reductionist holism that is beyond the sum of its constituents.

This representation of a complex system can be formalized through the

Definition. Complexity. An open thermodynamic system of many interdependent and interacting parts is *complex* if it lives in synthetic competitive cohabitation with its induced negative dual in a state of homeostatic, hierarchical, two-phase dynamic equilibrium of top-down, self-organizing, dispersive thermodynamic engine and a self-induced, bottom-up, emergent, concentrative anti-thermodynamic pump, coordinated and mediated by the environment ("universe").

2.2.1 Complexity: A Two-Phase Mixture of Bottom-Up Collaboration and Top-Down Competition

Consider Fig. 4 for the dual-pair $(W, \mathfrak{W})$ with reference to the formalization represented by Eqs. (10*a*, *b*). Fig. 4(*a, b*) defines four disjoint regions (I), (II), (III), (IV) characterized by the product $\iota\alpha \geq 0$ for W in (I) and (III) and $\iota\alpha \leq 0$ in (II) and (IV) for $\mathfrak{W}$. The significant feature is the complete specification of these regions in terms of the product and the direct linkages of region (I) with (II) through T_+ and of (III) with (IV) through T_-. Considering T_c as a variable with T_h given, produces the bounds of Eqs. (13*b*) and (15*b*) with the rather remarkable property that for the operational range $0 < T_c < T_h$, $T_\pm$ are composed of bifurcated components of $(T_+ = (1-\alpha)T_h,\, T_- = 0)$ at $T_c := 0$ and of $(T_+ = T_h,\, T_- = -\alpha T_h)$, at $T_c := T_h$; thus $T_\pm$ in the operational range are holistic expressions of themselves at the limiting values of 0 and T_h.

The non-trivial range $T_h < T_c < 0$, that makes sense only for negative T for $T_h := +\infty$, is graphed in Fig. 4(c) and (d). Of fundamental significance is the fact that the roots of Eq. (13*a*) form *continuous curves* in these regions, bifurcating as individual holistic components at $\alpha = \pm 1$: note how at these values the continuous curves changes character in disengaging from each other to form separate linear entities before "collaborating" once again in generating the profiles $T_\pm$ in the operating range. These adaptations of the engine-pump system-environment are substantive in the sense that these specific α-values denote physical changes in the global behaviour of the system (and reciprocally of the environment); they mark the critical and triple points to be pursued in Fig. 5. The two-phase complex surface denoted by $\alpha = \iota$ is to be distinguished from the non-complex general Pv region shown as $\alpha = \eta\zeta$. Since the ideal participatory universe satisfies a more involved *nonlinear* equation of state (12) compared to the simple *linear* relationship of an ideal gas, diagram 5(b) is more involved than the corresponding (a), with the transition at the triple point $\alpha = 1$ showing definite distinctive features as compared to the later. While panels (b) and (c) clearly establish that the triple point cannot be accessed from the $\iota = \alpha$ surface and requires a detour through the general $\alpha = \eta\zeta$, it also offers a fresh insight on the origin of the insular nature of the absolute zero temperature $T = 0$.

Among the noteworthy distinctions of Figs. 5(a) and (b), attention should be drawn to Eq. (12) and Fig. 5(b) which show that the 2-phase region $\iota = \alpha$ is distinguished by constancy of α — and hence of the product Pv — just as P and T separately remain constant in Fig. 5(a). At the critical point $v_f = v_g$ of distinguished specific volumes for passage to second order phase transition, $T_c = T_h$ requires T_+ to be equal to T_- which according to Eqs. (13*b*) and (15*b*) can happen only at $\alpha = -1$ corresponding to the $(P_{\mathrm{cr}}, T_{\mathrm{cr}})$ of figure (a). At the other unique adaptability of $+1$, the system passes into region (IV) from (III) just as (I) passes into (II) as $T_c \to T_h$. Observe from Eq. (13*a*) that the limits $T_c \to 0$ and $T_c \to T_h$ are reciprocally inclusive; hence

$$(T_c \to 0) \Longleftrightarrow (T_h \to \infty) \tag{17}$$

allows the self-organizing complex phase-mixture of concentration and dispersion to maintain its state as the condition of homeostatic equilibrium. In this limiting condition then, we are left with the two regions: (I) characterized by $\iota\alpha > 0$ of the complex real world W and (IV) characterized by $\iota\alpha < 0$ of the negative world $\mathfrak{W}$. The three phases of matter of the solid, liquid and gaseous phases of our perception manifests only in W, the negative world not admitting this distinction is a miscible concentrate in all proportions. The reciprocal implication of (17) at the big-bang degenerate singularity $\alpha = \pm 1$ at $t = 0$ [22], instantaneously

[9] The definition of *cybernetics* as the study of systems and processes that "interact with themselves and produce themselves from themselves" by Louis Kauffman remarkably captures this spirit.

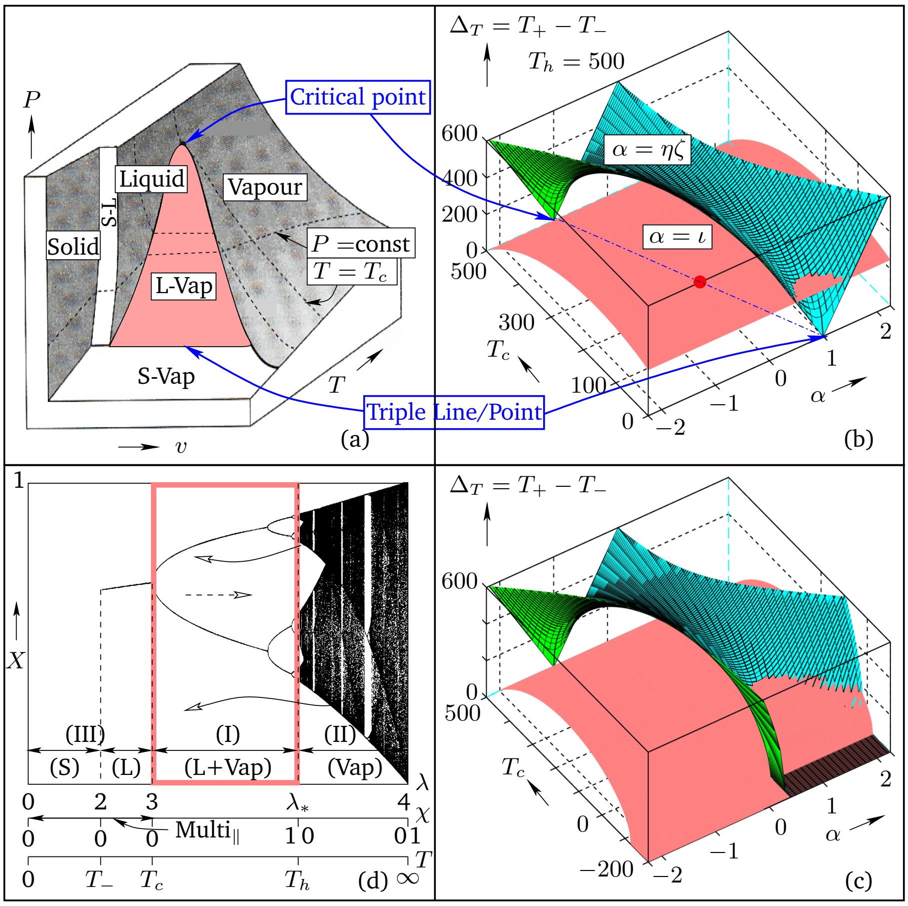


Figure 5: The 2-phase complex $\iota = \alpha$ region, (b) and (c), with the *critical point* $T_c = T_h$ at $\alpha = -1$ in (b), yielding to α-dependent general dependence $\alpha = \eta\zeta$ at low T_c. The *triple point* $\alpha = 1$, $T_c = 0$ is approachable only through this route. Compared to the normal transition of (a), self-organization in (b) occurs for $\alpha \leftrightarrow Pv$ =const, with P, v, T varying according to Eq. (12). $\Delta T = T_+ - T_-$ is taken as an indicator of first-order-second-order transition because of Eqs. (13*b*) and (15*b*).

causes the birth of the $(W, \mathfrak{W})$ duality at some *unique admissible* value of α and $0 < T_c < T_h$, that arise from the complexity criterion $\iota = \alpha$.

Figure 5(d) identifies the complex W on the bifurcation diagram of the logistic map $\lambda x(1-x)$ that we now turn to.

3 The Logistic Map $\lambda \mathrm{x}(1-\mathrm{x})$: A Nonlinear Qubit

A correspondence between the dynamics of the engine-pump system and the logistic map $\lambda x(1-x)$, with the direct iterates $f^i(x)$ corresponding to the "pump" $\mathfrak{W}$ and the inverse iterates $f^{-i}(x)$ to the "engine" W, is summarized in Table 1. The two-phase complex region (I), $\lambda \in (3, \lambda_*)$, $T_+ \in (T_c, T_h)$, $\iota \in (0, 1)$, is the outward manifestation of the tension between the regions (I), (III) on the one hand and (II), (IV) on the other: observe from Eq. (13*a*) and Fig. 4 that at the environment $T_c = (0, T_h)$ the two worlds merge at $\alpha = \pm 1$ bifurcating as individual components for $0 < T_c < T_h$. The logistic map — and its possible generalizations — with its rising and falling branches denoted (↑) and (↓), see Fig. 6, constitutes a perfect example of a *nonlinear qubit,* not represented as a (complex) linear combination: nonlinear combinations of the branches generate the evolving structures, as do the computational base $(1\ 0)^{\mathrm{T}}$ and $(0\ 1)^{\mathrm{T}}$ for the linear qubit. This qubit can be prepared efficiently by its defining nonlinear, non-invertible, functional representation, made to interact with the environment through discrete non-unitary time evolutionary iterations, with the final (homeostatic) equilibrium "measured" and recorded through its resulting complex expressions.

The effective power law $f(x) = x^{1-\chi}$[21] for

$$
\begin{aligned}
\chi &= 1 - \frac{\ln\langle f(x)\rangle}{\ln\langle x\rangle}, \qquad 0 \le \chi \le 1, && (18a)\\
\langle x\rangle &\triangleq 2^N \overset{\lambda=\lambda_*}{\longrightarrow} \infty && (18b)\\
\langle f(x)\rangle &\triangleq 2f_1 + \textstyle\sum_{j=1}^{N}\sum_{i=1}^{2^{j-1}} f_{i,i+2^{j-1}},\ N = 1,2,\cdots,\\
&= \{[(2f_1 + f_{12}) + f_{13} + f_{24}] + f_{15} + f_{26} + f_{37} + f_{48}\} && (18c)
\end{aligned}
$$

and the hierarchical levels $(N = 1)$, $[N = 2]$, $\{N = 3\}$, with $\langle x\rangle$ the 2^N microstates of the basic unstable fixed points resulting from the $N + 1$ macrostates $\{f^i\}_{i=0}^N$ constituting the net feedback $\langle f(x)\rangle$, bestows the complex system with its composite holism. Hence

$$
\chi_N = 1 - \frac{1}{N\ln 2}\ln\left[2f_1 + \sum_{j=1}^{N}\sum_{i=1}^{2^{j-1}} f_{i,i+2^{j-1}}\right] \tag{19}
$$

is the measure of chanoxity, for $f_i = f^i(0.5)$, $f_{i,j} = |f^i(0.5) - f^j(0.5)|$, $i < j$, and

$$
\chi = \iota = \alpha, \qquad \lambda \in (3, \lambda_* := 3.5699456) \tag{20}
$$

in Regions (I) and (III) can be taken as the definite assignment of thermodynamical perspective to the dynamics of the logistic map with $\iota\alpha = \chi^2$, χ being the *measure of chanoxity*, Eq. (18*a*).

$\iota; T; \alpha$	$\lambda; \chi$	x_{fp}
$(-\infty,\ \iota_c]; (-\infty, 0]; [\infty, 0)$	$(0, 1], (1, 2]; 0$	$(\bullet, -), (\circ, -)$
$\iota\alpha < 0$: Multifunctional Simple $\mathfrak{W}$ (IV: S)		
$(\iota_c,\ 0); (0, T_c]; (0, -\infty)$	$(2, 3); 0$	$(\circ, \bullet)$
$\iota\alpha > 0$: **Functional Simple W (III: L)**		
$(0, 1); (T_c, T_h); (\infty, 0)$	$[3, \lambda_*); [0, 1)$	$(\circ, \bullet/\circ)$
$\iota\alpha > 0$: **Functional Complex W (I: L+Vap)**		
$[1, \infty); [T_h, \infty); [0, -\infty)$	$[\lambda_*, 4), [4, \infty); \{0, 1\}$	$(\circ, \circ)$
$\iota\alpha < 0$: Multifunctional Chaotic $\mathfrak{W}$ (II: Vap)		

Table 1: Emergence of the "Participatory Universe", for $0 < T_c < T_h$ in W; $\iota_c = -T_c/(T_h - T_c)$: putting dynamics and thermodynamics together.

Table 1 shows that the dynamics of the logistic map undergoes a discontinuous transition from the monotonically increasing $0 \le \chi < 1$ in $3 \le \lambda < \lambda_*$ of region (I) to a disjoint world at $\chi = 0$ in the fully chaotic $\lambda_* \le \lambda < 4$ of (II) thereby *reducing the chaotic world to one of effective linear simplicity.* Eq. (20), Fig. 4(a), (b) demonstrate that the boundary $\text{Multi}_{||}(X)$ between $W := \text{map}(X)$ and $\mathfrak{W} := \text{Multi}(X)$ comprising the chaotic region $\lambda \in (\lambda_*, 4)$ can occur only for $\chi = 0 = \iota = \alpha$ at $T_c = 0$ and $T := T_- = 0$ (see Eq. (13*a*, *b*)). According to Table 1, the values $\chi = 0$ and $\chi = 1$ of regions (I) and (II) establishes one-one correspondences between $\lambda = 3$, $\lambda \in (\lambda_*, 4)$ and between $\lambda = \lambda_*$, $\lambda \ge 4$. The second interface at $T_c = T_h = \infty$ accounts for a boundaryless transition between these complimentary dual worlds. Hence $T_c \ge T_h$ is to be interpreted to imply $-\infty < T_c \le 0$ of negative temperatures that define $\mathfrak{W}$.

Index of Complexity

Equation (14) for $\iota = \alpha$ leads to

$$
\begin{aligned}
\iota_\pm &= \frac{T_h - 2T_c \pm \sqrt{T_h^2 + 4T_hT_c}}{4T_h - 2T_c} \qquad (21)\\
&= \begin{cases} (0.5, 0), & T_c = 0 \\ \pm\frac{1}{2}\left(\sqrt{5} \mp 1\right), & T_c = T_h \end{cases}
\end{aligned}
$$

at temperatures $T_\pm$ of Eq. (15*a*) denoted as $T_\bullet$ and $T_\circ$ in Fig. 4(a). The complexity σ of a system is expected to depend on both the irreversibility ι and the interaction α; thus the definition

$$\sigma_\pm \triangleq \frac{1}{\ln 2}\begin{cases} -\tilde{\iota}_- \left\{\iota_+ \ln \iota_+ + (1-\iota_+)\ln(1-\iota_+)\right\} \\ -\iota_+ \left\{\tilde{\iota}_- \ln \tilde{\iota}_- + (1-\tilde{\iota}_-)\ln(1-\tilde{\iota}_-)\right\} \end{cases} \tag{22}$$

with $\tilde{\iota}_- = \iota_-/\iota_c \in [0,1]$, ensures the expected two-state, logistic-like, $(\uparrow,\downarrow)$ signature at T_+ and T_-.

4 Quantum Mechanics: A Linear Representation of Chaos

• *Bell's inequalities represent, first of all, an experimental test of the consistency of quantum mechanics. Many experiments have been performed in order to check Bell's inequalities; the most famous involved EPR pairs of photons and was performed by Aspect and co-workers in 1982. This experiment displayed an unambiguous violation of CHSH inequality and an excellent agreement with quantum mechanics. More recently, other experiments have come closer to the requirements of the ideal EPR scheme and again impressive agreement with the predictions of quantum mechanics has always been found. If, for the sake of argument, we assume that the present results will not be contradicted by future experiments with high-efficiency detectors, we must conclude that* NATURE DOES NOT SUPPORT THE EPR POINT OF VIEW. In summary, THE WORLD IS NOT LOCALLY REALISTIC.

There is more to learn from Bell inequalities and Aspect's experiments than merely a consistency test of quantum mechanics. These profound results show us that entanglement is a fundamentally new resource, beyond the realm of classical physics, and that it is possible to experimentally manipulate entangled states. A major goal of quantum information science is to exploit this resource to perform computation and communication tasks beyond classical capabilities. VIOLATION OF BELL'S INEQUALITIES IS A TYPICAL FEATURE OF ENTANGLED STATES. Benenti et al. [2]

Reference to the above, it is natural to inquire if quantum mechanics is indeed a general theory that applies to everything from subatomic particles to galaxies as it is generally believed to be, that is if Nature is indeed governed by entanglements of linear superposition in Hilbert space, or is it an expression of the nonlinear holism of emergence, self-organization, and complexity that we have outlined above? What is clear is that some basic structure of holistic "entanglement" is involved in the expressions of Nature; what is not so clear and is the subject of our present concerns is the question of whether this is linear quantum mechanical or nonlinear, self-organizing-emergent, and complex.

Composite systems in QM are described by tensor products of vector spaces, a natural way of putting linear spaces together to form larger spaces. If V, W are spaces of dimensions n, m, $A\colon V_1 \to V_2$, $B\colon W_1 \to W_2$ are linear operators, then $C := \sum_i \alpha_i A_i \otimes B_i$ on the nm-dimensional linear space $V \otimes W$ defined by $C(|v\rangle \otimes |w\rangle) = \sum_i \alpha_i (A_i |v\rangle \otimes B_i |w\rangle)$, together with the bi-linearity of tensor products, endows $V \otimes W$ with standard properties of Hilbert spaces inherited from its components. Moreover, the state space of a composite system is the tensor product of the state spaces of the components.

In quantum mechanics, the basic unit of classical information of the *b*(inary)(dig)*it* of either "on $|\uparrow\rangle$" of "off $|\downarrow\rangle$", is replaced by the *qubit* of a normalized vector in two-dimensional complex Hilbert space spanned by the orthonormal vectors $|\uparrow\rangle := (1\ 0)^{\mathrm{T}}, |\downarrow\rangle := (0\ 1)^{\mathrm{T}}$. The qubit differs from a classical bit in that it can exist either as $|\uparrow\rangle$ or as $|\downarrow\rangle$ or as a superposition $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ (with $|\alpha|^2 + |\beta|^2 = 1$) of both. The distinguishing feature in the quantum case is a consequence of the linear superposition principle that allows the quantum system to be in any of the 2^N basic states simultaneously, leading to the non-classical manifestations of interference, non-locality and entanglement.

Entanglement is the new quantum resource that distinguishes it so fundamentally from the classical in the sense that with the qubit, the degeneracy of composite entangled states is hugely larger than the $2N$ possibilities for classical systems. An immediate consequence of this is that for physically separated and entangled S and E in state $(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ for example, a measurement of $|\uparrow\rangle$ on S reduces/collapses the entangled state to the separable $|\uparrow\downarrow\rangle$ so that a subsequent measurement on E in the same basis always yields the predictable result $|\downarrow\rangle$; if $|\downarrow\rangle$ occurs in S then E will be guaranteed to return the corresponding reciprocal value $|\uparrow\rangle$. System $|E\rangle$ has accordingly been altered by local operations on $|S\rangle$, with a measurement on the second qubit always yielding a predictable complimentary result from measurements on the first qubit. In the linear setting of quantum mechanics, multipartite systems modeled in 2^N-dimensional tensor products $\mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_N$ of 2-dimensional spin states, correspond to the 2^N "dimensional space" of unstable fixed points in the evolution of the logistic map. This formal equivalence illustrated in Fig. 6 while clearly demonstrating how holism emerges in 2^N-cycle complex systems for increasing complexity with increasing λ — the emergent 2^N-cycle are "entangled" in the basic ($\uparrow$) and ($\downarrow$) components as the system self-organizes to the graphically converged multifunctional limits indicated by the heavy lines: the parts surrendering their individuality to the holism of the periodic cycles also focuses on the significant differences between complex holism and quantum non-locality.

The converged holistic behaviour of complex "entanglement" reflects the fact that the subsystems have combined nonlinearly to form an emergent, self-organized system of the 2^1, 2^2 and 2^3 cycles in Fig. 6(a), (b) and (c) that cannot be decoupled without destroying the entire structure; contrast with the quantum entanglement and the notion of partial tracing for obtaining properties of individual components from the whole. Unlike the quantum case, the complex evolutions are not linearly superposed reductionist entities but appear as emergent, self-organized holistic wholes. In this sense complex holism represents a stronger form of "entanglement" than Bell's nonlocality: *linear systems cannot be chaotic, hence complex, and therefore holistic.* While quantum non-locality is a paradoxical manifestation of linear tensor products, complex holism is a natural consequence of the nonlinearity of emergence and self-organization.

Nature uses chaos as an intermediate step in attaining states that would otherwise be inaccessible to it. Well-posedness of a system is an extremely inefficient way of expressing a multitude of possibilities as this requires a different input for every possible output. The countably many outputs arising from the non-injectivity of f for a given input is interpreted to define complexity because in a nonlinear system each of these possibilities constitute a experimental result in itself that may not be combined in any definite predetermined manner. This multiplicity of possibilities that have no predetermined combinatorial property is the basis of the diversity of Nature.

The reduced density matrix plays a key role in *decoherence,* a mechanism by which open quantum systems interact with their environment leading to spontaneous suppression of interference and appearance of classicality, involving transition from the quantum world of superpositions to the definiteness of the classical objectivity. Partial tracing over the environment of the total density operator produces an "environment selected" basis in which the reduced density is diagonal. This irreversible decay of the off-diagonal terms is the basis of decoherence that effectively bypasses "collapse" of the state on measurement to one of its eigenstates. This derivation of the classical world from the quantum is to be compared with nonlinearly-induced emergence of complex patterns through the multifunctional graphical convergence route of the type in Fig. 6. Multiplicities inherent in this mode illustrated by the blue dots, liberated from the strictures of linear superposition and reductionism, allow interpretation of objectivity and definiteness as in classical probabilistic systems through a judicious application of the Axiom of Choice: *To define a choice function is to conduct an experiment.* Because of the drive toward ininal topology of maximal equivalence classes of open sets at chaos, the selection by choice function refers to the analogue of continuous quantum probability of the Bloch sphere rather than the discrete or randomized classical probability. Non-local entanglement and interference, the distinguishing features of this distinction, are more pronounced and pervasive in nonlinear complexity than in linear *isolated and closed,* quantum systems, with its origins in the noninvertible, maximal ill-posedness of the dynamics of the former compared to the bijective, reversible unitary Schrodinger evolution of the later. This identifying differentiation of quantum non-locality and complex holism forms the basis of the following inferences.

Unlike in the quantum-classical transition, complex evolving systems are in a state of homeostasis with the environment with "measurement" providing a record of such interaction; probing holistic systems for its parts and components is expected to lead to paradoxes and contradictions. A complex system represents a state of dynamic stasis between the opposites of bottom-up pump induced synthesis of concentration, order, and emergence, and top-down engine dominated analysis of dispersion, disorder, and self-organization, the pump effectively deceiving Second Law through entropy reduction and gradient dissipation. While quantum non-locality is a natural consequence of quantum entanglement that endows multi-partite systems with definite properties at the expense of the individual constituents, the effective power law $f(x) = x^{1-\chi}$ of Eqs. (18*a*),(*b*),(*c*) and the discussions of the effective linearity of the chaotic region in Table 1, suggests the integration of quantum mechanics with chanoxity by identifying $\langle x \rangle = 2^N$ of Eq. (19) with the dimension of the resulting Hilbert space leading to the conjecture that *quantum mechanics is an effective linear representation* $\chi = 0$ *of the fully chaotic, maximally illposed* $\text{Multi}_{\|}$ *boundary* $\lambda_* \leq \lambda < 4$ that manifests itself only through a bi-directional, contextually objective, inducement of W in adapting to the Second Law of Thermodynamics; Quantum Mechanics resides at the interfacial boundary between W and $\mathfrak{W}$ thereby possessing simultaneously the properties both of functional objectivity of the former and mutifunctional ubiquity of $\mathfrak{W}$. The opposites of the (pump) preparation of the state and the subsequent (engine) measurement collaborate to define the contextual reality of the present. This combined with the axiom of choice allows the inference that *quantum mechanical "collapse" of the wave function is a linear objectification of the measurement choice function,* the "measurement" process allowing the quantum boundary between the dual worlds of Table 1 to

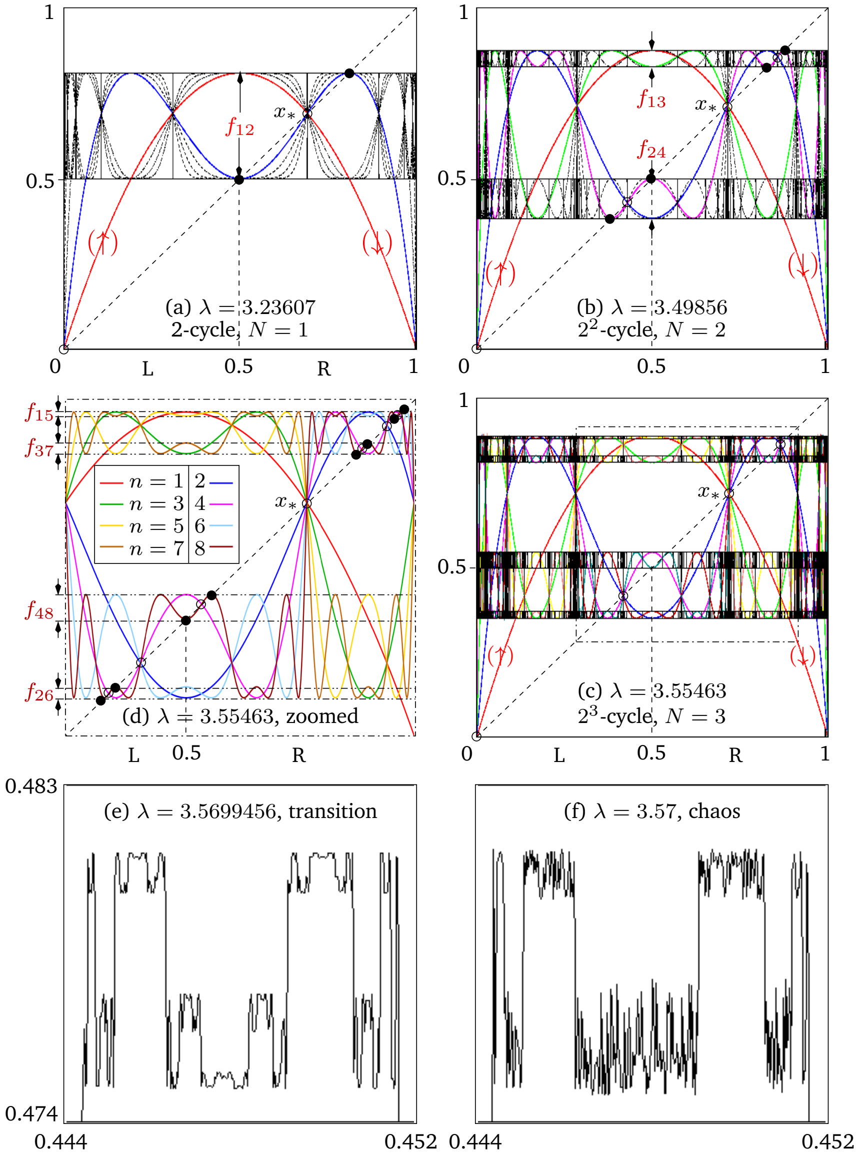


Figure 6: Complex entangled holism (a)-(d), generated by the logistic map $f(x) = \lambda x(1-x)$. The effective nonlinearity $0 \leq \chi \leq 1$ of the representation $f(x) = x^{1-\chi}$ rises with λ, as the system becomes more holistic with an increasing number of interacting parts of unstable fixed points shown unfilled, the stable filled points being the interacting, interdependent, parts of the evolved pattern. Using the convention of the text, these can be labeled left to right in (a), (b), (d) as $(0;1)$, $[(01,00);(10,11)]$, and $\{[(011,010),(000,001)];[(101,100),(110,111)]\}$ respectively. The resulting holistic patterns of one, two, and three hierarchical levels are entangled manifestations of these observables, none of which can be independently manipulated outside of the collaborative whole. Forward iterates $f^i(x)$ of entropy decrease, collaboration, concentration comprises the anti-thermodynamic arrow, the inverse iterates $f^{-i}(x)$ of entropy increase, competition, dispersion is its holistic opposite, with homeostasis being a dynamic equilibrium between them.

interact with the "apparatus" in W to generate the complex "reality" of the present [10].

Possibly the most ambitious projected utility of quantum entanglement and interference is that of *quantum computers* [2]. Any two-level quantum system — like the ground and an excited states of an ion — that can be prepared, manipulated, and measured in a controlled way comprises a qubit, a collection of N qubits with its 2^N dimensional wave function in a Hilbert space constituting a quantum computer. *Neglecting its coupling with the environment*, the *unitary* (hence invertible) time evolution of the computer is governed

[10] "While the linearity of quantum theory's unitary process gives that theory a particular elegence, it is that very linearity (or unitarity) which leads directly to the measurement paradox. Is it so unreasonable to believe that this linearity might be an approximation to some more precise (but subtle) nonlinearity? ··· Einstein's theory explained these deviations, but the new theory was by no means obtained by tinkering with the old; it involved a completely radical change in perspective. This it seems to me is the kind of change in the structure of quantum mechanics that we must look towards, if we are to obtain the needed nonlinear theory to replace the present-day conventional quantum theory" [15].

by the Schrodinger equation, with measurements disrupting this process. A *quantum computation* therefore consists of three basic steps: (i) preparation of the input state, (ii) implementation of the desired unitary transformation (quantum *gates*) acting on this state, and (iii) measurement of the output. In an ion-trap quantum computer for example, any linear array of ions constrained within a trap formed by static and oscillating electric fields is the quantum register. Ions are prepared in a specific qubit state by a laser pulses, the linear interaction between qubits being moderated by the collective vibrations of the trapped collection of ions.

The significant attributes of the programme for quantum computers that are in direct conflict with the defining features of chanoxism are the following. Isolation from the environment, invertible unitary interactions and the ability to selectively operate on constituent parts of the entangled state (of "Alice", for example, who "shares an e-bit with Bob") that in the ultimate analysis depend on the linear invertibility of unitary evolution, and superposition of quantum states. As none of these hold in complex holism, *being externally imposed classical interactions of the quantum system with its environment* and not self-generated, it can be hypothesized that *holistic computation, as the source of its linear quantum realization, is unlikely to be feasible*: unlike linear superpositions, any of the evolved holistic multifunctional entities in Fig. 6(a), (b), (c) cannot be decomposed or altered without adversely affecting the entire pattern.

The labeling of the interdependent, interacting, stable fixed points in Fig. 6 is according to the following criterion. The interval $[0,1]$ is divided into two equal parts at $\frac{1}{2}$ with 0 corresponding to L and 1 to R. At any stage of the iterative hierarchy generated by the unstable (unfilled) points with the $f_{i<j}$ shown, the stable points are labeled left to right according to the prescription of Table 2, for $\langle f(x)\rangle = \{[(2f_1+f_{12})+f_{13}+f_{24}]+f_{15}+f_{26}+f_{37}+f_{48}\}$ the mean value of f according to Eq. (18*c*). This gives the symbolic representation

$$N=1 \qquad (0;1) \tag{23a}$$

$$N=2 \qquad [(01,00);(10,11)] \tag{23b}$$

$$N=3 \qquad \{[(011,010),(000,001)];[(101,100),(110,111)]\} \tag{23c}$$

for the self-organized, emergent levels corresponding to $N=1,2,3$.

$f^i(0.5)$	L of unstable f.p.	R of unstable f.p.
convex up	0	1
concave up	1	0

Table 2: Rule for symbolic representation of the stable fixed points of Fig. 6 at each hierarchical level with f^i the i^{th} iterate of f_i in f_{ij}.

Specifically for $N=2$, the complex "entangled" holistic pattern of Fig. 6(b) clearly demostrates that the four components of Eq. (23*b*) cannot be decoupled into Bell states, being itself nonlinearly "entangled" rather than separated. The various operations historically performed on the respective qubits of the entangled pair to generate dense coding and teleportation ($N=3$) for example, are not meaningful on the nonlinear holistic entities; in fact it is possibly not significant to ascribe any specific qubit to the individual members of the strings as in Eq. (23*b*). These suggestive points of departure between linear quantum nonlocality generated by external operations and nonlinear self-evolved complex holism calls for a deeper investigation that we hope to perform subsequently.

Nevertheless, the phenomenal sucess of linear quantum mechanics to "classify and predict the physical world" begs a proper perspective. Our hypothesis is that nature operates in accordance with chanoxity only in its "kitchen" that forever remains beyond our direct perception; what we do observe physically is only a linearized, presentable, table-top version of this complexity, through the quantum linear interface of $W-\mathfrak{W}$. This boundary between the dual worlds, of course, carries signatures of both, which seems to explain its legendary observational success.

5 Black Hole and Gravity: The Negative World and its Thermodynamic Legacy

5.1 A Defining Example: The $(W, \mathfrak{W})$, (Top-Down, Bottom-Up), (Particle, "Wave") Duality

Consider the two-state paramagnet of N elementary $(\uparrow, \downarrow)$ dipoles with a magnetic field B in the $+z$-direction. Then with μ the magnetic moment and

$$E = -N\mu B \tanh\left(\frac{\mu B}{kT}\right) \tag{24}$$

the total energy of the system, the corresponding expressions for temperature, entropy, and specific heat plotted in Fig. 7 displays the typical unimodal, two-state, $(\uparrow, \downarrow)$ character of S that admits the following interpretation. In the normalized ground state energy $E = -1$ of all spins along the B-axis, the number of microstates is 1 and the entropy 0. As energy is added to the system some of the spins flip in the opposite direction till at $E = 0$ the distribution of the $\uparrow$ and $\downarrow$ configurations exactly balance, and the entropy attains the maximum of $\ln 2$. On increasing E further, the spins tend to align against the applied field till at $E = 1$ the entropy is again zero with all spins opposing the field for a single microstate and *negative* T. Traditionally, it is held that [1] "all negative temperatures are hotter than positive temperatures. Moreover, the coldest temperature is just above 0K on the positive side, and the hottest temperatures are just below 0K on the negative side". This view of $\mathbb{R}_-$ as a set of "super positives" is to be compared with what it really is: the negative world $\mathfrak{W}$.

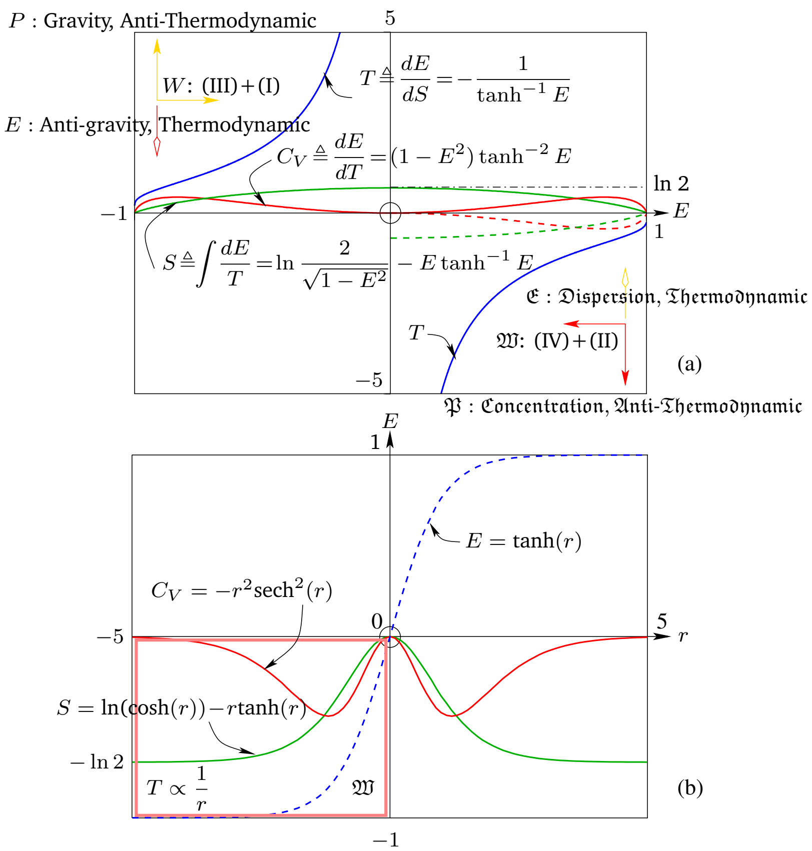


Figure 7: (a) Normalized ($N = \mu = B = 1 = k_B$) negative temperature, specific heat and entropy for a $(\uparrow, \downarrow)$ system. (b) The virial negative world $\mathfrak{W}$ of negative specific heat and entropy. Observe that the region of negative temperature is the plot of $-1/\tanh^{-1}(E)$, $0 < E < 1$.

Bidirectionally, quite a different interpretation based on the *virial theorem* relates the average kinetic energy of a system to its average potential energy, $2\mathfrak{T} = -\sum_{k=1}^{N} \mathbf{F}_k \cdot \mathbf{r}_k$ where $\mathbf{F}_k$ is the force on the k^{th} particle at $\mathbf{r}_k$. For power law potentials $\mathcal{V}(r) = cr^n$, the theorem takes the simple form

$$\mathfrak{T} = \frac{n\mathcal{V}}{2}, \tag{25}$$

which for $c=-1$, $n=-1$ of attractive gravitational systems, reduces to

$$\mathfrak{T}+E=0, \qquad E \text{ total energy}. \tag{26}$$

Since the potential energy decreases ($d\mathcal{V}/dr>0$) faster than increase in kinetic energy ($d\mathfrak{T}/dr<0$) the total energy decreases with bounding radius $dE/dr>0$. With $\mathfrak{T}\sim NT$, $\mathcal{V}\sim -NT$ it follows

$$T\propto\frac{1}{r}, \qquad \frac{dT}{dr}<0, \tag{27a}$$

and the gas gets hotter with shrinking radius. Hence

$$C_V \triangleq \frac{dE}{dT}=\left(\frac{dE}{dr}\right)\left(\frac{dr}{dT}\right)<0 \tag{27b}$$

and, from $dS(r)\triangleq dE/T=-dT/T$, the entropy becomes

$$S(r)\propto\ln r<0 \tag{27c}$$

as $r\to 0$ gravitationally. It is clear from Fig. 7 that only $E>0$ with its direction reversed, $1<E<0$, can qualify for the gravitational region of negative specific heat and entropy. Hence $E(r)=\tanh(r)$: $-\infty<r<0$, $-1<E<0$ applies only to the gravity-induced region of negative T and therefore of negative r, and

$$\begin{aligned} C_V(r) &\triangleq -r^2\text{sech}^2(r)\le 0 && (28a)\\ &= -r^2+r^4-\frac{2}{3}r^6+\frac{17}{45}r^8-\cdots, && (28b)\\ S(r) &\triangleq \int\frac{dE}{T}=\int C_V\left(\frac{dT}{dr}\right)\left(\frac{dr}{T}\right)=\int r\text{sech}^2(r)\,dr \\ &= \ln\cosh(r)-r\tanh(r)\overset{r\to\infty}{\longrightarrow}-\ln 2\le 0 && (28c)\\ &= -\frac{1}{2}r^2+\frac{1}{4}r^4-\frac{1}{9}r^6+\frac{17}{360}r^8-\cdots && (28d)\end{aligned}$$

are both negative and even functions of r, as predicted by the arguments above. In the gravitationally collapsed region, therefore, entropy is proportional to r^2 (surface) *rather than to* r^3 (volume) *at small* r, a characteristic feature of the black hole. This most noteworthy manifestation of viriality in the dynamics of a $(\uparrow,\downarrow)$ system, of the natural appearance of negative r, can be taken as a confirmation of the existence of a negative, gravitationally collapsed world, *that in fact constitutes a black hole*. In this negative multifunctional dual $\mathfrak{W}$, where "anti-second law"[11] requires heat to flow spontaneously from lower to higher temperatures with positive temperature gradient along increasing temperatures, the engine and pump interchange their roles *with order inducing compression of the system by the environment — rather than expansion against it as in* W *— being the thermodynamic direction in* $\mathfrak{W}$. For an observer in W heat flows from higher *negative* temperatures to lower *negative* temperature.[12]

The opposing arrow of $\mathfrak{W}$ translated to W, generate the full curves of Fig. 7; hence the entropy, specific heat and temperature are all positive as seen from $\mathfrak{W}$ ($1<E<0$) dashed in the figure. In this framework, entropy increases with energy in $\mathfrak{W}$, rather than negative temperatures acting "as if they are higher than positive temperatures": the temperature increases to infinity in $\mathfrak{W}$ with $E\to 0_+$ as it does in W for $E\to 0_-$ with the interconnection between the complimentary dual worlds through the equivalences at $E=\pm1$ and $T=\mp\infty$ allowing them to competitively collaborate as realized by the full curves. The maximum entropy of $\ln 2$ occurs at $E=0$ and the minimum at $E=\pm1$ when all spins are aligned unidirectionally in single microstates. *This manifestation of* $\mathfrak{W}$ *in* W *produces the characteristic two-state* $(\uparrow,\downarrow)$ *signature of complexity and holism through the induced contractive manifestation of gravity*

The intuitively pathological $T_h\le T_c$ of (II) in the fully-chaotic region $\lambda\ge\lambda_*$, Fig. 4 and Table 1 where no complex patterns are possible, can now be understood iff $T_h=\infty$ when (II) and (IV) merge in the single region of negative temperatures with its own "negative" dynamics in relation to W. Reciprocally at $T_c=0$, region (III) vanishes and with $T_h=\infty$ leads to the two surviving $\alpha\ge 0$ unshaded portions of Table 1, one for $\iota\alpha\le 0$ of the multifunctional (IV) of $\mathfrak{W}$ and the other functional $\iota\alpha>0$ of W (I). Since matter is born only in W as a gravitational materialization of the miscible mixture $\mathfrak{W}$, the boundary $\text{Multi}_{\|}(X)$ between the two worlds at $\chi=0$, $\lambda\in[\lambda_*,4)$ is an expression of functional-particle, maximally-multifunctional-"wave", duality that is inaccessible from W because the equivalence at $E=\pm1$ generates a passage between these antagonistic domains. The subsequent $T_c>0$, $T_h<\infty$ exposition of Fig. 4 is responsible for the complex structures and patterns of Nature.

[11] All qualifications are with respect to W.

[12] See the Appendix (A) for some additional consideration.

5.2 Gravitational Black Hole and the Negative World

The (self-induced) engine-pump system that forms the basis of our approach has a relativistic analogue in the Schwarzschild-de Sitter metric

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2 \tag{29a}$$

where

$$f = 1 - \left(\frac{2GM}{c^2}\right)\frac{1}{r} - \left(\frac{\Lambda}{3}\right) r^2, \quad \Lambda > 0. \tag{29b}$$

The zeros of f give the limiting values

$$r_{M,\Lambda} = \begin{cases} \dfrac{2GM}{c^2}, & \Lambda = 0 \\ \sqrt{\dfrac{3}{\Lambda}}, & M = 0 \end{cases} \tag{30}$$

of the Schwarzschild and cosmological radii r_M and r_Λ, respectively. Equation $f = 0$ solved for M and Λ

$$M(\Lambda) = \frac{c^2}{2G} r \left(1 - \frac{\Lambda}{3} r^2\right) > 0 \Rightarrow r < \sqrt{\frac{3}{\Lambda}} \triangleq r_\Lambda \tag{31a}$$

$$\Lambda(M) = \frac{3}{r^2}\left(1 - \frac{2GM}{c^2}\frac{1}{r}\right) > 0 \Rightarrow r > \frac{2GM}{c^2} \triangleq r_M; \tag{31b}$$

denote that $r_M < r < r_\Lambda$ corresponds to region (I) of the complex holistic world, see Fig. 8. Reciprocally,

$$r_\Lambda < r < r_M, \qquad M < 0 \tag{31c}$$

denotes the negative world $\mathfrak{W}$ of negative temperature, Fig. 8 being a detailed representation of this equivalence; in fact, taking $\Lambda = 10^{-52}\text{m}^2$, $r_\Lambda = 1.73 \times 10^{26}\text{m}$ is of the order of magnitude of the radius of the observable universe. The tilting of light cones at the removable singularity of the event horizon r_M, which prevents all future directed timelike or null worldline reaching $r > r_M$ from the interior, is the relativistic expression of this passage to $\mathfrak{W}$ through the $T_c = 0$, $\alpha = 1$ triple point, compare Fig 8(a) as $r_\Lambda \to \infty$. At the other extreme of $r_M \to 0$ for decreasing M, $T \to \infty$ as r approaches the physical singularity 0, with negative r (and M, from Eq. (31a)) denoting crossover to $\mathfrak{W}$ through the $T_c = T_h$, $\alpha = -1$ critical point. The gravitationally collapsed expression Eq. (27a)

$$T = \left(\frac{\hbar c}{k_\text{B}}\right)\frac{1}{r},$$

is the Hawking temperature[13], while the entropy Eq. (28c), that *for small* r reduces to

$$S = -\left(\frac{c^3}{\hbar G}\right) r^2,$$

is the negative of Hawking-Bekenstein entropy, but has the usual volumetric dependence of $\ln 2$ at full dispersion. The fully chaotic region $\lambda_* \leq \lambda \leq 4$ of the boundary $\text{Multi}_{\|}$ between W and $\mathfrak{W}$, as region (III) in Fig. 8, is the "skin" of the gravitational black hole $\mathfrak{W}$ that actually lies beyond the physical singularity $r = 0$, occupying all of $r < 0$ and identified as $M < 0$, in Eq. (31c). Gravity as we experience it in W, is the legacy of the thermodynamic arrow of $\mathfrak{W}$, see Fig. 7.

The zeros of f,

$$-r_\text{SdS} = \frac{c^2}{\left(3GM\Lambda^2c^4 + \sigma\right)^{1/3}} + \frac{\left(3GM\Lambda^2c^4 + \sigma\right)^{1/3}}{c^2\Lambda}, \tag{32a}$$

$$r_\pm = -\frac{r_\text{SdS}}{2} \pm i\sqrt{3}\left[\frac{c^2}{2\left(3GM\Lambda^2c^4 + \sigma\right)^{1/3}} - \frac{\left(3GM\Lambda^2c^4 + \sigma\right)^{1/3}}{2c^2\Lambda}\right] \tag{32b}$$

$\sigma := \Lambda c^4\sqrt{9G^2M^2\Lambda^2 - \Lambda c^4}$, have the interesting property of possessing only one *negative* real root; the two other complex conjugate pair merge to a single real value of multiplicity 2 for

$$\sigma = 0 \Rightarrow 9G^2M^2\Lambda = c^4 \tag{33a}$$

[13]**Note:** With T negative in $\mathfrak{W}$, r must also be so.

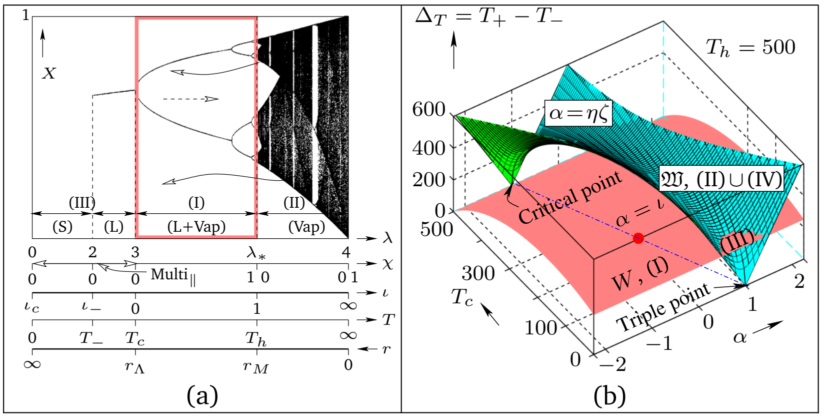


Figure 8: (a) Bifurcation profile of the Universe: Integration of the dynamical and thermodynamical perspectives. The complex phase (I) is a mixture of concentration and dispersion. $r_M = \sqrt{2GM/c^2}$ and $r_\Lambda = \sqrt{3/\Lambda}$ are the Schwarzschild and cosmological radii; $\Lambda = 1.3793 \times 10^{-52}$ m^{-2}, $G = 6.6742 \times 10^{-11}$ m^3/kg-s^2.

at the Nariai radius $r_\mathrm{N} = 3GM/c^2 = 1/\sqrt{\Lambda}$, permitting f to be factored like $f = -\left(\frac{\Lambda}{3r}\right)(r - r_\mathrm{N})^2(r + 2r_\mathrm{N})$, with $2r_\mathrm{N} = -r_\mathrm{SdS}$. Generally, f has two positive real zeros ρ_M and ρ_Λ satisfying $0 < \frac{2GM}{c^2} < \rho_M < \frac{3GM}{c^2} < \rho_\Lambda$ iff

$$0 < \frac{3GM}{c^2}\sqrt{\Lambda} < 1 \tag{33b}$$

with ρ_M monotonically increasing and ρ_Λ monotonically decreasing to the common value of r_N as $\Lambda \to \frac{c^4}{9G^2M^2}$. In this region, $f \leq 0$ everywhere so that r becomes a time coordinate and t a space coordinate, [16]. Specifically requiring $9G^2M^2\Lambda > c^4$ as we do, prescribes $M \gtrsim 4 \times 10^{52}$ kg of magnitude of the mass of the observable universe. As Fig. 8 illustrates, the positive real roots play no role in the complex forms of a horizonless W.

While bypassing the significance of the negative root, the Nariai solution by equating the cosmological and mass horizons $\rho_\Lambda = \rho_M$ at the double root, deny the emergent feedback system of the two opposing competitors of expansion (Λ) and compression (M) the privilege of collaborating with each other. Figure 8 and our approach by not insisting on this essentially unique reductionist behaviour, illustrate how gravity effectively moderates the Second Law dictate of *death* by allowing the system to "continually draw from its environment *negative entropy*". The degenerate singularities at $\alpha = \pm 1$ of ($T_c = 0$), ($T_c = T_h$, $T_h = \infty$) for ($\Lambda = 0$), $\left(\frac{4}{3}G^2M^2\Lambda = c^4,\ M = 0\right)$, has no acceptable commonality except for a negative $\mathfrak{W}$ with $M = -\infty$. The simultaneous requirement ($\Lambda = 0$, $M = -\infty$) regularizes the singularity through collaborative competition of gravitational collapse and de Sitter expansion in W: mutual support of the two opposites generates the complex holistic structures depicted in Fig. 8. The negative real root of Eq. (32*a*) adds additional justification of the negative world through negative M; observe however that Λ is not affected by this negativity of r, Eqs. (31*a*, 31*b*).

What is the significance of the negative root (32*a*) of negative mass M? Our assertion in Sec. 2.2.1 that the three phases of matter are born only in W at $t = 0$ and have no meaning in $\mathfrak{W}$ is supported by this distribution of the zeros, $\mathfrak{W}$ being characterized fully by just the vacuum energy Λ. $\mathfrak{W}$ induces in W two simultaneous effects (recall Figs. 2*b* and 7): its thermodynamic arrow of compression generates the dispersive thermodynamic arrow of W while its anti-thermodynamic expansion is responsible for the gravitational attraction in W. This, in the present approach, is how Nature's holism operates through unipolar gravity, with the anti-thermodynamic concentration in $\mathfrak{W}$ completing its bipolar credentials. Gravity is uniquely distinct from other known interactions in that it straddles $(W, \mathfrak{W})$ in establishing its domain, the other known forms reside within W itself.[14] It is this unique expression of the maximal multifunctional nonlinearity of $\mathfrak{W}$ in the functional reality of W that is responsible for the inducement of "neg-entropy" effects necessary for the sustenance of life.

[14]"Gravity seems to have a very special status, different from that of any other field. Rather than sharing in the thermalization that in the early universe applies to all other fields, gravity remained aloof, its degrees of freedom lying in wait, so that the second law could come into play as these begin to be taken up. Gravity just seems to have been different. However one looks at it, it is hard to avoid the conclusion that in those circumstances where quantum and gravitational effects must both come together, gravity just behaves differently from other fields. For whatever reason, Nature has imposed a gross temporal asymmetry on the behaviour of gravity in such circumstances." [15]

6 Conclusion: *Reality* is not *Flat*

In his remarkable explorations along *The Road to Reality*, Roger Penrose[15] repeatedly stresses his conviction of "powerful positive reasons to believe that the laws of present-day quantum mechanics are in need of a fundamental (though presumably subtle) change", basing his arguments on the "distinctly odd type of way for a Universe to behave" in the reversible unitarity of Schrodinger evolution **U** being inconsistently partnered with irreversible state reduction **R.** This leads him to posit that "perhaps there is a more general mathematical equation, or evolution principle, which has both **U** and **R** as limiting approximations", see footnote 10. In fact, "a gross time-asymmetry (is) a necessary feature of Nature's quantum-gravity union": gravity "just behaves differently from other fields". Specifically, "there is some connection between $\mathbf{R}$ and the Second Law", with quantum state reduction being an *objectively real process* arising from the difference of gravitational self-energy E_G[16] between different space-time geometries of the quantum states in superposition. Thus, all observable manifestations in Nature are interpreted to be *always* gravity induced, quantum superpositions decaying into one or the other state.

This point of view is operationally consistent with ours, recall Sec. 5.1 in particular, the details being however, conspicuously different. The homeostasy of top-down-engine and bottom-up-pump endows the state of dynamical equilibrium with the distinctive characteristic of competitively cohabitating opposites (Eq. 16) in its continual search for life and order. The reality of the natural world of *not* being in a "flat" [8] state of dispersive maximum entropy is infact the quest of open systems to stay alive by temporarily impeding this eventuality through self-organized competitive homeostasis. Hierarchical top-down-bottom-up complex holism does not support "flatness"; because of its antithetical stance toward self-organization and emergence, such a world is essentially a dead world. The survival of open living systems lies in its successfully guarding against this contingency through the expression of gravity as a realization of the multifunctional "quantum" $\mathfrak{W}$ on the materially tangible W.

A socially significant remarkable example of this competitive collaboration is the open source/free software dialectics, developed essentially by an independent, dispersed community of individuals. Wikipedia as an exceptional phenomenon of this collaboration, along with Linux the operating system, are noteworthy manifestations of the power and reality of self-organizing emergent systems. How are these bottom-up community expressions of "peer-reviewed science" — with bugs, security holes, and deviations from standards having to pass through peer-review evaluation of the system (author) in dynamic equilibrium of competitive collaboration with the reviewing environment — able to "outperform a stupendously rich company that can afford to employ very smart people and give them all the resources they need? Here is a posible answer: Complexity. Open source is a way of building complex things" [13]. Note also that "the world's biggest computer company (IBM) decided that its enginners could not best the work of an ad-hoc open-source collection of geeks (Apache Web server), so they threw out their own technology and decided to go with the geeks!" [8].

Which brings us to the main issue: Building anything, open-source or otherwise, requires investment of resources, financial and human. While the human incentive of open-sourcing for personal recognition through peer-review is a major deciding factor for the individual component, "collaborating for free in the open-source manner (as) the best way to assemble the best brains for the job" guarantees the collective ingredient needed for emergence of these complex systems that are far beyond the capacity of any single organization to handle. The blended model of revenue generation followed by most of the major open source groups contributes to the financial assets required for the self-generation of the backward pump as operationally viable, with the dispersive engine of a readily available market completing the engine-pump paradigm of chanoxity; *economics infact is about collectivism to inhibit human selfish individualism and promote evolution to a state of sustainable homeostatic, collective and societal holism.* The (social) unit "may be the individual or a collective of individuals. If it is a collective, could its behaviour be deduced from the sum of the behaviour of its components? Or could its behaviour be governed by other things than the sum of its components?" Unlike other customs in the analysis of social phenomena, the through and through individualistic character of neoclassical economics based almost entirely on the analysis of the behaviour of a single individual and his interaction with others "begins and ends with the individual, and sadly, there is barely any role to anything which is a reflection of the collective. $\cdots$ From the utility maximizing behaviour of individuals we derived the demand; from the profit maximizing behaviour of firms we derived the supply.

[15] "The usual perspective with regard to the proposed marriage betwen these two theories is that one of them, namely general relativity, must submit itself to the will of the other. $\cdots$ Indeed the very name 'quantum gravity' that is normally assigned to the proposed union, carries the implicit connotation that it is a standard *quantum* (field) theory that is sought. Yet I would claim that there is observational evidence that Nature's view of this union is very different from this! Her design for this union must be what, in our eyes, would be a distinctly non-standard one, and that an objective state reduction must be one of its important features." [15]

[16] *Gravitational self-energy* in a mass distribution is the amount of (binding) energy gained in assembling the mass from point masses dispersed at infinity.

The opposition of forces here is quite clear and well depicted by the demand and supply analysis (founded on Newtonian mechanics). Market is where the conflicting forces meet, and the most basic question is what might influence the outcome of an encounter between a consumer and a seller?" [23] Further insight into these economic considerations are considered in Appendix (B).

The science of collective holism is specifically addressed to issues such as these leading to an understanding of their true perspective.

APPENDIX

(A): Gravity and Entropy

Figure 9 adapted from Penrose [15], with the accompanying caption reproduced, is a vivid illustration of the special property of long range unipolar gravity[17], and further supports our arguments against considering negatives as "super positive". Panels (a) and (b) are from [15] with the identification of (b) added. Recalling Figs. 2*a*, 2*b*, (a) and (c) represent the engine-pump duality expressed in Fig. 7(a).

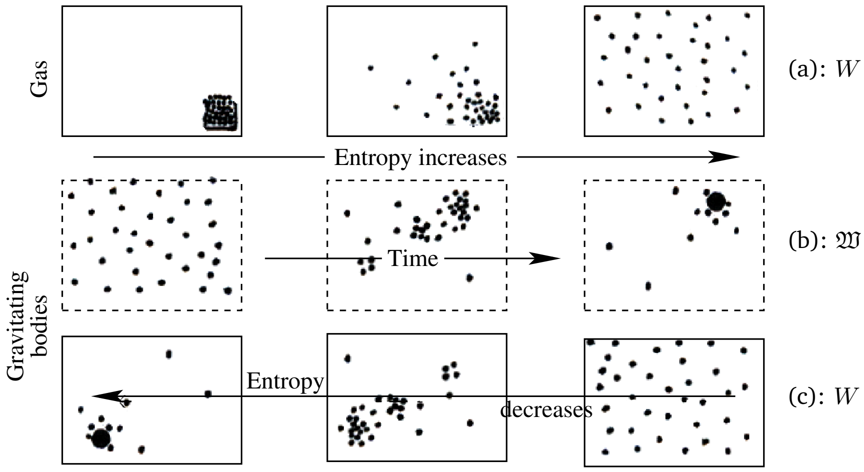


Figure 9: Increasing entropy with increasing time. (a) For gas in a box, initially tucked in one corner, entropy increases as the gas spreads throughout the box, finally reaching the uniform state of thermal equilibrium. (b) With gravity, things tend to be the other way about. An initial uniformly spread system of gravitating bodies represent a relatively low entropy, and clumping tends to occur as the entropy increases. From Penrose [15], p. 707.

(B): Economic Holism?

Modern individualistic, neo-classical Western economics, is a static Newtonian equilibrium theory, where supply by the firm equals the demand of the consumer. Linear stablity is central to this variant of economic thinking that has come under severe strain in recent times, *Economics Needs a Scientific Revolution, The Eonomy Needs Agent-Based Modelling* [3] reflecting some of the manifestations of this disillusionment. The linear mathematics of the neoclassical enterprise is founded in calculus with maximization and contraint-based optimization being the ground rules, see [9] for example, that "Western economics became obsessed with" [17]. These Marshallian linear static models seeking to maximize utility for the consumer and profit for firms, as epitomized in Pareto optimality[18], Nash equilibrium[19], Prisoner's Dilemma[20] for example, work as

[17] "Whereas with a gas, the maximum entropy of thermal equilibrium has the gas uniformly spread throughout the region in question, with large gravitating bodies maximum entropy is achieved when all the mass is concentrated in one place — in the black hole." [15]

[18] Given a set of alternative allocations for a collective of individuals, a change from one allocation to another that can make at least one individual better off without making any other worse, is called a Pareto improvement. An allocation is *Pareto optimal* when no further Pareto improvements can be made: *Pareto efficient situations are those in which any change to make any person better off is impossible without making someone else worse off.*

[19] Let (S, f) be a game with n players, where S_i is the strategy set for player i, $S = S_1 \times S_2 \times \cdots \times S_n$ is the set of strategy profiles and $f = (f_1(x), \cdots, f_n(x))$ is the payoff function. Let x_{-i} be a strategy profile of all players except player i. When each player $i \in 1, \cdots, n$ chooses strategy x_i resulting in strategy profile $x = (x_1, \cdots, x_n)$ then player i obtains payoff $f_i(x)$: the payoff depends on the strategy collectively chosen by all the players. *A strategy profile $x^* \in S$ is a Nash equilibrium if no unilateral deviation in strategy by any single player is profitable for him*, that is

$$\forall i, x_i \in S_i, x_i \neq x_i^* : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*).$$

[20] Two suspects — the only concern of each being to maximize his own advantage without any concern for the (collective) well-being

might well be expected with reasonable justification, as long as its canonized axioms of people with rational preferences acting independently with full and relevent information make sense. This framework of rationality of economic agents of individuals or company working to maximize own profits, of the "invisible hand" transforming this profit-seeking motive to collective societal benefaction, and of market efficiency of prices faithfully reflecting all known information about assests [3], can at best be relevent under severely restrictive conditions: "the supposed omniscience and perfect efficacy of a free market with hindsight looks more like propaganda against communism than plausible science. In reality, markets are not efficient, humans tend to be over-focused in the short-term and blind in the long-term, and errors get amplified, ultimately leading to collective irrationality, panic and crashes. Free markets are wild markets. Surprisingly, classical economics has no framework through which to understand 'wild' markets" (Bouchaud [3]). These "perfect world" models are meaningful only under "linear" conditions: "these successfully forecast a few quarters ahead as long as things stay more or less the same, but fail in the face of great change" (Farmer and Foley[3]), "as long as the influences on the economy are independent of each other, and the past remains a reliable guide to the future. But the recent financial collapse was a systemic meltdown, in which interwined breakdowns $\cdots$ conspired to destabilize the system as a whole. We have had a massive failure of the dominant economic model" (Buchanan[3]).

These authors advocate an agent-based computational modelling of economics ("the meltdown has shown that regulatory policies have to cope with far-from-equilibrium situations"), for simulating the interdependence and interactions of autonomous individuals with a view to assessing their effects on the system as a whole: the complex behaviour of adaptive system emerges from interactions among the components of the system and between the system and the environment. Individual agents are typically characterized as boundedly rational, presumed to be acting in what they perceive as their own interests such as economic benefit or social status, employing heuristics or simple decision-making rules. The computer keeps track of multiple agent interactions, monitoring a far wider range of nonlinear intercourse than conventional equilibrium models are capable of; "because the agent can learn from and respond to emerging market behaviour, they often shift their strategies, leading other agents to change their behaviour in turn. As a result prices don't settle down into a stable equilibrium, as standard economic theory predicts" (Buchanan[3]).

The essence of this cellular automata[21] based computer-graphics diagnosis of time evolution of the economy bears a strong formal resemblance with the engine-pump realism of chanoxity as summarized in Fig. 10. The competitive collaboration of the engine and its self-generated pump is identified as the tension between the consumer with its dispersive spending engine in conflict with the resource generating pump, in mutual feedback cycles attaining market homeostasis not through linear optimization and equilibrium of intersecting supply-demand curves, but through nonlinear feedback loops that generate entangled holistic structures like those of Fig. 6. Supply and demand in human society are not independent of each other: aggressive advertising for example can completely dominate the individual behaviour of these attributes. To take this into account, the interactive feedback between the opposites of engine consumption and pump production

of the other — are arrested by the police. The police have insufficient evidence for a conviction, and, having separated both prisoners, visit each of them to offer the same deal. If one testifies for the prosecution against the other ("competes" with the other) and the other remains silent ("cooperates" with the other), the betrayer goes free and the cooperating accomplice receives the full 10-year sentence. If both cooperate, they are sentenced to only six months in jail for a minor charge. If each competes with the other, both receives a five-year sentence. Each prisoner must choose to compete with the other or to cooperate. How should the prisoners act?

The *Prisoner's Dilemma* can be summarized as follows, with $(\uparrow), (\downarrow)$ denoting "collaboration", "competition" of one with the other:

$A \setminus B$	$(\uparrow)$	$(\downarrow)$
$(\uparrow)$	(6 mo (R), 6mo (R))	(10 ye (S), Free (T))
$(\downarrow)$	(Free (T), 10 ye (S))	(5 ye (P), 5 ye (P))

The Nash equilibrium of this game, which is not Pareto optimal $(\uparrow\uparrow)$, is $(\downarrow\downarrow)$ of 5 years each: competition dominates cooperation with competitors having a higher fitness than cooperators, compare Eq. 16 and Def. 2.2. The so-called pay-off matrix of benefits received by the parties defines a Prisoner's Dilemma when $T > R > P > S$.

In the *Iterated Prisoner's Dilemma,* when additionally $2R > S + T$, the participants have to choose their mutual strategy repeatedly with memory of their previous encounters, each getting an opportunity to "punish" the other for earlier non-cooperation. Cooperation may then arise as an equilibrium outcome, the incentive to defect being overcome by the threat of punishment leading to the possibility of a cooperative outcome. As the number of iterations increase, the Nash equilibrium tends to the Pareto optimum, the likelihood of cooperation increases, and a collective state of competitive-collaborating homeostasy emerges.

[21]Cellular automata (CA) are simple models of spatially extended decentralized systems comprising a number of individual component cells interacting with each other through local communications, with the state of a cell at any instant depending on the states of its neighbours. The division of CA into four classes [24] corresponding to the attractors of dynamical systems — Class 1: Stable Fixed Point, Class 2: Stable Limit Cycle, Class 3: Chaotic, Class 4: Complex — renders them attractive tools for graphical visualization of evolution like the emergence of altruistic or cooperative behaviour in Prisoner's Dilemma [14] from classical Darwinian competition of second-law dispersion.

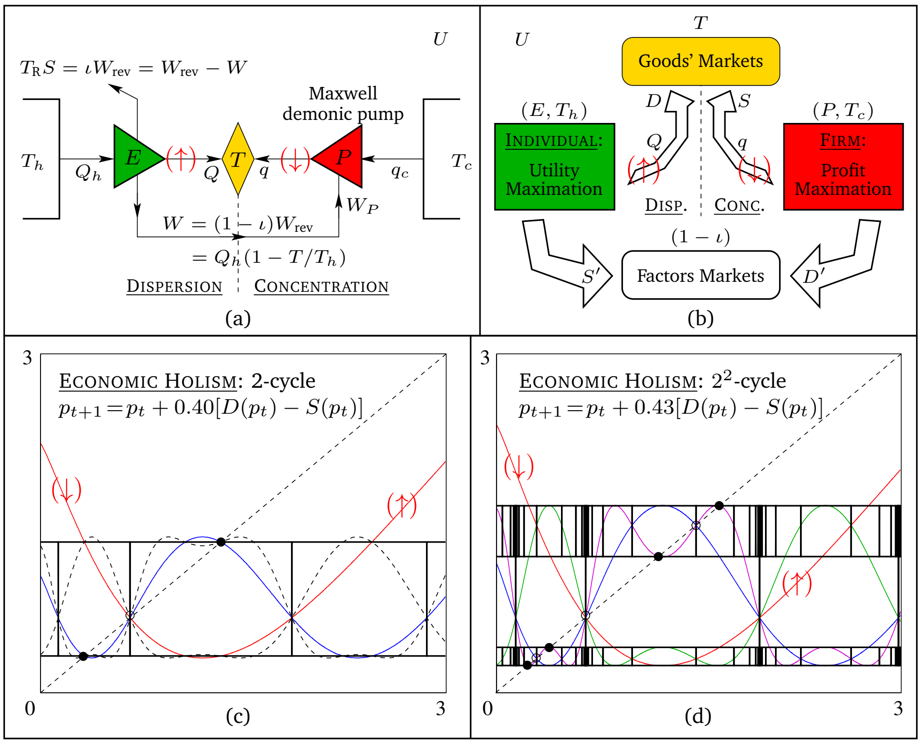


Figure 10: The economy as a complex system, U is the "universe". The presentation (b) of neo-classical economics is adapted from Witztum [23], (a) being a repeat of Fig. 3. According to this point of view, economics as the principal instrument of collective interaction in society, is to be distinguished from the exclusively individualistic stance of neo-classicalism. The Samuelson tatonnement of (c) and (d), to be compared with 6(a) and (b), show the emergence of economic complexity for *nonlinear* demand and supply profiles $D(p) = \frac{8.0}{1.1+p} - 1.75$, $S(p) = 10p^{1.5}e^{-p}$ respectively with p the commodity price.

can be modelled as a *product* of the supply and demand curves that now, unlike in its static manifestation of neo-classicalism, will evolve in time to generate a condition of dynamic equilibrium, see Fig. 10 for the different evolution strategy of Samuelson tatonnement [5] for *nonlinear* Walrasian demand and supply profiles. In the *linear* case, let $D(p) := 1 - \beta p$, $S(p) := \lambda p$, $\beta, \lambda > 0$, rescaled and normalized for $D(0) = 1$, $D(1) = 1 - \beta$, $S(0) = 0$ for $0 \leq p \leq 1$, be mappings on the unit square. Then supply and demand interact in the market as the shifted logistic $f_{DS}(p) = \lambda p(1 - \beta p)$ with a maximum $f_{DS}(p_m) = \frac{\lambda}{4\beta}$ at $p_m = \frac{1}{2\beta}$; note that at $\beta = 1$, f_{DS} reduces to the usual symmetric form $\lambda p(1-p)$ and at $\beta = \frac{1}{2}$, $p_m = 1$. Since we are interested only in the range $\frac{1}{2} \leq f_{DS} \leq 1$ for possible complex effects, let the slopes of the two opposites be related by $\beta = 0.25\lambda$ for the expected $f_{DS}(1) = 0$ at $\lambda = 4$. The market clearing condition $D(p^*) = S(p^*)$ at $p^* = \frac{1}{\beta+\lambda} = \frac{4}{5\lambda}$ apparently does not have any significance in the interactive evolution of $p_{t+1} = f_{\rm DS}(p_t)$ with fixed point $p_* = \frac{\lambda-1}{\beta\lambda} = \frac{4(\lambda-1)}{\lambda^2}$, except at the uninteresting "solid-state" (see Fig. 8(a)) $\lambda = 1.25$ for $p^* = p_*$. The time evolution of the p_m-shifted, demand-supply logistic function

$$f_{DS}(p) = 0.25\lambda p(4 - \lambda p) \tag{34}$$

is similar to the symmetric case, except for a right-shift of p_m for $2 \leq \lambda < 4$.

The remarkable similarity of this evolutionary profile with the logistic interaction is far too pronounced to be dismissed as incidental. In situations as in the Prisoner's Dilemma for example, the agents are infact not free to take unilateral decisions but are in entangled holistic states of competitive collaboration; thus a "good" individual in a stable "useful" state represented by the evolved holistic profile of Fig. 10(c), in his transition to "badness" "entangles" with an accomplice — the two (unfilled) unstable fixed points of figure (d) — with the four possible outcomes of footnote 20 denoted by the (filled) stable fixed points, leading to the iterated dilemma corresponding to the converged holism of (d). When the entanglement is weak (linear) however, it is possible to consider the dilemma in terms of the Bell states in the base $(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)/\sqrt{2}$ resulting in the Nash equilibrium $(\downarrow\downarrow)$. Carrying this type of reasoning a step further, it is conceivable that globalization has effectively transformed the world economy into a single-celled monolith from its complex multi-cellular form, with the inevitable consequence that it is incapable of any self-organization to a meaningful homeostatic form.

Nonlinear self-organization and emergence are fascinating demonstrations of dynamical homeostasis of opposites, apparenly the source and sustenance of Nature's diversity.

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