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Is nature quantum non-local, complex holistic, or what?, I: Theory & analysis

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ABSTRACT

Is Quantum Mechanics a general theory that applies to everything from subatomic particles to galaxies; is Nature governed by entanglements of linear superposition in Hilbert space or is it an expression of the nonlinear holism of emergence, self-organization, and complexity? This paper explores these issues in the context of extensions of the recently formulated approach to Chaos-Nonlinearity-compleXity [A. Sengupta, Chaos, nonlinearity, complexity: A unified perspective, in: A. Sengupta (Ed.), Chaos, Nonlinearity, and Complexity: The Dynamical Paradigm of Nature, in: StudFuzz, vol. 206, Springer-Verlag, Berlin, 2006, pp. 270–352] and proposes that Nature manifests itself only through a bi-directional, contextually objective adaptation of the Second Law of Thermodynamics. The mathematics developed is that of convergence in topological spaces, multifunctions, exclusion topology, and discontinuities, jumps, and multiplicities induced by difference equations rather than the smoothness and continuity of the Newtonian world of differential equations.

In the second Part of this paper [A. Sengupta, Is Nature Quantum Non-local, Complex Holistic, or What? II—Applications, Nonlinear Analysis: RWA (2009), in press (doi:10.1016/j.nonrwa.2008.09.002)] to be referenced "II-", we examine the bi-directionality of a self-organized, emergent, engine-pump system with specific reference of the role of gravity as the compressive agent responsible for generation and sustenance of the delicate homeostatic balance in life, structures, and patterns appearing in this Participatory Universe. Holism of "systems and processes that interact with themselves and produce themselves from themselves" calls for radically different techniques and philosophy from that adapted in the mainstream reductionist Newtonian paradigm that we are accustomed to.

1. Introduction

A scientific, self-contained and unified approach to Chaos-Nonlinearity-compleXity was formulated in [2] under the acronym ChaNoXity. This new perspective of the dynamical evolution of Nature is based on the irreversible multifunctional multiplicities generated by the equivalence classes from iteration of noninvertible maps, eventually leading to chaos of maximal ill-posedness, see [3] for details. In this sense, the iterative evolution of difference equations is in sharp contrast to the smoothness, continuity, and reversible development of differential equations that cannot lead to the degenerate irreversibility inherent in the classes of maximal ill-posedness. Unlike evolution of differential equations, difference equations update their progress at each instant with reference to its immediate predecessor, thereby satisfying the crucial requirement of adaptability that constitutes the defining feature of complex systems. Rather than the smooth continuity of differential equations, Nature takes advantage of jumps, discontinuities, and singularities to choose from the vast multiplicity of possibilities that rejection of such regularizing constraints entail. The immediate upshot of this

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shift in perspective from differential to difference (and possibly to differential-difference) equations, lies in the greater flexibility the evolving system has in presenting its produce of emergence and self-organization through a judicious application of the Axiom of Choice. Non-locality and holism, the natural consequences of this paradigm, are to be compared with the reductionist determinism of classical Newtonian reversibility suggesting striking formal correspondence with superpositions, qubits and entanglement of quantum theory. Although it is premature to make any definitive commitments in this regard, this paper aims at making this possibility at least plausibly admissible thereby contributing to the query: *Is Nature governed by entanglements of linear reductionist superposition in Hilbert space or by the nonlinear holism of emergence, self-organization, and complexity?* Could the awesome complex diversity of Nature be so breathtaking in its omnipresence if it were constrained to operate in a unique, smooth, and completely predictable one–one fashion? Or is it a case of both these alternatives with the former being really an *excellent effective representation* of the actuality of the later?[1]

The Hamilton–Jacobi formulation of classical mechanics is closest in approach to quantum mechanics. In essence, a classical system with N memory bits can access any of the 2^N basic states $e_i = (0, \ldots, 1_i, \ldots, 0)^{\mathrm{T}}$, $i = 0, \ldots, 2^{N-1}$, with probability $p_i \geq 0$, $\sum_{i=0}^{2^N-1} p_i = 1$. If the state i was obtained after a measurement then the system continues to be in this state into the future. If p_{ji} is the probability for the transition $i \to j$ then $(p_{ji})_{2^N \times 2^N}$ is the *stochastic transition matrix* that yields the state vector in a succeeding time step j from that at i. In its *deterministic form*, an observable $\mathscr{O}$ is represented by a two-valued real function $f_{\mathscr{O}} : \Omega \to \mathbb{R}$ mapping the $6N$ dimensional phase space Ω, with the possible 0-no/1-yes response of the query $f_{\mathscr{O}}^{-}(\Delta) \overset{?}{\subseteq} \Omega$ defining $f_{\mathscr{O}}$.

The quantum state-space of any *isolated* physical system, in comparison, is represented by the complex Hilbert space $\mathscr{H}$, and the system is completely described by its unit state vector. The time evolution of a *closed* quantum system is governed by a unitary transformation $\Psi(t_2) = U\Psi(t_1)$ of the Schrodinger equation $i\hbar\,\partial\,|\Psi\rangle\,/\partial t = H\,|\Psi\rangle$. Linear subspaces of $\mathscr{H}$ correspond to the inverse images $f_{\mathscr{O}}^{-}(\Delta) \subseteq \Omega$, and continuous probabilities replace the binary yes/no possibilities of classical mechanics. Arbitrary superpositions is the principal distinguishing feature of quantum mechanics having no classical analogue: whereas a system in a classical ensemble of states can only be in one of the states unknown to the observer, a quantum superposition endows the system with a ubiquitous omnipresence that translates into *entangled correlations* between different subsystems. This leads to properties of the whole that cannot be traced to the individual parts: "the (quantum-mechanical) whole is different from the sum of its parts" [8].

The long range, all-pervading *contractive* attribute of gravity appears to be in direct conflict with the *expansive* dispensation of dispersion, degradation, and the capacity to be useful as mandated by the Second Law of Thermodynamics. The very fact that open systems exist in Nature in apparent defiance of this "supreme among the laws of Nature" appears to point to the bi-directionality of chanoxity [2] as a possible mechanism of why this attractive attribute of Nature has successfully resisted attempts at integration with its other forces: gravity is a component that endows Nature with the bi-directional yang-yingism required for its existence as a complex multifaceted system.

This paper hopes to elaborate on these issues to a reasonable degree of admissibility; it hopes to establish the Question

Question: Is nature interactively nonlinear and holistic, or is it additively linear and reductionist?

2. Chaos-nonlinearity-complexity

2.1. Chaos is maximal non-injectivity[2]

Chaos responsible for complexity is the eventual outcome of *non-reversible* iterations of one-dimensional *non-injective* maps [3]; noninjectivity leads to irreversible nonlinearity and one-dimensionality constrains the dynamics to evolve with the minimum latitude thereby inducing emergence of new features as required by complexity. In this sense, chaos is the maximal ill-posed irreversibility of the maximal degeneracy of multifunctions; features that cannot appear through differential equations. The mathematics involve topological methods of convergence of nets and filters with the multifunctional graphically converged adherent sets effectively enlarging the functional world in the outward manifestation of Nature. Chaos therefore is more than just an issue of whether or not it is possible to make accurate long-term predictions of the system: chaotic systems must necessarily be sensitive to initial conditions, topologically mixing with dense periodic orbits [10]; this however is not sufficient.

[1] (a) "The experimental evidence against Bell inequality tells us that any theory quantum mechanics is derived from must be non-local. It is then natural to hypothesize that this non-local theory is a cosmological theory", [4]; see also [5,6].
(b) "We shall see that the only way to understand the quantum-mechanical computations and to justify their undeniable success, is to realize that quantum mechanics is nothing but an approximation of Quantum Field Theory (QFT) in the limit when its "field densities" become extremely small. Naturally the eclipse of quantum mechanics as a basic physical theory will also mean the eclipse of the Copenhagen view as a new, unsavory image of the physical world, upon which so much bad philosophy has been recently built" [7].

[2] And thermodynamic irreversibility, by (Eq. (22)), (II-8) – citations of the type (II-xx) are with reference to Sengupta [9] – and the discussions therein.

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2.1.1. Graphical convergence

Let $(X, \mathcal{U})$ and $(Y, \mathcal{V})$ be Hausdorff spaces and $(f_\alpha)_{\alpha\in\mathbb{D}} : X \to Y$ be a net of piecewise continuous functions, not necessarily one–one, onto, or with the same domains and range, and suppose that for each $\alpha \in \mathbb{D}$ there is a finite set $I_\alpha = \{1, 2, \dots P_\alpha\}$ such that the "inverse" f_α^- satisfying $f_\alpha^- f_\alpha f_\alpha^- = f_\alpha^-$ and $f_\alpha f_\alpha^- f_\alpha = f_\alpha$[3] has P_α functional injective branches possibly with different domains; obviously I_α is a singleton iff, f is an injective. For each $\alpha \in \mathbb{D}$, define functions $(g_{\alpha i})_{i\in I_\alpha} : Y \to X$ such that

$$f_\alpha g_{\alpha i} f_\alpha = f_{\alpha i}^I \qquad i = 1, 2, \dots, P_\alpha,$$

where $f_{\alpha i}^I$ is a basic injective branch of f_α on some subset of its domain: $g_{\alpha i} f_{\alpha i}^I = 1_X$ on $\mathcal{D}(f_{\alpha i}^I)$, $f_{\alpha i}^I g_{\alpha i} = 1_Y$ on $\mathcal{D}(g_{\alpha i})$ for each $i \in I_\alpha$.

In terms of the residual and cofinal subsets $\mathrm{Res}(\mathbb{D}) = \{\mathbb{R}_{\not\succeq} \in \mathcal{P}(\mathbb{D}) : \mathbb{R}_\alpha = \{\beta \in \mathbb{D} \text{ for all } \beta \succeq \alpha \in \mathbb{D}\}\}$ and $\mathrm{Cof}(\mathbb{D}) = \{\mathbb{C}_{\not\succeq} \in \mathcal{P}(\mathbb{D}) : \mathbb{C}_\alpha = \{\beta \in \mathbb{D} \text{ for some } \beta \succeq \alpha \in \mathbb{D}\}\}$ of a directed set $\mathbb{D}$, with x and y in the equations below being taken to belong to the required domains, define subsets $\mathcal{D}_-$ of X and $\mathcal{R}_-$ of Y as

$$\mathcal{D}_- = \{x \in X : ((f_\nu(x))_{\nu\in\mathbb{D}} \text{ converges in } (Y, \mathcal{V}))\} \tag{1}$$

$$\mathcal{R}_- = \{y \in Y : (\exists i \in I_\nu)((g_{\nu i}(y))_{\nu\in\mathbb{D}} \text{ converges in } (X, \mathcal{U}))\}. \tag{2}$$

Thus:

$\mathcal{D}_-$ is the set of points of X on which the values of a given net of functions $(f_\alpha)_{\alpha\in\mathbb{D}}$ converge pointwise in Y. Explicitly, this is the subset of X on which subnets of functions combine to form a net of functions converging pointwise to a limit function $F : \mathcal{D}_- \to Y$ in $\mathrm{map}(X, Y)$.

$\mathcal{R}_-$ is the set of points of Y on which the values of the nets in X generated by the injective branches of $(f_\alpha)_{\alpha\in\mathbb{D}}$ converge pointwise in Y. Explicitly, this is the subset of Y on which subnets of injective branches of $(f_\alpha)_{\alpha\in\mathbb{D}}$ in $\mathrm{map}(Y, X)$ combine to form a net of functions that converge pointwise to a family of limit functions $G : \mathcal{R}_- \to X$. Depending on the nature of $(f_\alpha)_{\alpha\in\mathbb{D}}$, there may be more than one $\mathcal{R}_-$ with a corresponding family of limit functions on each of them. To simplify the notation, we will usually let $G : \mathcal{R}_- \to X$ denote all the limit functions on all the sets $\mathcal{R}_-$.

If we consider cofinal rather than residual subsets of $\mathbb{D}$ then corresponding $\mathbb{D}_+$ and $\mathbb{R}_+$ can be expressed by Eqs. (1) and (2) with $\nu \in \mathrm{Cof}(\mathbb{D})$ rather than in $\mathbb{D}$. Since $\mathcal{D}_+$ and $\mathcal{R}_+$ differ from $\mathcal{D}_-$ and $\mathcal{R}_-$ only in having cofinal subsets of D replaced by residual ones, and since residual sets are also cofinal, it follows that $\mathcal{D}_- \subseteq \mathcal{D}_+$ and $\mathcal{R}_- \subseteq \mathcal{R}_+$. The sets $\mathcal{D}_-$ and $\mathcal{R}_-$ serve for the convergence of a net of functions just as $\mathcal{D}_+$ and $\mathcal{R}_+$ are for the convergence of subnets of the nets (*adherence*). The later sets are needed whenever subsequences are to be considered as sequences in their own right as, for example, in dynamical systems theory in the case of ω-limit sets.

Definition 1 (*Graphical Convergence of a Net of Functions*). A net of functions $(f_\alpha)_{\alpha\in D} : (X, \mathcal{U}) \to (Y, \mathcal{V})$ converges *graphically* if either $\mathcal{D}_- \neq \emptyset$ or $\mathcal{R}_- \neq \emptyset$; in this case let $F : \mathcal{D}_- \to Y$ and $G : \mathcal{R}_- \to X$ be the entire collection of limit functions. The graph of the *graphical limit* $\mathscr{M}$ *of the net* (f_α) denoted by $f_\alpha \xrightarrow{\mathbf{G}} \mathscr{M}$, is the subset of $\mathcal{D}_- \times \mathcal{R}_-$ that is the union of the graphs of the function F and the multifunction G^-

$$\mathbf{G}_{\mathscr{M}} = \mathbf{G}_F \bigcup \mathbf{G}_{G^-}$$

where

$$\mathbf{G}_{G^-} = \{(x, y) \in X \times Y : (y, x) \in \mathbf{G}_G \subseteq Y \times X\}.$$

2.1.2. Boundary $\mathrm{Multi}_{\|}(X, Y)$ of $\mathrm{map}(X, Y)$

In this section we show how the usual topological treatment of pointwise convergence of functions to functions can be generalized to generate the boundary $\mathrm{Multi}_{\|}(X, Y)$ between $\mathrm{map}(X, Y)$ and $\mathrm{multi}(X, Y)$; here $\mathrm{map}(X, Y)$ and $\mathrm{multi}(X, Y)$ are respectively proper functional and non-functional subsets of all the relations $\mathrm{Multi}(X, Y)$ between X and Y

$$\mathrm{Multi}(X, Y) = \mathrm{map}(X, Y) \bigcup \mathrm{Multi}_{\|}(X, Y) \bigcup \mathrm{multi}(X, Y);$$

see Eq. (6g). The generalization we seek defines neighborhoods of $f \in \mathrm{map}(X, Y)$ to consist of those functional relations in $\mathrm{Multi}(X, Y)$ whose images at any point $x \in X$ lies not only arbitrarily close to $f(x)$ but whose inverse images at $y = f(x) \in Y$ contain points arbitrarily close to x. Thus the graph of f must not only lie close enough to $f(x)$ at x in V, but must additionally be such that $f^-(y)$ has at least branch in U about x; hence f is constrained to cling to f as the number of points on its graph increases with convergence and, unlike in the situation of simple pointwise convergence, no gaps in the graph of the limit is permitted not only on the domain of f, but on its range too.

[3] $f^{-i}f^i(x) = [x]$; hence $f^{-i}f^i = \mathbf{1}_{X/\sim}$.

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Denote the resulting *topology of pointwise biconvergence* generated on $\mathrm{map}(X, Y)$ by $\mathcal{T}$. Thus for any given integer $I \geq 1$ the open sets of $(\mathrm{map}(X, Y), \mathcal{T})$, $i = 1, 2, \ldots, I$, are

$$B((x_i), (V_i); (y_i), (U_i)) = \left\{g \in \mathrm{map}(X, Y) : (g(x_i) \in V_i) \wedge \left(g^-(y_i) \bigcap U_i \neq \emptyset\right)\right\}, \tag{3}$$

where $(x_i)_{i=1}^I \in X$, $(V_i)_{i=1}^I \subset Y$ and $(y_i)_{i=1}^I \in Y$, $(U_i)_{i=1}^I \in X$ are chosen arbitrarily. A local base at f, for $(x_i, y_i) \in \mathbf{G}_f$, is the set of functions of (3) with $y_i = f(x_i)$ and the collection of all local bases

$$B_\alpha = B((x_i)_{i=1}^{I_\alpha}, (V_i)_{i=1}^{I_\alpha}; (y_i)_{i=1}^{I_\alpha}, (U_i)_{i=1}^{I_\alpha}), \tag{4}$$

for every choice of $\alpha \in \mathbb{D}$, is a base ${}_T\mathcal{B}$ of $(\mathrm{map}(X, Y), \mathcal{T})$.

In a manner similar to Eq. (3), the open sets of $(\mathrm{Multi}(X, Y), \widehat{\mathcal{T}})$ is defined as

$$\widehat{B}((x_i), (V_i); (y_i), (U_i)) = \left\{\mathcal{G} \in \mathrm{Multi}(X, Y) : \left(\mathcal{G}(x_i) \bigcap V_i \neq \emptyset\right) \wedge \left(\mathcal{G}^-(y_i) \bigcap U_i \neq \emptyset\right)\right\} \tag{5}$$

with $\mathcal{G}^-(y) = \{x \in X : y \in \mathcal{G}(x)\}$ and $(x_i)_{i=1}^I \in \mathcal{D}(\mathcal{G})$, $(V_i)_{i=1}^I$; $(y_i)_{i=1}^I \in \mathcal{R}(\mathcal{G})$, $(U_i)_{i=1}^I$ chosen as above. The topology $\widehat{\mathcal{T}}$ of $\mathrm{Multi}(X, Y)$ is generated by the collection of all local bases $\widehat{B}_\alpha$ for every choice of $\alpha \in \mathbb{D}$, and it is not difficult to see from Eqs. (3) and (5), that the restriction $\widehat{\mathcal{T}}|_{\mathrm{map}(X,Y)}$ is $\mathcal{T}$; denote convergence in the topology of pointwise biconvergence in $(\mathrm{Multi}(X, Y), \widehat{\mathcal{T}})$ by $\rightrightarrows$.

Theorem 1. *Let* $(f_\alpha)_{\alpha\in\mathbb{D}}$ *be a net of functions in* $\mathrm{map}(X, Y)$. *Then*

$$f_\alpha \xrightarrow{\mathbf{G}} \mathscr{M} \Longleftrightarrow f_\alpha \rightrightarrows \mathscr{M}.$$

Observe that the boundary of $\mathrm{map}(X, Y)$ in the topology of pointwise biconvergence is a "line parallel with the Y-axis"; denote this boundary[4] by

$$\mathrm{Multi}_{\|}((X, Y), \widehat{\mathcal{T}}) = \mathrm{Cl}(\mathrm{map}((X, Y), \widehat{\mathcal{T}})) - \mathrm{map}((X, Y), \mathcal{T}).$$

2.1.3. *Chaos in* $\mathrm{Multi}((X, Y), \widehat{\mathcal{T}})$

The definitional requirement of a maximal multifunctional extension $\mathrm{Multi}(X, Y) \supseteq \mathrm{map}(X, Y)$ of the function space $\mathrm{map}(X, Y)$ is to be understood as follows. Let f be a noninjective map in $\mathrm{Multi}(X)$ and $P(f)$ the number of injective branches of f. Denote by

$$F = \{f \in \mathrm{Multi}(X) : f \text{ is a noninjective function on } X\}$$

the resulting basic collection of noninjective functions in $\mathrm{Multi}(X)$.

(i) For every α in some directed set $\mathbb{D}$, let F have the extension property

$$(\forall f_\alpha \in F)(\exists f_\beta \in F) : P(f_\alpha) \leq P(f_\beta)$$

(ii) For $f_\alpha, f_\beta \in \mathrm{map}(X)$, let

$$P(f_\alpha) \leq P(f_\beta) \Leftrightarrow f_\alpha \preceq f_\beta$$

with $P(f) := 1$ for the smallest f, define a partial order $\preceq$ in $\mathrm{Multi}(X)$. This is actually a preorder in which functions with the same number of injective branches are equivalent.

(iii) Let

$$C_\nu = \{f_\alpha \in \mathrm{Multi}(X) : f_\alpha \preceq f_\nu\},$$

be chains of non-injective functions of $\mathrm{Multi}(X)$ and

$$\mathcal{X} = \{C \in \mathcal{P}(F) : C \text{ is a chain in } (F, \preceq)\}$$

denote the corresponding chains of F with $\mathcal{C} = \{C_\alpha, C_\beta, \ldots, C_\nu, \ldots\}$ a chain in $\mathcal{X}$. By Hausdorff Maximal Principle, there

[4] For a subset $A \subseteq X$,

$$\mathrm{Bdy}(A) = \left\{x \in X : (\forall N \in \mathcal{N}_x)\left(N \bigcap A \neq \emptyset\right) \wedge \left(N \bigcap (X - A) \neq \emptyset\right)\right\} : \text{Boundary of } A \tag{6a}$$

$$\mathrm{Der}(A) = \left\{x \in X : (\forall N \in \mathcal{N}_x)\left(N \bigcap (A - \{x\}) \neq \emptyset\right)\right\} : \text{Derived set of } A \tag{6b}$$

$$\mathrm{Iso}(A) = A - \mathrm{Der}(A) = \mathrm{Cl}(A) - \mathrm{Der}(A) : \text{Isolated points of } A \tag{6c}$$

$$\mathrm{Int}(A) = \{x \in X : (\exists N \in \mathcal{N}_x)(N \subseteq A)\} : \text{Largest open set of } X \text{ contained in } A \tag{6d}$$

$$\mathrm{Ext}(A) = \mathrm{Int}(X - A) = X - \mathrm{Cl}(A) : \text{Largest open set of } X \text{ contained in } X - A \tag{6e}$$

$$\begin{aligned}\mathrm{Cl}(A) &= \left\{x \in X : (\forall N \in \mathcal{N}_x)\left(N \bigcap A \neq \emptyset\right)\right\} : \text{Smallest closed set of } X \text{ containing } A \\ &= A \bigcup \mathrm{Bdy}(A) = A \bigcup \mathrm{Der}(A) = \mathrm{Iso}(A) \bigcup \mathrm{Der}(A) = \mathrm{Int}(A) \bigcup \mathrm{Bdy}(A)\end{aligned} \tag{6f}$$

$$X = \mathrm{Int}(A) \bigcup \mathrm{Bdy}(A) \bigcup \mathrm{Ext}(A). \tag{6g}$$

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exists a maximal chain

$$\sup_{\mathcal{C}}(\mathcal{C}) = {}_{\rightarrow}C = \{f_\alpha, f_\beta, f_\gamma, \ldots\}$$

of F in $\mathcal{X}$. Compare Fig. 2b with the obvious identification of f_α as the elements of the sets and ${}_{\rightarrow}X \leftrightarrow {}_{\rightarrow}C$ and $X_{\leftarrow} \leftrightarrow x$.

Zorn's Lemma now applied to this maximal chain yields its supremum as the maximal element of ${}_{\rightarrow}C$, and thereby of F. Note that as in the case of the algebraic Hamel basis, the existence of this maximal non-functional element emerged purely set theoretically as the "limit" of a net of functions with increasing non-linearity, without recourse to any topological arguments. Because it is not a function, this supremum does not belong to the functional towered chain with itself as a fixed point; this maximal chain does not possess a largest, or even a maximal, element although it does have a supremum. The supremum is a contribution of the inverse functional relations (f_α^-) because the net of increasingly non-injective functions implies a corresponding net of multivalued functions increasingly ordered by $f_\alpha \preceq f_\beta \Leftrightarrow f_\beta^- \preceq f_\alpha^-$. Thus the inverse relations which are as much an integral part of graphical convergence as are the direct, have a smallest element belonging to the multifunctional class. Clearly, this smallest element as the required supremum of the increasingly non-injective tower of functions defined in (ii), serves to complete the significance of the tower by capping it with a "boundary" element that can be taken to bridge the classes of functional and non-functional relations on X.

Definition 2 (*Chaotic Map*). Let A be a non-empty closed set of a compact Hausdorff space $(X, \mathcal{U})$. A function $f \in \text{Multi}(X)$ is *maximally non-injective* or *chaotic* on $\mathcal{D}(f) = A$ w.r.t. to $\preceq$ if (a) for any f_i there exists an f_j satisfying $f_i \preceq f_j$ for every $i < j \in \mathbb{N}$, and (b) the dense set $\mathcal{D}_+ := \{x \in X : (f_\nu(x))_{\nu \in \text{Cof}(\mathbb{D})}\}$ converging in X is a countable collection of isolated singletons.[5]

The significance of this ill-posed approach to chaos [3] in the discrete one dimensional evolution of non-injective maps defined by the dual binary increasing and decreasing $(\uparrow, \downarrow)$ components of positive and negative slopes, has a direct significance to complexity, holism, and chanoxity in the sense that

(a) The multifunctional extension Multi(X) being a superspace of map(X), elements $\Phi \in \text{Multi}(X) \notin \text{map}(X)$ are in a special privileged position in the evolutionary dynamics of $f \in \text{map}(X)$. Any $\Phi \notin \text{map}(X)$ being a multi in both the forward and backward directions of increasing and decreasing times (iterates) – unlike F which are multis only for backward times into the past through its "mind" – has the privilege of *symmetric* access to all its resources in both direction that would be a source of major embarrassment in X if this here and now were to be also multifunctionally ubiquitous. This spatial omnipresence throughout X – the signature of emergence in nonlinear complex systems – bears comparison with quantum entanglement and non-locality. The chaotic states forming the boundary between the functional and multifunctional subspaces of Multi(X) marks a transition from the less efficient functional world to the more efficient multifunctional.

(b) The collective macroscopic cooperation between map(X) and its closure $\text{map}(X) \bigcup \text{Multi}_{\|}(X)$ generates the equivalence classes in map(X) through fixed points and periodic cycles of F. As all the points in a class are equivalent under F, a net or sequence converging to any one must necessarily converge to the every other in the set. This implies that the cooperation between map(X) and Multi(X) conspires to alter the topology of X to large equivalence classes whose Φ-images are the open sets in $\mathcal{R}(F)$, the inverse images in turn constituting the topology of X in $\mathcal{D}(F)$. This dispersion throughout the domain of F of initial localizations suggests increase in entropy (and disorder) with increasing chaoticity; complete chaos therefore corresponds to the state of maximum entropy of the second law of thermodynamics.

(c) Since chaos requires an enlargement of the function space to include multifunctions with the boundary $\text{Multi}_{\|}(X, Y)$ between the two, the significance of the extension lies [3] in dispersion of the driving gradients, as required by the Second Law of Thermodynamics. Physically, a map $f : (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ can be regarded as an interaction between X and Y with the algebraic and topological properties of f determining the nature of this interaction. While a simple bijection merely sets up a correspondence between elements of X and Y, continuity establishes a stronger correspondence between the privileged class of "open" sets. The special category of functions that are both image and pre-image continuous – the *ini*(tial-fi)*nal* functions – forces these well-defined and definite groups in $(X, \mathcal{U})$ and $(Y, \mathcal{V})$ to interact with each other through f; this is not possible with simple continuity as there may be open sets in X that are not derived from those of Y and non-open sets in Y whose inverse images are open in X. *It is our contention that the driving force behind the evolution of a system represented by the input–output relation $f(x) = y$ is the attainment of a ininal triple state (X, f, Y) for the system,* with the pre-image and image continuous $f : X \rightarrow Y$ endowing X with the smallest possible topology of f-saturated sets and Y the largest topology of all their images, reducing thereby to a simple non-bijective "homeomorphism". In the case of iterations on $(X, \mathcal{T})$, $\mathcal{T}$ acquires the very special initial-final character of dead-state of maximum entropy, as the commutative diagram in Fig. 1 seeks to illustrate.

Chaos as manifest in its attractors is a direct consequence of the increasing nonlinearity of the map with increasing iteration: under the right conditions this is the natural outcome in the difference of behavior of a function f and its multi-inverse f^- under their successive applications. Equivalence classes of fixed points, stable and unstable, are of defining significance in determining the eventual behavior of an evolving dynamical system and *it is postulated that chaoticity on a set*

[5] The reason for defining convergence with respect to cofinal rather than residual subsets is that we need adherent sets of convergent subnets rather than limit sets of complete nets.

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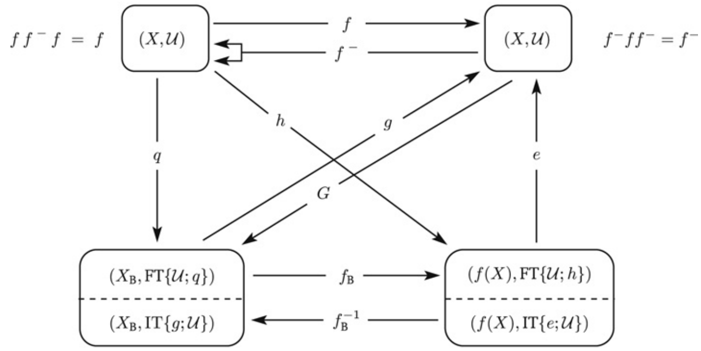

Fig. 1. The driving force behind attainment of homeostatic symmetry. f is a non-injective, non-surjective map on topological space $(X, \mathcal{U})$ that is studied through its bijective homeomorphic restriction f_B. The topology $\mathcal{U}$ on X is transformed to large f-equivalence classes on its domain and its images on the range to generate on X a topology that is both initial and final with respect to f.

X leads to a reformulation of the open sets of X to equivalence classes generated by the unfolding iterates of f. Such a redefinition of open sets of equivalence classes allow the evolving system to temporally access an ever increasing number of states that can be considered to be the governing criterion for the cooperative or collective behavior of the system. In the limit of infinite iterations leading to the boundary multifunction $\mathrm{Multi}_{\|}$, the generated open sets constitute a basis for a topology on $\mathcal{D}(f)$, and the basis for the topology of $\mathcal{R}(f)$ are the corresponding $\mathrm{Multi}_{\|}$-images of these equivalent classes: *it is our contention that the motive force behind evolution toward chaos is the drive toward homogenization of the state of the dynamical system that supports ininality of the limit multi*. In the limit of infinite iterations, the open sets of the range $\mathcal{R}(f) \subseteq X$ are the images that graphical convergence generates at each of these inverse-stable fixed points. X therefore has two topologies imposed on it by the dynamics of f: the first of equivalence classes generated by $\mathrm{Multi}_{\|}$ in the domain of f and the second as $\mathrm{Multi}_{\|}$-images of these classes in the range of f. Neighborhoods of points in this topology cannot be arbitrarily small as they consist of all members of the equivalence class to which any element belongs; hence a sequence converging to any of these elements necessarily converges to all of them, and the eventual objective of chaotic dynamics is to generate a topology in X with respect to which elements of the set can be grouped together in as large equivalence classes as possible in the sense that if a net converges simultaneously to points $x \neq y \in X$ then $x \sim y$. This topological homogenization of X responsible for chaos is the causative impulse for the dispersive manifestation of increasing entropy as required by the Second Law.

This scenario indicates a strong case for a fresh look at some of the contentious issues of quantum mechanics like non-locality, entanglement and decoherence in the complex systems perspective. We attempt to do this in the following.

2.2. ChaNoXity: The dynamics of $(\uparrow, \downarrow)$ opposites

• *In many real-world (situations), the second law can be obstructed or hindered for millions of years. Blockage of the second law is absolutely necessary for us to be alive and happy. Not one of the complex chemical substances in our body, and few in the things we enjoy, would exist for a microsecond if the second law was not obstructed. Its tendency is never eliminated but, fortunately for us, there is a huge number of compounds in which it is blocked for our lifetimes and even far longer.*

Let us see how we humans use the second law for our purposes. Whenever we run any kind of engine, we are using the second law for our benefit: Taking energy inside of substances that tend to spread out, but cannot because of (the activation energy) E_a, giving it the necessary energy, having the diffusing energy in the form of hot expanding gases push a piston that turns crankshafts, gears and wheels, with the exhaust gases, still fairly hot, but no longer available for any more piston-pushing in this engine. LAMBERT [11]

The basic undertaking in our approach to a formal theory of complex systems [2] consists in establishment of a rigorous exposition of bi-directionality as the main tool in their evolution, capable of providing a rational understanding of why and how Nature apparently defeats the all-embracing outreach of the Second Law. This is achieved by identifying the multifunctional world $\mathrm{Multi}(X)$ as a dual "negative" $\mathfrak{W}$ of the real world W of $\mathrm{map}(X)$ as follows.

Let W be a set and suppose that for every $w \in W$ there exists a negative dual $\mathfrak{w} \in \mathfrak{W}$ with the property that

$$\mathfrak{W} \triangleq \left\{\mathfrak{w} : \{w\} \bigcup \{\mathfrak{w}\} = \emptyset\right\} \tag{7a}$$

defines the negative, or exclusion, set of W. This means that for every subset $A \subseteq W$ there is a complementary neg(ative)set $\mathfrak{A} \subseteq \mathfrak{W}$ associated with (generated by) A such that

$$\begin{aligned} &A \bigcup \mathfrak{G} \triangleq A - G, \quad G \leftrightarrow \mathfrak{G} \\ &A \bigcup \mathfrak{A} = \emptyset. \end{aligned} \tag{7b}$$

Table 1
Competitive directions of inclusion and exclusion topologies.

World	Directed set $\mathbb{D}$	Direction $\preceq$
W	$N : N \in \mathcal{N}_w$	$M \preceq N \Leftrightarrow N \subseteq M$
	$\{(N, t) : (N \in \mathcal{N}_w)(t \in N)\}$	$(M, s) \preceq (N, t) \Leftrightarrow N \subseteq M$
	$\{(N, \beta) : (N \in \mathcal{N}_w)(w_\beta \in N)\}$	$(M, \alpha) \preceq (N, \beta) \Leftrightarrow (\alpha \leq \beta)$ $(N \subseteq M)$
$\mathfrak{W}$	$\{\mathfrak{N} : \mathfrak{N} \in \mathcal{N}_{\mathfrak{w}}\}$	$\mathfrak{M} \preceq \mathfrak{N} \Leftrightarrow \mathfrak{M} \subseteq \mathfrak{N}$
	$\{(\mathfrak{N}, \mathfrak{t}) : \mathfrak{N} \in \mathcal{N}_{\mathfrak{w}}\}\{(\mathfrak{t} \in \mathfrak{N})\}$	$(\mathfrak{M}, \mathfrak{s}) \preceq (\mathfrak{N}, \mathfrak{t}) \Leftrightarrow \mathfrak{M} \subseteq \mathfrak{N}$
	$\{(\mathfrak{N}, \beta) : \mathfrak{N} \in \mathcal{N}_{\mathfrak{w}}\}\{(\mathfrak{w}_\beta \in \mathfrak{N})\}$	$(\mathfrak{M}, \alpha) \preceq (\mathfrak{N}, \beta) \Leftrightarrow (\alpha \leq \beta)$ $(\mathfrak{M} \subseteq \mathfrak{N})$

Hence a neg-set and its generator act as relative discipliners of each other in restoring a measure of order in the evolving confusion, disquiet and tension, with the intuitive understanding of the set-negset pair *undoing*, *controlling*, or *stabilizing* each other. The complementing neg-element is an unitive inverse of its generator, with $\emptyset$ the corresponding identity and G the physical manifestation of $\mathfrak{G}$. Thus if $r > s \in \mathbb{R}_+$, the physical manifestation of any $-s \in \mathfrak{R}_+(\equiv \mathbb{R}_-)$ is the smaller element $r - s \in \mathbb{R}_+$. The left hand side of Eq. (7b), read in the more familiar form $a + (-b) = a - b$ with $a, b \in \mathbb{R}_+$ and $-b := \mathfrak{b} \in \mathfrak{R}_+$, represents a "+" operation which in the actual bi-directional physical-world manifests on the right as a retraction in the "−" direction.

Compared with the directed set $(\mathcal{P}(W), \subseteq)$[6], the direction of *increasing supersets* induced by $(\mathcal{P}(W), \supseteq)$ proves useful in generating a co-topology $\mathcal{U}_-$ on $(W, \mathcal{U}_+)$ as follows. Let $(w_0, w_1, w_2, \ldots)$ be a sequence in W converging to $w_* \in W$, and consider the backward arrow induced at w_* by the directed set $(\mathcal{P}(W), \supseteq)$ of increasing supersets at w_*. As the reverse sequence $(w_*, \ldots, w_{i+1}, w_i, w_{i-1}, \ldots)$ does not converge to w_0 unless it is eventually in every neighborhood of this initial point, we define an additional *exclusion topology* $\mathcal{U}_-$ on $(W, \mathcal{U}_+)$, where the w-*exclusion topology* consists together with W, all the subsets $\mathcal{P}(W - \{w\})$ that *exclude* w; a proper subset of W is open in this topology iff, it does not contain w [12]. Since $\mathcal{N}_w = \{W\}$ and $\mathcal{N}_{v \neq w} = \{\{v\}\}$ are the neighborhood systems at w and any $v \neq w$ in this exclusion topology, it follows that while every net must converge to the defining point of its own topology, only the eventually constant $\{v, v, v, \ldots\}$ converges to a $v \neq w$. All directions with respect to w are consequently rendered equivalent; hence the directions of $\{1/n\}_{n=1}^{\infty}$ and $\{n\}_{n=1}^{\infty}$ are equivalent in $\mathbb{R}_+$ as they converge to 0 in its exclusion topology, and this basic property of the exclusion topology induces an opposing direction in W.

With respect to a sequence $(w_i)_{i \geq 0}$ in $(W, \mathcal{U}_+)$ converging to $w_* = \bigcap_{i \geq 0} \mathrm{Cl}(N_i)$ in W, let there exist an increasing sequence of negelements $(\mathfrak{w}_i)_{i \geq 0}$ of $\mathfrak{W}$ that converges to $\mathfrak{w}_*$ in the $\mathfrak{w}_*$-inclusion topology $\mathfrak{U}$ of $\mathfrak{W}$ generated by the $\mathfrak{W}$-images of the neighborhood system $\mathcal{N}_{w*}$ of $(W, \mathcal{U}_+)$. Since the only manifestation of neg-sets in the observable world is their regulation of W, the $\mathfrak{W}$-increasing sequence $(\mathfrak{w}_i)_{i \geq 0}$ converges to $\mathfrak{w}_*$ in $(\mathfrak{W}, \mathfrak{U})$ if and only if, the sequence $(w_0, w_1, w_2, \ldots)$ converges to w_* in $(W, \mathcal{U}_+)$. Affinely translated to W, this means that the $\mathfrak{w}_*$-inclusion arrow in $(\mathfrak{W}, \mathfrak{U})$ induces an w_0-exclusion arrow in $(W, \mathcal{U}_+)$ generating an additional topology $\mathcal{U}_-$ in W that opposes the arrow converging to w_*. This *direction of increasing supersets of* $\{w_*\}$ *excluding* w_0 associated with $\mathcal{U}_-$ is to be compared with the *natural direction of decreasing subsets containing* w_* in $(W, \mathcal{U}_+)$. We take the reference natural direction in $W \cup \mathfrak{W}$ to be that of W pulling the inclusion sequence $(w_0, w_1, w_2, \ldots)$ to w_*; hence, the decreasing subset direction in $\mathfrak{W}$ of the inclusion sequence $(\mathfrak{w}_0, \mathfrak{w}_1, \mathfrak{w}_2, \ldots, \mathfrak{w}_*)$ appears in W as an exclusion sequence converging to w_0 because any sequence in an exclusion space must necessarily converge to the defining element in its own topology. A comparison of their respective neighborhoods is shown in Table 1.

Although the backward sequence $(w_j)_{j=\ldots,i+1,i,i-1,\ldots}$ in $(W, \mathcal{U}_+)$ does not actually converge to w_0, the effect of $(\mathfrak{w}_i)_{i \geq 0}$ of $\mathfrak{W}$ on W is to regulate the evolution of the forward arrow $(w_i)_{i \geq 0}$ to an effective state of stasis of dynamical equilibrium, that becomes self-evident in considering for W and $\mathfrak{W}$ the sets of positive and negative reals, and for w_*, $\mathfrak{w}_*$ a positive number r and its negative $-r$. The existence of a negelement $\mathfrak{w} \in \mathfrak{W}$ for every $w \in W$ requires all forward arrows in W to have a matching forward arrow in $\mathfrak{W}$ that actually *appears backward when viewed in* W. It is this opposing complimentary dualistic effect of the apparently backward-$\mathfrak{W}$ sequences on W – responsible by (7b) for moderating the normal uni-directional evolution in W – that is effective in establishing a stasis of dynamical balance between the opposing forces generated in the composite of a compound system and its environment. Obviously, the evolutionary process ceases when the opposing influences in W due to itself and its moderator $\mathfrak{W}$ balance, marking a state of dynamic equilibrium. Summarizing, the exclusion topology of large equivalence classes in $\mathrm{Multi}(X)$ successfully competes with the normal inclusion topology of $\mathrm{map}(X)$ to generate a state of dynamic homeostatic equilibrium in W that permits out-of-equilibrium complex composites to coexist despite the privileged omnipresence of the Second Law.

An additional technical support of the hypothesis of Eqs. (7a) and (7b) is provided by the inverse and direct limits summarized in Figs. 2a and 2b, [13]. The *direct* (or *inductive*) limit is a general method of taking limits of a "directed families of objects", while the *inverse* (or *projective*) limit allows "glueing together of several related objects", the precise nature of which is specified by morphisms between them, see [2] for details related to the present application.

[6] $\mathcal{P}(W)$ is the power set of subsets of W.

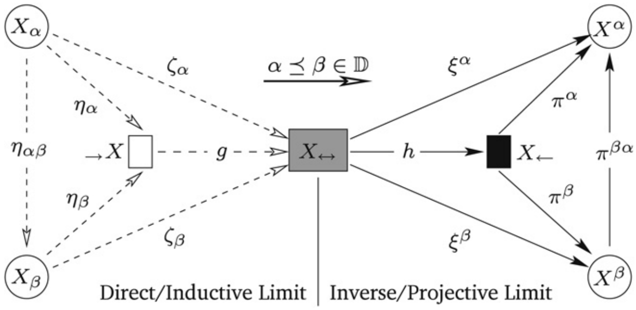

Fig. 2a. Direct and inverse limits of direct and inverse systems $(X_\iota, \eta_{\iota\kappa})$, $(X^\iota, \pi^{\kappa\iota})$. Induced homeostasis is attained between the two adversaries by the respective arrows opposing each other as shown in the next figure where expansion to the atmosphere is indicated by decreasingly nested subsets. At the cosmological level, the negative gravitational repulsive pressure of dark energy causing expansion of the universe is believed to more than offset its positive gravitational attraction.

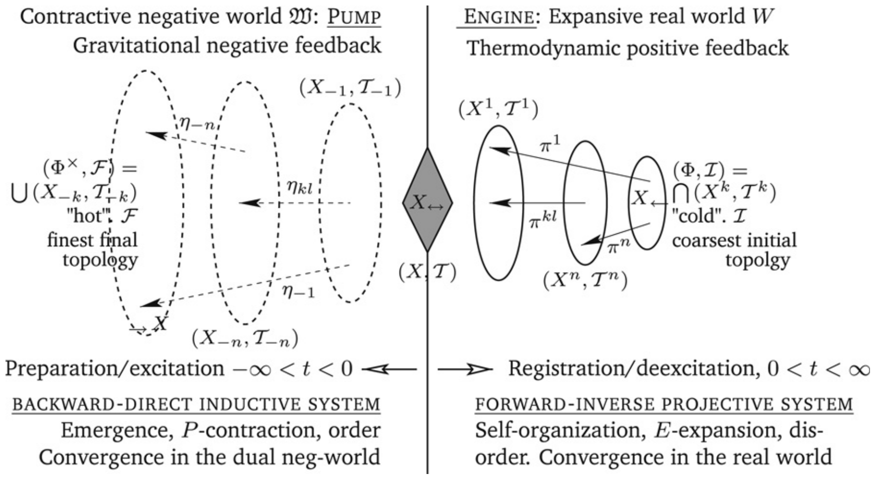

Fig. 2b. Intrinsic arrows of time based on inverse-direct limits of inverse-direct systems. Intrinsic irreversibility follows since the thermodynamic forward-inverse arrow is the natural arrow in $\mathbb{R}_+$ equipped with the usual inclusion topology, while the backward-direct positive arrow of $\mathbb{R}_-$ manifests itself as a dual "negative" exclusion topology in $\mathbb{R}_+$. Notice that although E and P are born in $[T_h, T]$ and $[T, T_c]$ respectively, they operate in the domain of the other in the true spirit of competitive-collaboration, see Fig. 3 and Eq. (14). The entropy decreases on contraction since the position uncertainty decreases faster than the increase of momentum uncertainty.

These figures illustrate the basic features of the limits defined in terms of the family of continuous maps $\eta_{\alpha\beta} : X_\alpha \to X_\beta$ and $\pi^{\beta\alpha} : X^\beta \to X^\alpha$ connecting the family of spaces as indicated and satisfying $\eta_{\alpha\alpha}(x) = x \in X_\alpha$, $\eta_{\alpha\gamma} = \eta_{\beta\gamma} \circ \eta_{\alpha\beta}$ and $\pi^{\alpha\alpha}(x) = x \in X^\alpha$, $\pi^{\gamma\alpha} = \pi^{\beta\alpha} \circ \pi^{\gamma\beta}$ for all $\alpha \preceq \beta \preceq \gamma$. The basic content of these commutative diagrams is the existence of the limits

$$_{\to}X \triangleq \coprod_{\kappa} X_\kappa / \sim_{\mathrm{D}} \tag{8a}$$

$$X_{\leftarrow} \triangleq \left\{ x \in \prod_{\kappa} X^\kappa : p^\alpha(x) = \pi^{\beta\alpha} \circ p^\beta(x) \text{ for all } \alpha \preceq \beta \in \mathbb{D} \right\} \tag{8b}$$

where $\coprod$ is a disjoint union,[7] $\sim_{\mathrm{D}}$ denotes the equivalence class

$$[x_\alpha] = \{x_\beta \in X_\beta : \exists\, \gamma \succeq \alpha,\ \beta \text{ such that } \eta_{\alpha\gamma}(x_\alpha) = \eta_{\beta\gamma}(x_\beta)\}$$

of x_α possessing a common successor, and

$$p : \coprod_{\kappa} X_\kappa \to\ _{\to}X$$

[7] For a family of sets $(X_{\kappa \in \mathbb{D}})$ the disjoint union is the set $\coprod_{\alpha \in \mathbb{D}} X_\alpha \triangleq \bigcup_{\alpha \in \mathbb{D}} \{(x, \alpha) : x \in X_\alpha\}$ of ordered pairs, with each X_α being canonically embedded in the union as the pairwise disjoint $\{(x, \alpha) : x \in X_\alpha\}$, even when $X_\alpha \cap X_\beta \neq \emptyset$. If $\{X_k\}_{k \in \mathbb{Z}_+}$ is an increasing family of subsets of X, and $\eta_{mn} : X_m \to X_n$ is the inclusion map for $m \leq n$, then the direct limit is $\bigcup X_k$.
For $\{X^k\}_{k \in \mathbb{Z}_+}$ a decreasing family of subsets of X with $\pi^{nm} : X^n \to X^m$, the inclusion map, the inverse limit is $\bigcap X^k$.

$$p^{\alpha} : \prod_{\kappa} X^{\kappa} \to X^{\alpha}$$

are projections. The restriction $\eta_{\kappa} : X_{\kappa} \to {}_{\to}X$ of p maps each element to its equivalence class; hence

$$_{\to}X = \bigcup_{\kappa} \eta_{\kappa}(X_{\kappa}) \tag{9}$$

implies that $_{\to}X$ is not empty whenever at least one X_{α} is not empty with the algebraic operations on $_{\to}X$ being defined via these maps. If the directed family is disjoint with each X_{α} the domain of an injective branch of f that partitions $\mathcal{D}(f)$, then the direct limit $_{\to}X$ is isomorphic to the basic set X_{B} of X, [3] with $\eta_{\alpha\beta}(x_{\alpha})$ the element of $[x_{\alpha}]_f$ in X_{β}. The projections p^{α} of the product onto its components, make $X_{\leftarrow}$ a subspace of $\prod X^{\kappa}$ such that a point $x = (x^{\kappa}) \in \prod X^{\kappa}$ is in $X_{\leftarrow}$ iff, its coordinates satisfy $x_{\alpha} = \pi^{\beta\alpha}(x_{\beta})$ for all $\alpha \preceq \beta \in \mathbb{D}$. The sets $(\pi^{\alpha})^{-1}(U)$, $U \subseteq X^{\alpha}$ open, is a topological basis of $X_{\leftarrow}$, and all pairs of points of $X_{\leftarrow}$ obeying $x_{\alpha} = \pi^{\beta\alpha}(x_{\beta})$ for $\alpha \preceq \beta$ are identical iff, their images coincide for every α. The restriction $\pi^{\alpha} : X_{\leftarrow} \to X^{\alpha}$ of p^{α} is the continuous *canonical morphism* of $X_{\leftarrow}$ into X^{α} with two points of $X_{\leftarrow}$ being identical iff their images coincide for every α.

The inverse and direct limits are therefore generated by opposing directional arrows whose existence follow from very general considerations; thus for example existence of the union of a family of nested sets entails the existence of their intersection, and conversely. In the context of Hilbert spaces, Fig. 2b specializes to *rigged Hilbert spaces* $\Phi \subset \mathcal{H} \subset \Phi^{\times}$ useful in quantum mechanics as

$$\Phi^{\times} \triangleq \bigcup_{k} \mathcal{H}_{-k} \supset \cdots \supset \mathcal{H}_{-1} \supset \mathcal{H} \supset \mathcal{H}^{1} \supset \cdots \supset \bigcap_{k} \mathcal{H}^{k} \triangleq \Phi$$

with Φ the space of physical states prepared in actual experiments, and $\Phi^{\times}$ are antilinear functionals on Φ that associates with each state a real number interpreted as the result of measurements on the state. Mathematically the space of test functions Φ and the space of distributions $\Phi^{\times}$ enlarge the Hilbert space $\mathcal{H}$ to a rigged Hilbert space, $(\Phi, \mathcal{H}, \Phi^{\times})$.

Example 1 (*Dialectics of Positive and Negative Feedback*). A feedback mechanism is called *positive* (to be denoted by $(\uparrow)$) if the resulting reaction acts in the same direction as the triggering action reinforcing the change or trend, *negative* $(\downarrow)$ if they oppose each other. Thus the concentrating pump of the direct iterates of the logistic map is the positive feedback promoting emergence and changes in the system, the dispersive engine of the inverse iterates of negative feedback organizes this emergence into a homeostatic whole; if negative feedback loops hold a system stable, then positive loops allow systems to explore their environment and follow new development paths. Feedback is the basis of exploration in cybernetic systems: it denotes the information that the system uses to adjust its behavior to achieve a desired goal; it is the "study of systems and processes that interact with themselves and produce themselves from themselves".

The logistic parameter λ is the "strength" of the positive feedback of the logistic system.

(a) *Ecology*.In the logistic map $n_{t+1} = \lambda n_t(1 - n_t)$ describing the dynamics of non-overlapping generations of population [14] with λ the reproductive rate of the average number of offsprings surviving to adulthood, absence the negative feedback $(1 - n_t)$ results in a linear growth of the population by the factor of $\lambda \in (0, 4)$ in every successive generation. With the feedback present, however, the resultant nonlinearity regulates the size of a population according to the standard "complicated" prescriptions of the logistic equation for $\lambda \in [3, \lambda_*)$ when the population size progressively oscillates between these increasing number of stable values, extinction for $\lambda \in (0, 1)$, a unique constant size when $\lambda \in [1, 3)$, and an erratic behavior marked by the absence of any regular pattern for $\lambda \geq \lambda_*$ when the system not only fails to reach a stable limit but also does not frequent any set of stable fixed points. See [2], Fig. 10.10*c*.

(b) *Biology*. Blood sugar levels in biological systems are regulated by a negative feedback by release of insulin upon increase of its level due to intake of food, converting thereby the glucose to glycogen and fat. Malfunction of the pancreas results in the dangerous and fatal consequences of diabetes. Satiation of hunger and thirst also represents positive–negative feedback loops that maintains optimum levels of life supporting constituents in living systems.

As another example of the deadly consequence of the failure of biological homeostasy in living systems, mention must be made of the dysfunctional cell bifurcation system leading to uncontrolled positive feedback, growth, and cancer: abnormal growth of cells occur because of malfunctioning of the mechanism that controls cell growth and differentiation, and the level of cellular differentiation is sometimes used as a measure of cancer progression. A cell is constantly faced with problems of proliferation, differentiation, and death. The bidirectional control mechanism responsible for this decision is a stasis between cell regeneration and growth on the one hand and restraining inhibition on the other. Under healthy and normal conditions, cells grow and divide to form new cells only when the body needs them; in its normal functional form, Nature chooses the bifurcation-differentiation route to express itself as multicellular complex systems rather than nonadaptive unicellular monoliths. Mutations can sometimes disrupt this orderly process, however. New cells form when the body does not need them, and old cells do not die when they should. This cancerous bifurcation, which is ultimately a disease of genes, is represented by the chaotic region $\lambda \geq \lambda_*$ where no stabilizing effects exist. Typically, a series of several mutations are required in a process involving both oncogenes that promote cancer when "switched on" by a mutation, and tumor suppressor genes that prevent cancer unless "switched off" by a mutation.

Camazine et al. [15] give the following definitions in the context of pattern-formation in biological systems

- *Self-organization* is a process in which pattern at the global level of a system emerges solely from numerous nonlinear interactions among lower-level components of a system. The rules specifying interactions among the system's components are executed using only local information, without reference to the global pattern
- *Emergence* is a process by which a system of nonlinearly interacting subunits acquires qualitatively new properties that cannot be understood as the simple addition of their individual contributions.

(c) *Social and Economic.* Bionomics is a theory that studies economics using the principles of biology as a self-organizing ecosystem. Economics, that "branch of social science that deals with the production distribution and consumption of goods and services and their management", is after all the brainchild of human society invented for interaction and communication with each other; outside of its societal institutions, monetary capital loses all its commanding power, privileges and dispensation. "Where mainstream economics is based on concepts borrowed from classical Newtonian physics, bionomics is derived from the teachings of modern evolutionary biology. Where orthodox thinking describes the economy as a static, predictable engine, bionomics sees the economy as a self-organizing information ecosystem. Where the traditional view sees organizations as production machines, bionomics sees organizations as intelligent social organisms. Where conventional business strategy focuses on physical capital, bionomics holds that organizational learning is the ultimate source of all profit and growth" [16]. Mainstream neoclassical economics operating on the guiding principles that (a) people have rational preferences among outcomes, (b) individuals maximize utility and firms maximize profits, (c) people act independently on the basis of full and relevant information, attempts to "present economics as a science. As a result the field of economics has become substantially detached from real-world behavior and has tended toward a closed theoretical discipline disconnected from the world it tries to explain. Instead of being a science, I suggest that economics should become a systems profession" [17].

Economics as "a science which studies human behavior as a relationship between ends and scarce means which have alternative uses[8]" should properly be considered to be integrated with human society with the individual consumer and the collective firm serving not merely to maximize utility and profits respectively, but co-existing as a dynamical, interdependent and interacting, holistic order. The systems approach exemplified by the top–down engine, bottom–up pump paradigm suggests that a planned-and-controlled firm-pump actively monitored and tuned, collaborating with the consumer-engine, generates the best synthesis of the opposites of individualism and collectivism in society. Emergence and adaption, and not the "stable clockwork mechanism of the heavens described by Newton" of neoclassical vintage, is implied just as efficiency in a world of limited resources demands differentiation, diversity, individuality and decentralization. Adam Smith, widely regarded as the father of modern economic culture, claimed that selfish behavior of individuals is "somehow transmuted by capitalist social relations into public benefaction".[9] However, "since workers and capitalists meet as antagonists in the market, there is no reason for capitalists to share the increases in labor productivity with workers as high wages. The selfish pursuit of gain by capitalists may create the *potential* for broad social benefits through the accumulation of capital and the widening division of labor [pump: bullish, concentrative]. But society as a whole can only achieve these potential gains by going beyond capital accumulation to *distribute* the resulting wealth" [engine: bearish, dispersive] [18].

In the social, economic and management sciences, the holistic attitude maintains that the "lawlike statements" that formal mainstream analysis generate often do not adequately explain "the nature of social reality". For example, whereas classical growth theory talks about the effect on output of changes in rates of saving, holists analyze the causes behind mobilization of resources in its social, political, cultural, beside economic, manifestations: holists, acknowledging the organic unity of human system, study the whole rather than a part taken out of context. The context of a particular event is important "because the character of any given part is largely conditioned by the whole to which it belongs and by its particular function and location in the larger system. *Thus, reality for holists is viewed as a process of evolutionary change driven by the dynamic interaction between the parts and the whole*", [20]. For more, see Section 3.

(d) *Management.* The traditional view to management is through a linear cause-effect relationship; thus poor sales in a business might be attributed simply to inadequacy of resources and/or incompetence of the workforce. While this approach is probably adequate for simple "linear" problems, it is unlikely to be so in the complex real-life situations where a "system behavior" would be more appropriate. This requires that "we move away from looking at isolated events and their causes, and start to look at the organization as a system made up of an interdependent group of interacting parts forming a unified

[8] Lionel Robbins, An Essay on the Nature and Significance of Economic Science, 2nd. Edition, MacMillan and Co., Ltd., London (1945).

[9] Smith's *Invisible Hand*: The theoretical foundation on which economics is based asserts "the apparently self-contradictory notion that capitalism transforms selfishness into its opposite: regard and service for others. Thus by being selfish within the rules of capitalist property relations, Smith promises, we are actually being good to our fellow human beings. The logical fallacy (of this 'amazing argument') is that neither Smith nor any of his successors has been able to demonstrate rigorously and robustly how private selfishness turns into public altruism" [18].

"If [the unit as the prime mover of the social world] is a collective of individuals, could its behavior be deduced from the sum? Or could its behavior be governed by other things than the sum of its components? ··· Modern economics is fundamentally an individualistic theory. It is a theory based almost entirely on the analysis of the behavior of a single individual and his interaction with others. Any group analysis is viewed as a consequence of individuals' interactions. The group as such has no significance.

"The crux of the debate was the famous dilemma about the consistency of thermodynamics with Newtonian classical mechanics. In mechanics all actions are reversible; in thermodynamics the direction of time is unique. How can atoms which are subject to the Newtonian principle of reversibility constitute the foundation of matter which is subject to the laws of thermodynamics? ··· Thus it is quite conceivable that the atoms of a gas will act according to Newtonian mechanics but the *statistical* behavior will be subject to the laws of thermodynamics. ··· In society, the collective may be working according to rules different from those controlling its individual components" [19].

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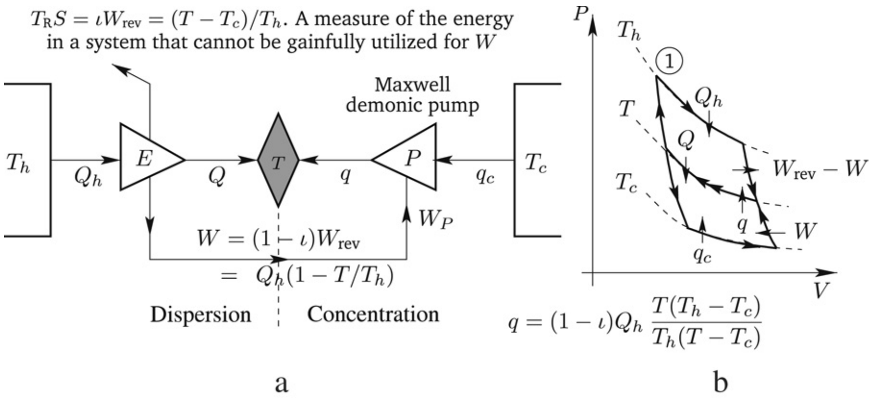

Fig. 3. Reduction of the dynamics of opposites to an equivalent engine-pump thermodynamic system. The fraction $W = (1 - \iota)W_{\rm rev}$ of the available maximum reversible work $W_{\rm rev} = \eta_{\rm rev} Q_h := (1 - T_c/T_h)\, Q_h$ of a reversible engine operating between $[T_c, T_h]$ is gainfully utilized internally to self-generate a heat pump P to inhibit, through gradient dissipation, the entropy that would otherwise be produced by the system. This permits decoupling natural irreversibility to a reversible engine-pump dual that uses the fraction $1 - \iota$ of the available exergy in sustaining the Maxwell-demon P, the entropic fraction ι defining "the information contained in a physical system that is invisible to us", [1]. The induced pump P prevents the entire internal resource $T_h - T_c$ from dispersion at $\iota = 1$ by stimulating a temperature difference $T_h - T$ at some $\iota < 1$ that defines the homeostatic temperature $T_c < T < T_h$. E and P interact with each other in the spirit of competitive collaboration at the induced interface T.

pattern. When we face a management problem we tend to assume that some external event caused it. With a systems approach, we take an alternative viewpoint — namely that the internal structure of the system is often more important than external events in generating the problem" [21]. Kirkwood gives the following self-explanatory example of collaboration of positive and negative loops in stabilizing the sales growth of a new product, compare (e) below and Fig. 3(b).

$$\begin{array}{ccc} & \text{Market} & \\ & \text{Saturation} & \\ & (\downarrow)(\uparrow) & \\ & \leftarrow \textbf{SALES} \leftarrow & \\ \swarrow & & \nwarrow \\ \text{Income} & (\uparrow) & \text{Motive} \\ \searrow & & \nearrow \\ & \rightarrow \text{Morale} \rightarrow & \end{array} \tag{10}$$

The concentrating "Sales $\rightarrow$ Income $\rightarrow$ Morale $\rightarrow$ Motive $\rightarrow$ Sales" loop is ($\uparrow$), while the dispersive loop "Sales $\rightarrow$ Market Saturation $\rightarrow$ Sales" is ($\downarrow$). The positive loop leads to an early exponential growth, but after a delay the negative loop dominates the behavior of the system leading to an overall S-shaped pattern of the eventual "goal seeking behavior" for sales of the new product. "Most growth processes have limits on their growth. At some point, some resource limit will stop the growth. As Eq. (10) illustrates growth of sales for a new product will ultimately be slowed by $\cdots$ the lack of additional customers who could use the product", sums up the interplay of the opposites in maintaining a balance of growth and decay leading to homeostasy.[10]

(e) *Nuclear Reactor Control.* A fission nuclear reactor which generates energy by repeated fission (bifurcation) of a heavy nucleus is controllable due to the presence of a small fraction of delayed neutrons β. If all neutrons produced in fission were to be born prompt ("instantaneously"), it would be impossible to control the reactor due to the large response times that would be needed to keep pace with the very rapid generation of neutrons. For large reactivities ρ, neglect of all delayed neutron concentrations leads to the exponentially growing neutron density $n(t) = n_0 \mathrm{e}^{(\rho-\beta)t/\ell}$. A self-induced feedback $\rho_f(\theta) = \alpha\theta$, where α is a temperature coefficient of reactivity and θ the reactor temperature, in $\rho = \rho_0 - \rho_f$ results in the altogether different behavior $n(\rho) \propto (\rho_0 - \beta)^2 + (\rho - \beta)^2$, typically logistic-like comprising rising and falling branches with the generated feedback actually halting the growth of $n(\rho)$ and forcing it to decrease and eventually vanish at $\rho = \rho_o$ and at $2\beta - \rho_0$. The resulting variation of the neutron density $n(t) = n_0\, \mathrm{sech}^2(\omega t)/2$ with characteristic rising and falling components bears no resemblance to the no-feedback exponential $\mathrm{e}^{(\rho-\beta)t/\ell}$.

[10] "In terms of organizations interacting with their environments, traditional mechanistic organization and management theory views a firm moving through its business environment as a separate, fixed and rigid machine that scans its environment, calculates its options, and plans its responses. It relies on information feedback, forecasting and prediction, calculations, contingency planning and rationality. This artificial separation of an organization from its environment and greater society within which it is embedded and dependent, is quite dysfunctional. A more holistic view sees a successful firm as an integrated, flexible, adaptive organism, as a self-restructuring system, capable of self-reshaping and adapting to a variety of environmental variations and needs. These organizations are expected to rely more on flexible response patterns, distributed sensing and scanning, and a continuous process of integrating with its surroundings. Such an organization is viewed as an integral part of its environment, simultaneously creating it and being created by it, taking from it and giving back to it", [22].

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These examples demonstrates the effectual creation of the engine-pump dualism of bidirectionality that encapsulates the essence of chanoxity. The rising temperature in the reactor as a result of increasing power would, for example, proceed uncontrolled to a possible disaster if it were not to be self-regulated by the creation of the dual negative feedback of a delayed-neutron engine.

2.3. Thermodynamics of ChaNoXity

A complex system can hence be represented as

$$\left.\begin{array}{r}\text{Backward-Direct Arrow}\\ P\text{-synthesis of concentration}\\ \text{order, entropydecreasing,}\\ \text{bottom–up } emergence_{\rightarrow} C\end{array}\right\} \bigoplus \left\{\begin{array}{l}\text{Forward-Inverse Arrow}\\ E\text{-analysis of dispersion,}\\ \text{disorder, entropy increasing,}\\ \text{top–down self-organization } C_{\leftarrow}\end{array}\right. \tag{11}$$
$$\Longleftrightarrow$$
$$\text{Synthetic cohabitation of opposites, } \mathscr{C} = C_{\leftrightarrow}$$

where $\bigoplus$ denotes a non-reductionist sum of the components of a top–down engine and its complimentary dual bottom–up pump. The interpretation of Eq. (11) in a global system-environment context is as follows. Concentration by the pump of increasing order, decreasing entropy (less available states), increasing temperature and internal energy, leads to a net dumping of heat to its environment by the First Law. The resulting input of work from the expanding environment operating the pump must be accompanied by an increase of entropy to offset the decrease from the pump resulting in a net increase of disorder and dispersion,[11] the homeostasy being attained neither by the maximization of entropy nor minimization of free energy [23]. A complex system behaves in an organized collective manner with properties that cannot be identified with any of the individual parts but arise from the structure as a whole: these systems cannot dismantle into their components without destroying themselves. Analytic methods cannot simplify them as such techniques do not account for characteristics that belong to no single component but relate to the parts taken together, with all their interactions. This analytic base must be integrated into a synthetic whole with new perspectives that the properties of the individual parts fail to add up to. Complexity is therefore a dynamical, interactive, interdependent, hierarchical homeostasy of *P-emergent, ordering instability of positive feedback* competitively collaborating with the negative and adaptive, *E-self-organized, disordering and stabilizing feedback* generating a non-reductionist structure that is beyond the sum of its constituents. We have attempted to provide, in the above context, an essentially thermodynamic analysis of the defining role of the (gravity) pump in generating complex structures in nature.

To investigate further the question of equivalence between W and $\mathfrak{W}$, consider the reduction of the inverse-direct model as a coupled thermodynamic engine-pump system in which heat transfer between temperatures $T_h > T_c$, is reduced to a engine E-pump P combination operating respectively between temperatures $T_c < T = T_{\leftrightarrow} < T_h$, as shown in Fig. 3. We assume that a complex adaptive system is distinguished by the full utilization of the fraction $W := (1-\iota)W_{\text{rev}} = (1-\iota)\eta_{\text{rev}}Q_h = (1-\iota)(1-T_c/T_h)Q_h$ of the work output of an imaginary reversible engine running between temperatures T_h and T_c, to self-generate the demonic pump P working in competitive collaboration with a reversible engine E, where the irreversibility factor

$$\iota \triangleq \frac{W_{\text{rev}} - W}{W_{\text{rev}}} \tag{12a}$$

accounts for that part ιW_{rev} of available energy (exergy) $W_{\text{rev}} = (U-U_0) + P_{\text{R}}(V-V_0) - \Sigma_{j=1}^{J}\mu_{j,0}(N_j - N_{j,0}) - T_{\text{R}}(S-S_0) \triangleq W + T_{\text{R}}\Delta S$ (with $_{\text{R}}$ the infinite reservoir with which the system attains equilibrium $_0$), that cannot be gainfully utilized but must be degraded in increasing the entropy of the universe. Hence,

$$\iota = \left(\frac{T_{\text{R}}}{W_{\text{rev}}}\right) S \tag{12b}$$

yields the effective entropy

$$S = \frac{W_{\text{rev}} - W}{T_{\text{R}}} \tag{13}$$

in terms of the parameter $\iota \in [0, 1]$ and available internal energy $W_{\text{rev}} = Q_h(T_h - T_c)/T_h$. The self-induced demonic pump effectively decreases the engine temperature-gradient $T_h - T_c$ to $T_h - T$, $T_c \le T < T_h$, thereby inducing a degree of dynamic stability to the system. Fig. 3 represents the essence of competitive collaboration: the entropic dispersion of E is proportional to the domain $T - T_c$ of P and the anti-entropic concentration of P depends on $T_h - T$ of E. Thus an increase in ι can occur only at the expense of P which naturally tends to inhibit this tendency, reciprocally any decrease in ι invites opposition from E.

[11] For the cyclic engine-pump paradigm that we adapt, entropy change follows the net work transfer $dQ = dW$, the constant volume heating and cooling of rigid bodies from changes in temperature $dQ = dU$ not being relevant.

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Let the irreversibility ι be computed on the basis of dynamic equilibrium through the definition of the equilibrium temperature $T(\iota)$

$$W := Q_h\left(1-\frac{T}{T_h}\right) \triangleq (1-\iota)Q_h\left(1-\frac{T_c}{T_h}\right)$$

of the engine-pump system; hence

$$\begin{aligned}\iota(T) &= \frac{(T_h-T_c)-(T_h-T)}{T_h-T_c}\\ &= \frac{T-T_c}{T_h-T_c}\end{aligned} \tag{14}$$

where $T_h - T_c$ represents the internal energy that is divided into the non-entropic $T_h - T$ free energy A internally utilized to generate the pump P, and a reduced $T - T_c$ manifestation of entropic dispersion by E, with the equilibrium temperature T serving to actually establish this recursive definition.[12] The generated pump is a realization of the energy available for useful, non-entropic, work arising from reduction of the original entropic gradient $T_h - T_c$ to $T - T_c$. The irreversibility $\iota(T)$ can be considered to have been adapted by the engine-pump system in a manner that the induced instability due to P balances the imposed stabilizing effort of E to the best possible advantage of the system and its surroundings. Hence a measure of the energy in a system that cannot be utilized for work W but must necessarily be dumped to the environment is given by the "generalized entropy"

$$TS = \iota W_{\text{rev}} = W_{\text{rev}} - W \tag{15a}$$

$$= U - A \tag{15b}$$

which the system attains by adapting itself to a state that optimizes competitive collaboration for the greatest efficiency consistent with this competitiveness. This distinguishing feature of non-equilibrium dynamics compared with the corresponding equilibrium case lies in the mobility of the defining temperature T: for self-organizing systems the dynamics adapts to the prevailing situation by best adjusting itself *internally* for maximum global advantage.

For the realization Fig. 3(b), the original dispersion and entropy production in (T_c, T_h) is reduced by the induced pump to (T, T_h), with the engine now rejecting heat $Q - q = (T/T_h)Q_h - \alpha Q_h$ at temperature T. The expansion phase $1-2-3$ consists of isothermal $1-2: Q_h$ dispersive entropy increase and adiabatic $2-3: (W_{\text{rev}} - W)$ cooling, while the compressive stage comprising isothermal entropy decrease $3-4: -(Q-q)$ and adiabatic heating $4-1: -W$ results in a net reduction of entropy thereby contributing to the sustenance of life, structures and patterns.

Define the equilibrium steady-state representing $X_{\leftrightarrow}$ of optimized $E-P$ adaptability $\alpha := \eta_E\zeta_P$

$$\alpha(T) = \left(\frac{T_h - T}{T_h}\right)\left(\frac{T}{T-T_c}\right) \triangleq \frac{q}{Q_h} \tag{16}$$

of the generated pump per unit of the total heat input, be the product of the efficiency of a reversible engine operating between (T, T_h) and the coefficient of performance of a reversible pump in (T_c, T). Fig. 4 illustrating this tension for $T_h = 480$ K and $T_c = 300$ K, shows that the engine-pump duality has the significant property of supporting more than one condition based on

$$\begin{aligned}T_{\pm}(\alpha) &= \frac{1}{2}\left[(1-\alpha)T_h \pm \sqrt{(1-\alpha)^2T_h^2 + 4\alpha T_c T_h}\right]\\ &= \begin{cases}((1-\alpha)T_h,\ 0), & T_c = 0,\ \text{maximum temperature gradient}\\ (0,\ 0), & T_c = 0,\ \alpha = 1\\ (T_h,\ -\alpha T_h), & T_c = T_h,\ \text{zero temperature gradient}\\ (T_h, T_{h,}), & T_c = T_h,\ \alpha = -1\end{cases}\end{aligned} \tag{17}$$

for any given value of α. Fig. 4(c) then allows identification of the real and its dual negative worlds in terms of the irreversibility-adaptability product expressed in terms of the COP ζ of a reversible pump operating between (T_c, T_h)

$$\begin{aligned}\iota\alpha(T) &= \frac{T(T_h-T)}{T_h(T_h-T_c)}\\ &= \zeta\theta(1-\theta), \quad \zeta = \left(1-\frac{T_c}{T_h}\right)^{-1}, \theta \triangleq \frac{T}{T_h}\end{aligned} \tag{18}$$

[12] The definition equation (14) of ι so formally corresponds to the quality $x = (v - v_f)/(v_g - v_f)$ of a two-phase liquid–vapor mixture that it is tempting to examine the possibility of holism representing a phase transition between concentration and dispersion. In this sense, regions (IV), (III), (I) and (II) of Table 2 correspond to the solid (S), liquid (L), liquid–vapour (L + Vap), and vapor (Vap) phases respectively.

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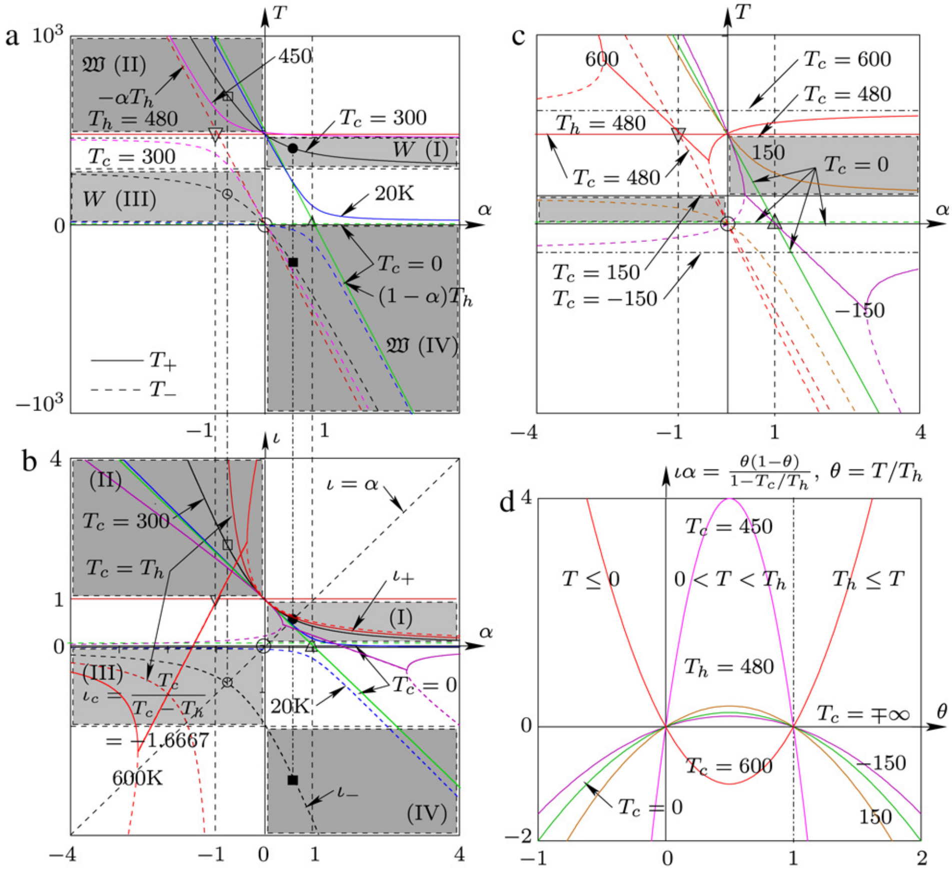

Fig. 4. The interactive "participatory universe" for $T_h = 480$ K. The real world evolutionary dynamics in W shown by the shaded regions defined by $\iota\alpha > 0$ for $T_+ \in (T_c, T_h)$ and $T_- \in (0, T_c]$ is partnered by the complementary negworld effects of $\mathfrak{W}$ of $\iota\alpha < 0$ for $T_+ \in [T_h, \infty)$ and $T_- \in (-\infty, 0]$. Observe how the two parts (II) and (IV) of $\mathfrak{W}$ actually collaborate to produce the real W for $T \in (0, T_h)$, $\alpha \in (-1, 1)$. Panel (d) shows the logistic-like quadratic variation of $\iota\alpha$ with temperature leading to the demarcations of (a) and (b). (b) and (d) also illustrate that while $T = 0$ indeed serves to define absolute temperatures at the maximum gradient $T_c = 0$, the more interesting $T = T_h$ for zero gradient $T_c = T_h$ leads to a singularly catastrophic reversal of roles of the dispersive engine and contractive pump, see Example 4. The straight lines connecting the $T < T_c$ and $T > T_h$ segments in (c) correspond to complex roots.

as ($\iota_c = -T_c/(T_h - T_c)$)

$$\frac{\iota\alpha > 0,\ W}{0 < T_c < T_h :} \quad \begin{array}{l} \alpha > 0,\ \iota \in (0, 1),\ T_+ \in (T_c, T_h), \\ \alpha < 0,\ \iota \in (\iota_c, 0),\ T_- \in (0, T_c). \end{array} \tag{19a}$$

$$\underline{\iota\alpha \le 0,\ \mathfrak{W}} \quad \begin{cases} \underline{T_c \le 0} : \alpha > 0,\ T_\pm \in (-\infty, T_c),\ [0, T_h], \\ \qquad\quad \alpha < 0,\ T_- \in (T_c, 0],\ T_+ \in [T_h, \infty) \\ \underline{T_h \le T_c} : \alpha < 0,\ T_\pm \in (T_c, \infty),\ [0, T_h], \\ \qquad\quad \alpha > 0,\ T_+ \in [T_h, T_c),\ T_- \in (-\infty, 0]. \end{cases} \tag{19b}$$

The real world, with ι and α of the same sign, attains concentration-dispersion homeostasy in collaborative competition with the dual neg-world identified by negative $\iota\alpha \le 0$. The displayed difference in the temperature profiles of W and $\mathfrak{W}$ is essentially a reflection of the multifunctional symmetry of the later compared with the functional asymmetry of W; thus T_+ and T_- appear as continuous curves in $\mathfrak{W}$ for $T_c \le 0$, $T_h \le T_c$ bifurcating as separate branches at these defining temperatures to give birth to W. Observe how unlike in W, each of the components of the dual negative world span both the parts and reaches out to the entire temperature range in $(-\infty, \infty)$; it is this global outreach of $\mathfrak{W}$ that distinguishes it from the real world and endows it with the omnipresence to induce complex structures and emergence in W. The bi-directional dispersion-concentration tuning of the dynamics of a non-equilibrium system as implied by Eq. (16), means that any displacement from its steady state motivates opposition from one of its constituents inducing the system back to the state of dynamic equilibrium. The non-equilibrium system X can therefore be expected to oscillate about this equilibrium $T_\leftrightarrow$ by responding to changes in its environmental gradients. The temperatures (17) serve to parametrize the functional and multifunctional components of the universe: *complex systems by definition represent homeostasy between them with respect to convergence and hence their induced topologies.*

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Table 2
Emergence of the "Participatory Universe" for $0 < T_c < T_h$ in W. The structure of Eq. (19b) simplifies as indicated here in accordance with Fig. 4, with the only components $T_+ \in [T_h, \infty)$ and $T_- \in (-\infty, 0]$ surviving in this physically meaningful, real-world case. For an interpretation of $T_c \geq T_h$ and the corresponding relation of (II) to (IV), see Example 4. The shaded rows vanish for $T_c = 0, T_h = \infty$; the remaining pair constitute the universe at $t = 0$.

ι; T; α	λ; χ	x_{fp}
$(-\infty, \iota_c]$; $(-\infty, 0]$; $[\infty, 0)$	$(0, 1]$, $(1, 2]$; 0	$(\bullet, -)$, $(\circ, -)$
$\iota\alpha \leq 0$: Multifunctional Simple$\mathfrak{W}$(IV: S)		
$(\iota_c, 0)$; $(0, T_c]$; $(0, -\infty)$	$(2, 3)$; 0	$(\circ, \bullet)$
$\iota\alpha > 0$: Functional Simple$W$ **(III: L)**		
$(0, 1)$; (T_c, T_h); $(\infty, 0)$	$(\mathbf{3}, \boldsymbol{\lambda_*})$; $(0, 1]$	$(\circ, \bullet/\circ)$
$\iota\alpha > 0$: Functional Complex$W$ **(I: L+Vap)**		
$[1, \infty)$; $[T_h, \infty)$; $[0, -\infty)$	$[\lambda_*, 4)$, $[4, \infty)$; $\{0, 1\}$	$(\circ, \circ)$
$\iota\alpha \leq 0$: Multifunctional Chaotic$\mathfrak{W}$(II: Vap)		

What is the most advantageous state of this emergent, self organizing system? With the self-induced equilibrium $T_\leftrightarrow$ denoting an effective T_c and T_h for the engine and pump respectively, the adaptability α and irreversibility ι define the entropy $S_\leftrightarrow = \iota W_{\mathrm{rev}}/T_{\mathrm{R}}$ that represents a dynamical balance of the dispersing and concentrating opposites generated within the system. The nature of these indices illustrated in Fig. 4(b) suggests that the balancing condition

$$\iota(T) = \alpha(T) \tag{20}$$

can be taken to define the most appropriate

$$T_\pm = \frac{T_h(T_h + T_c) \pm (T_h - T_c)\sqrt{T_h^2 + 4T_cT_h}}{2(2T_h - T_c)} = \begin{cases} (0.5T_h, 0), & T_c = 0 \\ (T_h, T_h), & T_c = T_h \end{cases} \tag{21}$$

value of the equilibrium steady-state $T_\leftrightarrow \in [T_c, T_h]$; in fact, $q := (1 - \iota)Q_h(T(T_h - T_c))/(T_h(T - T_c)) = \iota Q_h$ following the definition of α, corroborates the pump's proportionality to $T - T_c$. Thus, in the example of Fig. 4 of $T_h = 480$ K and $T_c = 300$ K, $Q_h = 75$ kJ min^{-1}, $W_{\mathrm{rev}} = 28.125$ kJ min^{-1}, the dynamical irreversibility and temperature of $\iota = 0.5894$ and $(T_+, T_-) = (406.09, 161.18)$ satisfying Eqs. (20) and (21) are both less than the $\iota = 0.7033$, $T_\leftrightarrow = 426.59$ K for $\alpha := \eta_{\mathrm{rev}} = 1 - T_c/T_h$ that was taken in [2], leading to a corresponding decrease in the entropy production rate from $S_\leftrightarrow = 0.0659$ kJ/min K to $S_\leftrightarrow = 0.0553$ kJ/min K which is only 58.94% of the maximum entropy that would otherwise be produced in a non self-organizing, expansion only, case.[13]

A correspondence between the dynamics of the engine-pump system and the logistic map $\lambda x(1-x)$ — in which the direct iterates $f^i(x)$ correspond to the "pump" $\mathfrak{W}$ and the inverse iterates $f^{-i}(x)$ to the "engine" W — is summarized in Table 2 with the fixed points $x_{\mathrm{fp}} = (0, (\lambda-1)/\lambda > 1/2)$ designated as $\bullet$ stable and $\circ$ unstable and $\iota_c = -T_c/(T_h - T_c)$. The complex region (I), $\lambda \in (3, \lambda_*)$, $x_{\mathrm{fp}} = (\circ, \bullet/\circ)$ corresponding to $T_+ \in (T_c, T_h)$ for $\iota \in (0, 1)$, is the outward manifestation of the tension between the regions (I), (III) on one hand and (II), (IV) on the other: observe from Eq. (17) and Fig. 4 that at environments of $T_c = 0, T_h$ the two worlds respectively merge at $\alpha = +1, -1$ bifurcating as individual components for $0 < T_c < T_h$: Fig. 4 is a graphical representation of these regions characterized by the positive/negative values of $\iota\alpha$. The value $\iota\alpha = 0$ for $T = 0,\ T_h$ demarcates the real world W from its negative dual $\mathfrak{W}$, substantiating the classification as a typical manifestation of the irreversible adaptability of a thermodynamic system to its surroundings. Hence

$$\chi = \iota = \alpha, \qquad \lambda \in (3, \lambda_* := 3.5699456) \tag{22}$$

in Region (I) can be taken as a definite assignment of thermodynamical perspective to the dynamics of the logistic map as a link to the generated complexity with $\iota\alpha = T(T_h - T)/T_h(T_h - T_c) = \chi^2$, χ being the *measure of chanoxity* introduced in [2], see Section II-2.1. Thus in Example 3, $\iota_+ = 0.6129$ and $\iota_- = 0.01587$ generate 2^4- and 2^1-cycle complexities; the near reversible dead-state of the later is the source of its discomfort.

Eq. (22) and Fig. 4(a), (b) show that the boundary $\mathrm{Multi}_{\|}(X)$ between $W := \mathrm{map}(X)$ and $\mathfrak{W} := \mathrm{Multi}(X)$ comprising the chaotic region $\lambda \in [\lambda_*, 4)$ can occur only for $\chi = 0 = \iota = \alpha$ at $T_c = 0$ and $T := T_- = 0$ (see Eq. (17)): according to Table 2, the values $\chi = 0$ and $\chi = 1$ of regions (I) and (II) establishes a one–one correspondence between $\lambda = 3$ and $\lambda \in (\lambda_*, 4)$ and between $\lambda = \lambda_*$ and $\lambda \geq 4$. The second interface at $T_c = T_h = \infty$ accounts for a boundaryless transition between these complimentary dual worlds, refer Example 4 for the equivalence of $T = \pm\infty$; hence $T_c \geq T_h$ is to be interpreted to imply $-\infty \leq T_c \leq 0$ of negative temperatures defining $\mathfrak{W}$. Fig. 5 summarizes the relationship between W and $\mathfrak{W}$.

[13] Clearly, the entropy in all of our considerations is the internally generated component S_i in the Prigogine decomposition $dS = dS_e + dS_i$, with the Second Law $dS_i \geq 0$ implying the universal irreversibility of spontaneous processes. In the statistical treatment of thermodynamics, the simple microcanonical ensemble is the natural descriptor of dS_i since by definition it is just an *isolated* collection of systems with a common energy E that may exist in many equally probable microstates W. It is a simple matter to recognize this as the natural setting of the dynamics of unimodal maps such as the logistic by identifying the microstates corresponding to an energy E with the equivalence class of inverse images $W = \{x_i\}_{i=1}^I$ corresponding to some $\{f(x_i) = f(x_j)\}_{i,j=1}^I$.

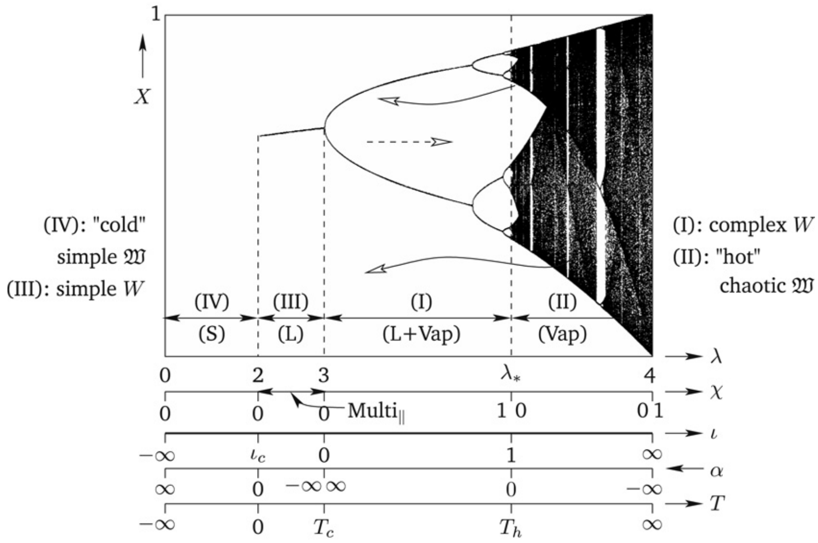

Fig. 5. Bifurcation profile of the Universe: integration of the dynamical and thermodynamical perspectives. The antagonistic directions are shown as the thermodynamic forward-inverse solid arrow of the expansive real world and gravitational backward-direct dashed arrow of the contractive dual world $\mathfrak{W}$. The complex phase (I) is a mixture of contentration and dispersion.

Example 2 (*Life*). *Life viewed as a "system of inferior negative feedbacks subordinated to a superior positive feedback"* [24] *has been defined* [2] *to be*"a special complex system of activating mind (gravitational negative feedback) and restraining body" (thermodynamic positive feedback).[14] Consider as an example of (11) for the living human system at a constant temperature of $T = 37$ °C. The question we ask is: why are we most comfortable at an environment of $T_c = 295$ K to 300 K but find temperatures approaching our stable equilibrium so very difficult/impossible to bear? What is responsible for this asymmetry? Our source of constant energy input T_h is the sun, the food we eat and the air we breathe. With the environment at a temperature $T_c < T$, we are effectively P-controlled, thereby inhibiting the expansive E effects responsible for minimum order. With increasing T_c, the level of discomfort increases because the contribution of E increases relative to P, with the second-law gradually asserting itself, until at $T_c = T$ we are reversibly integrated with the environment at $\alpha = \infty$. However, $\alpha = \pm\infty$ are equivalent because $\iota(\infty) = \iota(-\infty)$ and the system undergoes a transition to T_- operating in the completely unnatural environment (III) $T_c > T$. Observe additionally from Fig. 4 how the real functional W is actually enveloped by the multifunctional neg-$\mathfrak{W}$: not only are $\alpha = \pm\infty\iota$-equivalent, $\iota = \mp\infty$ being $\iota\iota^-$-equivalent generate a cyclic $\iota - \alpha$ organization of Nature. Thus what we actually experience in W is only a small fraction of the "reality" of $W \cup \mathfrak{W}$.

This profile of emergent living systems can be appreciated by realizing that the role of medicines during illness is to externally induce a necessary measure of orderliness in the decaying structure of the system which, once the state of dynamic stability has been restored, is no longer necessary and must be withdrawn. When the internal pump cannot however be rejuvenated to an acceptable state of repair – as in ailments like hypertension and diabetes – a continual supply of external stimulants is essential to artificially provide the foundation that natural resources can no longer produce.

2.3.1. Index of complexity

To enumerate a complex system with a quantitative measure based on the top down, bottom up engine-pump paradigm, use of the homeostasy condition equation (20) $\iota = \alpha$ leads to

$$\begin{aligned}\iota_{\pm} &= \frac{T_h - 2T_c \pm \sqrt{T_h^2 + 4T_hT_c}}{4T_h - 2T_c}\\ &= \begin{cases}(0.5, 0), & T_c = 0\\ \pm\frac{1}{2}\left(\sqrt{5} \mp 1\right) & T_c = T_h\end{cases}\end{aligned} \tag{23}$$

at the corresponding temperatures $T_{\pm}$ of Eq. (21) denoted as $T_\bullet$ and $T_\circ$ in Fig. 4(a). The complexity σ of a system is expected to depend on both the irreversibility of the system in its surroundings and on their mutual interaction α; thus the definition

[14] Often the characteristic of "life" distinguishing itself from non-life far-from-equilibrium dissipative structures is taken to be the sole focus of the former toward survival and reproduction: "the only purpose of (life) is to reproduce itself in as many copies as possible" [24]. Since survival and reproduction, however, are essentially physical manifestations of an excited mental instability, we take this to define the positive feedback of "life".

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of complexity

$$\sigma_{\pm} = \begin{cases} -\dfrac{\tilde{\iota}_-}{\ln 2}\{\iota_+ \ln \iota_+ + (1-\iota_+)\ln(1-\iota_+)\} \\ -\dfrac{\iota_+}{\ln 2}\{\tilde{\iota}_- \ln \tilde{\iota}_- + (1-\tilde{\iota}_-)\ln(1-\tilde{\iota}_-)\} \end{cases} \tag{24}$$

with $\tilde{\iota}_- = \iota_-/\iota_c \in [0, 1]$, ensures the expected two-state $(\uparrow, \downarrow)$, logistic-like, increasing–decreasing signature at T_+ and T_- respectively. Observe that $\sigma_\pm = 0$ at $T_c = 0$ because $\iota_- = 0$.

Example 3 (*Complexity of Life*). Consider the human life form at its equilibrium temperature $T = T_+ = 310$ in an environment of $T_c = 295$. The generator of sustenance provided by food and heat comprising the source of high temperature

$$T_h = \frac{1}{2T}\left(2T^2 + T_c^2 - TT_c + (T - T_c)\sqrt{4T^2 + T_c^2}\right) = 319.4743 \tag{25a}$$

obtained from (20), yields $\iota_+ = 0.6129$, $\iota_- = -1.3994$, $\iota_c = -12.0535$, $\tilde{\iota}_- = 0.1161$ for $T_- = 260.7503$. Therefore from Eq. (24)

$$\sigma_+ = 0.1118 \tag{25b}$$

is the effective complexity of life form corresponding to an entropy of 0.6674 bits at the irreversible adaptability $(\iota_+, \tilde{\iota}_-)$. If now, as in the summer season, $T_c = 315$ say, is actually higher than the equilibrium temperature $T = T_-$ of the living system, then $T_h = 318.1486$ gives $\iota_+ = 0.6174$ at $T_+ = 316.9439$ and $\iota_- = -1.5880$, $\iota_c = -100.0432$, $\tilde{\iota}_- = 0.01587$ corresponding to the stable T. Hence

$$\sigma_- = 0.07259 \tag{26a}$$

is the effective complexity of life for an entropy of 0.0815 bits at the irreversible adaptability $(\tilde{\iota}_-, \iota_+)$, the relatively smaller value of σ explaining the increased discomfiture levels we experience in region (III) in an environment close to our body temperature.

These calculations can be verified with Eq. (18) for T.

Remark 1. Although it is possible to establish an overall correspondence between the dynamics of discrete and continuous systems [2], a careful consideration reveals some notable fundamentally distinctive characteristics between the two, which ultimately depends on the greater number of space dimensions available to the differential system. This has the consequence that continuous time evolution governed by differential equations is well-defined and unique, unlike the discrete case when ill-posedness and multifunctionality forms its defining character with the system being severely restrained in its manifestation, not possessing a set of equivalent yet discernible possibilities to choose from. It is our premise that the "kitchen of Nature" functions in an one-dimensional iterative analogue, not just to take advantage of the multiplicities inherent therein, but more importantly to format its dynamical evolution in a hierarchical canopy, so essential for the evolution of an interactive, non-trivial, complex structure. The 3-dimensional serving space of the real world only provides a convenient and attractive presentation of nature's produce of its uni-dimensional kitchen.

Example 4 (*The Negative Multifunctional world $\mathfrak{W}$: Temperature, Specific Heat, Entropy, Gravo-thermal Catastrophe*). Consider the two-state paramagnet of N elementary $(\uparrow, \downarrow)$ dipoles with a magnetic field B in the $+z$-direction. Then, with μ the magnetic moment and $E = -(N\mu B)\tanh(\mu B/kT)$ the total energy of the system [25],

$$S = Nk \ln\left[2\cosh\left(\frac{\mu B}{kT}\right)\right] - \frac{N\mu B}{T}\tanh\left(\frac{\mu B}{kT}\right) \tag{27a}$$

$$\frac{1}{T} \triangleq \frac{dS}{dE} = -\frac{k}{\mu B}\overset{-1}{\tanh}\left(\frac{E}{N\mu B}\right) \tag{27b}$$

$$C_V \triangleq \frac{dE}{dT} = T\frac{dS}{dT} = Nk\left[1 - \left(\frac{E}{N\mu B}\right)^2\right]\overset{-2}{\tanh}\left(\frac{E}{N\mu B}\right) \tag{27c}$$

are the expressions for temperature, entropy, and specific heat. The plot of these functions of Fig. 6 shows the typical unimodal, two-state, $(\uparrow, \downarrow)$ character of S that admits the following interpretation. In the normalized ground state energy $E = -1$ of all spins along the B-axis, the number of microstates is 1 and the entropy 0. As energy is added to the system some of the spins flip in the opposite direction until at $E = 0$ the distribution of the $\uparrow$ and $\downarrow$ configurations exactly balance and the entropy attains the maximum of ln 2. On increasing E further, the spins tend to align against the applied field till at $E = 1$ the entropy is again zero with all spins opposing the field for a single microstate and negative T. According to traditional wisdom [25], "the coldest temperature is just above 0 Kon the positive side, and the hottest temperatures are

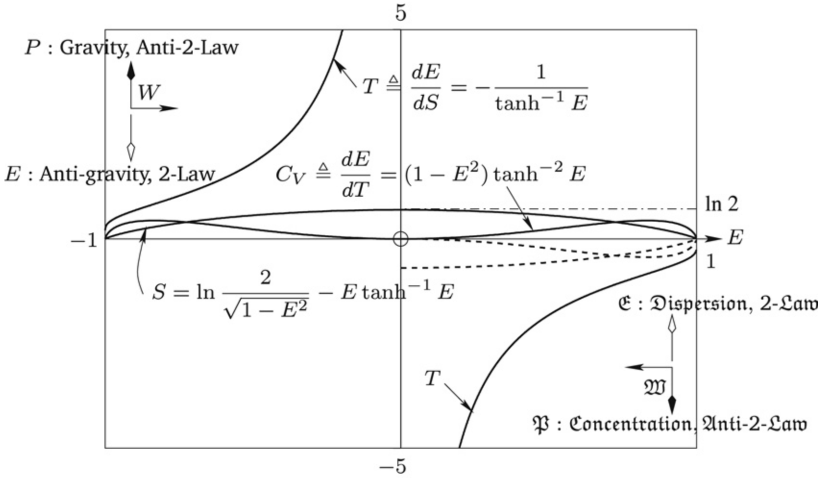

Fig. 6. Normalized ($N = \mu = B = 1 = k$) negative temperature, specific heat and entropy for a ($\uparrow, \downarrow$) system, with the natural thermodynamic arrows displayed open-headed. The symmetry of the two worlds in their respective domains permit them to collaborate as indicated by the full S and C curves in the upper plane. This is how the neg-world manifests itself in the real to generate complexity.

just below 0 Kon the negative side". This view of $\mathbb{R}_-$, as a set of "super positives", is to be compared with what it really is that defines the negative world $\mathfrak{W}$.

The bidirectional route admits quite a naturally different interpretation based on the virial theorem that relates the average kinetic energy $\mathscr{T}$ of a system to its average potential energy, $2\mathscr{T} = -\sum_{k=1}^{N} \mathbf{F}_k \cdot \mathbf{r}_k$ where $\mathbf{F}_k$ is the force on the kth particle located at position $\mathbf{r}_k$. For forces deriving from a potential $\mathscr{V}(r) = cr^n$, the virial theorem takes the particularly simple form

$$\mathscr{T} = \frac{n\mathscr{V}}{2} \tag{28}$$

with $n = -1$ denoting gravitational systems in the form

$$\mathscr{T} + E = 0$$

for a total energy E. For a bounded collapsing gas, the theorem then gives

- The potential energy decreases ($\mathrm{d}\mathscr{V}/\mathrm{d}r > 0$) faster than kinetic energy increases ($\mathrm{d}\mathscr{T}/\mathrm{d}r < 0$) as the total energy decreases with the bounding radius, $\mathrm{d}E/\mathrm{d}r > 0$. With $\mathscr{T} \sim NT$, $\mathscr{V} \sim -NT$; hence $\mathscr{V}(r) \sim -N^2 r^{-1} \Rightarrow T(r) \sim Nr^{-1}$ and $\mathrm{d}T/\mathrm{d}r \sim -r^{-2} < 0$ shows that the gas gets hotter on concentration. Hence the specific heat $C_V := \mathrm{d}E/\mathrm{d}T < 0$ is negative, and the entropy $S(r) = \int \mathrm{d}E/T \sim \ln r$ decreases to negative values as $r \to 0$ under its own gravity.
- While $\mathrm{d}E/\mathrm{d}r > 0$, $\mathrm{d}S/\mathrm{d}r > 0$ is also positive as the position uncertainty of the constituents decreases faster than the momentum uncertainty grows; hence temperature $T := \mathrm{d}E/\mathrm{d}S > 0$.
- For isothermal compression only the position uncertainty decreases with a consequent decrease in entropy, $\mathrm{d}S/\mathrm{d}r > 0$; for adiabatic compression only kinetic energy increases with $\mathrm{d}T/\mathrm{d}r \sim -r^{-2} < 0$.
- Applied to Fig. 6 the characteristic property $T > 0$, $C_V < 0$ of gravitational systems implies that the only region that can qualify for negative specific heat is $E > 0$ *with its direction reversed*, $1 < E < 0$.

In the negative multifunctional dual $\mathfrak{W}$, where the "anti-second law"[15] requires heat to flow spontaneously from lower to higher temperatures with positive temperature gradient along increasing temperatures, the engine and pump interchange their roles with order inducing compression of the system by the environment – rather than expansion against it as in W – being the thermodynamic direction (shown by open arrow-heads) in $\mathfrak{W}$: the equivalence class of dispersed microstates for a definite macro quite naturally induces the concentrated macrostate in a multifunctional world. The figure-of-8 cycle of Fig. 3(b), characterized by the parameter α representing an effective (T_c, T_h)-pump, therefore suggests that the expansion of the universe and the "unresolved conundrum of the cosmological constant problem" is an apparent complimentary manifestation of the global gravitational concentration of Nature's $E - P$ cycle against the second law E-expansion as indicated in the figure: the observed cosmological constant is hugely smaller than the theoretical because the inhibitory effect of the complimentary $\mathfrak{W}$ pump reduces the effective irreversibility ι to a vanishingly small, near-reversible, 0 value. It is assumed that each of the 4 expansion and compression strokes follow the basic isothermal-adiabatic route leading

[15] All qualifications here are with respect to W.

to the respective entropy and temperature changes inherent in these processes. With the positive directions of heat and work being out of and into $\mathfrak{W}$, energy "absorbed" in $\mathfrak{W}$ appears to be emitted from W, to be compared with the "precisely because the paramagnets emitted energy (at negative absolute temperatures) rather than absorbed it, they were hotter than paramagnets at any positive temperature. Indeed all negative temperatures are hotter than all positive temperatures", [25]. It is further argued that since a decrease in energy results in increasing entropy when $T < 0$ – as compared with the normal increasing energy with increasing entropy – when two such systems are brought in thermal contact, maximization of entropy requires negative temperatures to be "hotter" than the positive as heat must necessarily flow from higher to lower temperatures only.

Returning to Fig. 6, the opposing arrow of $\mathfrak{W}$ translated to W according to the principles of Section 2.2, generate the full curves; hence the entropy, specific heat and temperature are all positive in the perspective of $\mathfrak{W}(1 < E < 0)$ as indicated dashed in the figure. This permits the entropy to increase with energy in $\mathfrak{W}$ and avoids the need for "negative temperatures to behave as if they are higher than positive temperatures"; with increasing $E > 0$ temperatures actually decreasing in $\mathfrak{W}$ leading to negative specific heat and entropy. The negative temperatures increase to infinity as $E \to 0_+$ just as the positives do in W for $E \to 0_-$, with the interconnection between the complimentary dual worlds through the equivalences at $E = \pm 1$ and $T = \mp\infty$ allowing them to competitively collaborate as indicated by the full curves in the figure; the maximum entropy of $\ln 2$ occurs at $E = 0$ and the minimum at $E = \pm 1$ when all spins are aligned unidirectionally in single microstates. This is the manifestation of $\mathfrak{W}$ in W responsible for the characteristic two-state $(\uparrow, \downarrow)$ signature leading to complexity and holism.

The intuitively pathological $T_h \le T_c$ of (II) in the fully-chaotic region $\lambda \ge \lambda_*$, Fig. 4 and Table 2 where no complex patterns are possible – that is in $\mathfrak{T}_c \trianglelefteq \mathfrak{T} \trianglelefteq \mathfrak{T}_h$, with relations in $\mathfrak{W}$ denoted by $\trianglelefteq$ to distinguish from the $\le$ in W – can now be considered admissible iff $T_h = \infty$ when (II) and (IV) merge in the single region of negative temperatures with its own "negative" dynamics in relation to W. At the other extreme of $T_c = 0$, region (III) vanishes and taken with $T_h = \infty$ leads to the two surviving $\alpha \ge 0$ unshaded portions of Table 2, one for $\iota\alpha \le 0$ of the multifunctional (IV) of $\mathfrak{W}$ and the other functional $\iota\alpha > 0$ of W (I). The boundary $\mathrm{Multi}_{\|}(X)$ of the interface between the two worlds at $\chi = 0, \lambda \in [\lambda_*, 4)$ is inaccessible from W because the equivalence at $E = \pm 1$ generates a passage betwen these antagonistic domains.[16] We believe that the limiting behavior presented here represents a possible scenario of the creation of the universe, with the subsequent $T_c > 0$, $T_h < \infty$ dynamics of Fig. 4 responsible for the complex structures and patterns of Nature, see Section II-3 for further details.

It is not difficult to verify [26,27] that a negative C_V system (like $\mathfrak{W}$) can achieve thermal equilibrium with a normal system (like W) iff, their combined heat capacity is negative which corresponds to the fact that the product $\iota\alpha$ is a logistically moderated *pump* rather than an *engine*. Under unfavorable conditions on the value of this product there may be no collaboration between the opposites with the expansive engine of W disjoint and in perpetual competition with the compressive pump of $\mathfrak{W}$: with the resulting concentration and dispersion working in irreconcilable antagonism, "catastrophic collapse" of separation and divorce with universal entropy increase is likely. Only when λ is reduced to the complex region (I) will it be possible to reverse this tendency to a "high temperature equilibrium" state with a considerably reduced size and an increased entropy [27]. A study of the different characteristics of the temperature profiles for $T_c \lessgtr T_h$ in Fig. 4(c) supports this view.

The expansive dispersion of $\mathfrak{E}$-gravity in $\mathfrak{W}$ provides the necessary low entropy P-fuel in W that maintains the operation of its engine E and thereby the form and substance of Nature.

Remark 2. It is tempting to speculate that the other long-range potential with which we are intrinsically familiar does not play the same unique role as gravity because the coulomb potential sets up both its complementary arrows in W alone. Gravity is exceptional specifically because its reciprocal directions define the universal dyad $(\mathfrak{W}, W)$.

The Henon map, unlike the one-dimensional logistic, is well-defined and invertible, and we examine in the Appendix of Part II the significance of the foregoing delineation of the kitchen-dynamics and its presentation by Nature – and of our maximal non-injectivity criterion of chaos – in the iterative dynamics of this two-dimensional chaotic map.

> • *Did we beat the second law? No way. But by using the second law – taking the energy from spontaneous "downhill" reactions and transferring much of it to force a nonspontaneous process to go "uphill" energy-wise and make something – we got what we wanted. Living creatures are essentially energy processing systems that cannot function unless a multitude of "molecular machines", biochemical cycles, operate synchronically in using energy to oppose second law predictions. All of the thousands of biochemical systems that run our bodies are maintained and regulated by feedback subsystems, many composed of complex substances. Most of the compounds in the feedback systems are also synthesized internally by thermodynamically nonspontaneous reactions, effected by utilizing energy ultimately transferred from the metabolism of food. When these feedback subsystems fail – due to inadequate energy inflow, malfunction from critical errors in synthesis, the presence of toxins or competing agents such as bacteria or viruses – dysfunction, illness, or death results: energy can no longer be processed to carry out the many reactions we need for life that are contrary to the direction predicted by the second law.*
>
> LAMBERT [11]

[16] Is the Bose–Einstein condensate a physical realization of this interface?

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3. Discussions and conclusions

To complete the paper, we briefly mention two variant themes in the reductionism-holism debate: that of "exo-physics from without, endo-physics from within" [28] and of "holistic versus analytic cognition" [29]. In the traditional exophysical point of view, a complete description of the world is possible from outside without interacting with it; endophysics (rightly) differs with this philosophy in maintaining that the view from within a system generally differs from that obtained by an external observer: "All these studies lead us to think that evolution through formation and chaotic development of self-organization processes might be a general principle nature obeys in all its manifestations" [30]. The following passages from Buccheris's essay on endophysics substantiate this belief and illustrate the intra-disciplinary character of the holistic paradigm in Nature.

> *The history of mankind and the process of human civilization may also be considered parts of the evolution of the universe and characterized by the same self-organizing tendency. As a matter of fact, human societies are open, many-body systems interacting with the environment from which they receive resources and information. They are characterized by complex interactions between their members. These interactions create coherences which generate and maintain complex collective patterns.*
>
> *Quantum physics has been the most important element of disruption on the realistic view of nature. The collapse of the wave function describes a clearly irreversible event not foreseen by the Schrodinger equation, because the latter is linear and time symmetric and as such strictly deterministic. The predictions of the Schrodinger equation, however, hold only for closed systems – i.e., systems not interacting with their environment – in particular for the entire universe. The collapse of the wave function and its implied irreversibility is therefore relative to the point of view of the observer, an open system interacting with the rest of the universe. From the observer's point of view, the evolution of the external environment is neither deterministic nor reversible. The very fact that the probabilistic interpretation of quantum mechanics requires the presence of a semi-classical observer, considered external to the unitary evolution of the universe described by the Schrodinger equation, shows the rootedness of the exophysical attitude.*
>
> *The process of increasing knowledge in terms of a subjective-objective-subjective loop, closely connected with the features of our individual subjectivity, is a main characteristic of the endophysical viewpoint. As for the cosmological aspect, an endophysical theory has to consider the whole universe as an evolving, self-referential system, that must include "subjects" interacting with the external world, the "object".*

This endophysical "interfacial" world between the subject observer and the object environment represents the constitutive realism of the dynamical engine-pump, homeostatical, bi-directional, feedback loop; the multifunctional graphically converged limits correspond to the interface between the expansive Second-Law determined, inverse-iterative "Observer" and the contractive anti-Second-Law generated, direct-iterated "The Rest" [28].

In the eminently readable essay *Culture and Systems of Thought: Holistic versus Analytic Cognition* [29], Nisbett et al. find East Asians to be *holistic* attending to the entire field and assigning causality to it, making relatively little use of categories and formal logic and relying on "dialectic" reasoning, whereas Westerners are more *analytic*, paying attention primarily to the object and the categories to which it belongs and using rules including formal logic, to understand its behavior. The authors define *holistic thought* as "involving an orientation to the context or field as a whole, including attention to relationships between a focal object and the field and a preference for predicting events on the basis of such relationships. Holistic approaches rely on experience-based knowledge rather than on abstract logic and are dialectical, meaning that there is emphasis on change, a recognition of contradictions and of the need for multiple perspectives and a search for the *Middle Way* between opposing propositions". *Analytic thought*, in contrast, involve "detachment of the object from its context, a tendency to focus on attributes of the object to assign it to categories, and a preference for using rules about the categories to explain and predict the object's behavior. Inferences rest in part on the practice of de-contextualizing structure from content, the use of formal logic, and avoidance of contradictions".

Taking the ancient Chinese and Greek civilizations as defining examples of these social contradictions, the authors maintain that the Chinese do not seem to have been motivated to seek the first principles underlying their mathematical procedures or scientific assumptions but were more interested in a "dialectic which involves reconciling, transcending, or even accepting contradictions. In the Chinese intellectual tradition there is no necessary incompatibility in the opposite of a state of affairs existing simultaneously with the state of affairs itself" resulting "in a focus on particular instances and concrete cases". The Greeks favored the "epistemology of logic and abstract principles and (often) viewed concrete perception and direct experimental knowledge as unreliable and incomplete at best, and downright misleading at worst. Thus they were prepared to reject the evidence of the senses when it conflicted with reason. ⋯ Ironically, important as the Greek discovery of formal logic was for the development of science, it also impeded it in many ways. Importantly, there was never developed in Greece the critical concept of zero (which) was rejected as an impossibility on the grounds that nonbeing is logically self-contradictory!"

Without 0 and the negatives, "real" life would of course be hopelessly "incomplete"; with only increase the Second Law would rule supreme, nothing taking advantage of its ubiquity in generating the fascinations of Nature. It is hoped that the Hegelian yang-yin dialectic of *thesis-antithesis-synthesis* essentially expressed in the engine-pump duality will induce serious

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introspection of the "reconciliatory unification" of the bests of synthetic holism and logical analyticity that chanoxity, as the dialectical dynamics of opposites, strives to achieve.[17]

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[17] **Epilogue:** Is $\mathfrak{W}$ [the "Invisible Hand" of] "God"?