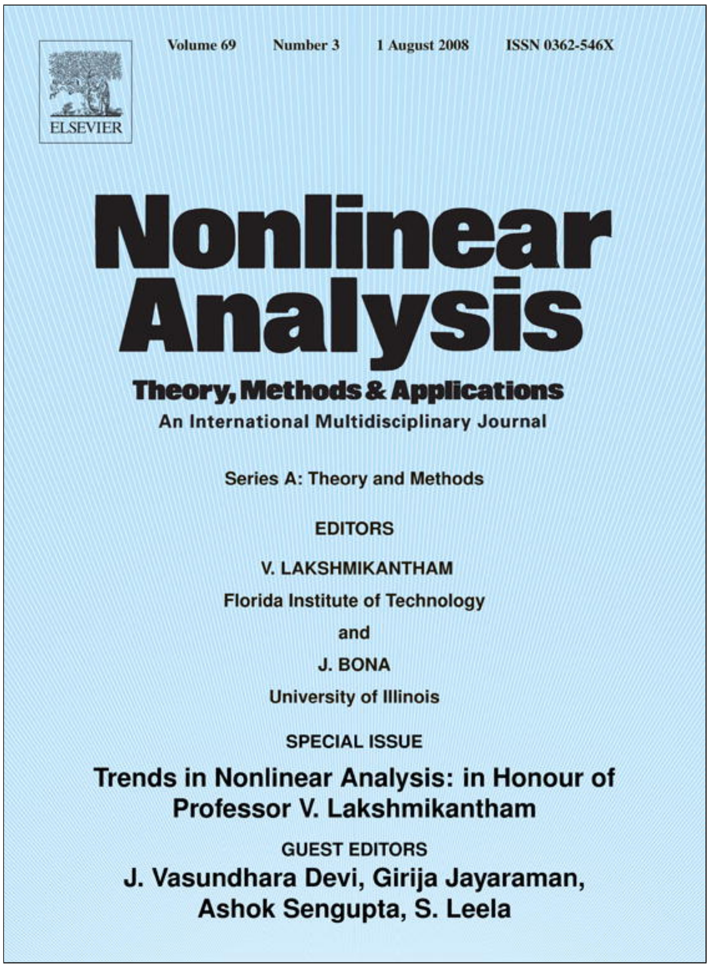
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Quantum non-locality and complex holism

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Abstract

Quantum Mechanics is supposed to be a general theory that applies to everything from subatomic particles to galaxies. If this were indeed to be so then Nature would be governed by the well-established reductionist principles of linearity and superpositions and the non-linear holism of emergence, self-organization, and complexity that we manifestly observe all around us would be absent. This paper seeks to explore these issues in relation to the theory of ChaNoXity [A. Sengupta, Chaos, nonlinearity, complexity: A unified perspective, in: A. Sengupta (Ed.), Chaos, Nonlinearity, and Complexity: The Dynamical Paradigm of Nature, in: Stud Fuzz, vol. 206, Springer-Verlag, Berlin, 2006, pp. 270–352] and proposes that Quantum Mechanics is an effective linear representation of the fully chaotic, maximally ill-posed negworld that manifests itself only through a bi-directional, contextually objective, inducement of the real world in adapting to the strictures of the Second Law of Thermodynamics.

Keywords: Chaos; Nonlinearity; Complexity; Holism; Quantum entanglement

1. Introduction

A self-contained and unified approach to Chaos–Non-linearity–compleXity was formulated in [7] under the acronym ChaNoXity. This new perspective of the dynamical evolution of Nature is based on the irreversible multifunctional multiplicities generated by the equivalence classes from iteration of non-invertible maps, eventually leading to chaos of maximal ill-posedness, see [6] for details. In this sense, the iterative evolution of difference equations is in sharp contrast to the smoothness, continuity, and reversible development of differential equations which cannot lead to the degenerate irreversibility inherent in the classes of maximal ill-posedness. Unlike evolution of differential equations, difference equations update their progress at each instant with reference to its immediate predecessor, thereby satisfying the crucial requirement of adaptability that constitutes the defining feature of complex systems. Rather than the smooth continuity of differential equations, Nature takes advantage of jumps, discontinuities, and singularities to choose from the vast multiplicity of possibilities that rejection of such regularizing constraints entails. The immediate upshot of this shift in perspective from differential to difference (and possibly to differential–difference) equations, lies in the greater flexibility the evolving system has in presenting its produce of emergence and self-organization through a judicious application of the Axiom of Choice. Non-locality and holism, the

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natural consequences of this paradigm, are to be compared with the reductionist determinism of classical Newtonian reversibility suggesting striking formal correspondence with superpositions, qubits and entanglement of quantum theory. Although it is premature to make any definitive commitments in this regard, this paper aims at making this possibility at least plausibly admissible thereby contributing to the query: *Is Nature governed by entanglements of linear superposition in Hilbert space or by the non-linear holism of emergence, self-organization, and complexity?* Could the awesome complex diversity of Nature have been so breathtaking in its omnipresence if it were constrained to operate in a unique, smooth, and completely predictable one–one fashion? Or is it a case of both these alternatives with the former being really an *excellent effective representation* of the actuality of the latter?[1]

The Hamilton–Jacobi formulation of classical mechanics is closest in approach to quantum mechanics. In essence, a classical system with N memory bits can access any of the 2^N basic states $e_i = (0, \ldots, 1_i, \ldots, 0)^{\mathrm{T}}$, $i = 0, \ldots, 2^{N-1}$, with probability $p_i \geq 0$, $\sum_{i=0}^{2^N-1} p_i = 1$. If the state i was obtained after a measurement then the system continues to be in this state into the future. If p_{ji} is the probability for the transition $i \to j$ then $(p_{ji})_{2^N \times 2^N}$ is the *stochastic transition matrix* that yields the state vector in a succeeding time step j from that at i. In its *deterministic form*, an observable $\mathcal{O}$ is represented by a two-valued real function $f_{\mathcal{O}} : \Omega \to \mathbb{R}$ mapping the $6N$-dimensional phase space Ω, with the possible 0-no/1-yes response of the query $f_{\mathcal{O}}^{-}(\Delta) \overset{?}{\subseteq} \Omega$ defining $f_{\mathcal{O}}$. The quantum state-space of any *isolated* physical system, in comparison, is represented by the complex Hilbert space $\mathcal{H}$, and the system is completely described by its unit state vector. The time evolution of a *closed* quantum system is according to a unitary transformation $\Psi(t_2) = U\,\Psi(t_1)$ described by the Schrodinger equation $i\hbar\,\partial\,|\Psi\rangle\,/\partial t = H\,|\Psi\rangle$. Linear subspaces of $\mathcal{H}$ correspond to the inverse images $f_{\mathcal{O}}^{-}(\Delta) \subseteq \Omega$, and continuous probabilities replace the binary yes/no possibilities of classical mechanics.

Arbitrary superposition is the principal distinguishing feature of quantum mechanics without any classical analogue: whereas a system in a classical ensemble of states can only be in one of these states unknown to the observer, a quantum superposition endows the system with an ubiquitous omnipresence that translates into *entangled correlations* between different subsystems. This leads to properties of the whole that cannot be traced to the individual parts: "the (quantum-mechanical) whole is different from the sum of its parts".

This paper hopes to elaborate on these issues to a reasonable degree of admissibility.

2. Chaos–Non-linearity–compleXity

2.1. *Chaos is maximal non-injectivity*

Chaos responsible for complexity is the eventual outcome of *non-reversible* iterations of one-dimensional *non-injective* maps [6]; non-injectivity leads to irreversible non-linearity and one-dimensionality constrains the dynamics to evolve with the minimum latitude thereby inducing emergence of new features as required by complexity. In this sense chaos is the maximal ill-posed irreversibility of maximal degeneracy of multifunctions; features that cannot arise through differential equations. The mathematics involve topological methods of convergence of nets and filters with the multifunctional graphically converged adherent sets effectively enlarging the functional world in the outward manifestation of Nature. Chaos therefore is more than just an issue of whether or not it is possible to make accurate long-term predictions of the system: chaotic systems must necessarily be sensitive to initial conditions, topologically mixing with dense periodic orbits; this however is not sufficient.

The definitional requirement of a maximal multifunctional extension $\mathrm{Multi}(X, Y) \supseteq \mathrm{Map}(X, Y)$ of the function space $\mathrm{Map}(X, Y)$ is to be considered as follows. Let f be a non-injective map in $\mathrm{Multi}(X)$ and $P(f)$ the number of injective branches of f. Denote by $F = \{f \in \mathrm{Multi}(X) : f \text{ is a non-injective function on } X\}$ the resulting basic collection of non-injective functions in $\mathrm{Multi}(X)$.

(i) For every α in some directed set $\mathbb{D}$, let F have the extension property $(\forall f_\alpha \in F)(\exists f_\beta \in F) : P(f_\alpha) \leq P(f_\beta)$.

(ii) For $f_\alpha, f_\beta \in \mathrm{Map}(X)$, let $P(f_\alpha) \leq P(f_\beta) \Leftrightarrow f_\alpha \preceq f_\beta$ such that $P(f) := 1$ for the smallest f, defines a partial order $\preceq$ in $\mathrm{Multi}(X)$. This is actually a preorder in which functions with the same number of injective branches are equivalent.

[1] "The experimental evidence against Bell inequality tells us that any theory quantum mechanics is derived from must be non-local. It is then natural to hypothesize that this non-local theory is a cosmological theory", Smolin [9]; see also [2,10].

(iii) Let $C_\nu = \{f_\alpha \in \text{Multi}(X) : f_\alpha \preceq f_\nu\}$, be chains of non-injective functions of Multi(X) and $\mathcal{X} = \{C : C$ is a chain in $(F, \preceq)\}$ denote the corresponding chains of F with $\mathcal{C}_\text{T} = \{C_\alpha, C_\beta, \ldots, C_\nu, \ldots\}$ a chain in $\mathcal{X}$. By the Hausdorff Maximal Principle, there exists a maximal chain $\sup_{\mathcal{C}_\text{T}}(\mathcal{C}_\text{T}) = C_\leftarrow = \{f_\alpha, f_\beta, f_\gamma, \ldots\}$ of F in $\mathcal{X}$.

Zorn's Lemma now applied to this maximal chain yields its supremum as the maximal element of ${}_\to C$, and thereby of F. Note that as in the case of the algebraic Hamel basis, the existence of this maximal non-functional element emerged purely set theoretically as the "limit" of a net of functions with increasing non-linearity, without recourse to any topological arguments. Because it is not a function, this supremum does not belong to the functional towered chain with itself as a fixed point; this maximal chain does not possess a largest, or even a maximal, element although it does have a supremum. The supremum is a contribution of the inverse functional relations (f_α^-) because the net of increasingly non-injective functions implies a corresponding net of increasingly multivalued functions ordered inversely by the relation $f_\alpha \preceq f_\beta \Leftrightarrow f_\beta^- \preceq f_\alpha^-$. Thus the inverse relations which are as much an integral part of graphical convergence as are the direct relations, have a smallest element belonging to the multifunctional class. Clearly, this smallest element as the required supremum of the increasingly non-injective tower of functions defined in (ii), serves to complete the significance of the tower by capping it with a "boundary" element that can be taken to bridge the classes of functional and non-functional relations on X.

Chaotic map. Let A be a non-empty closed set of a compact Hausdorff space $(X, \mathcal{U})$. A function $f \in \text{Multi}(X)$ is *maximally non-injective* or *chaotic* on $\mathcal{D}(f) = A$ w.r.t. to $\preceq$ if (a) for any f_i there exists an f_j satisfying $f_i \preceq f_j$ for every $i < j \in \mathbb{N}$, and (b) the set $\mathcal{D}_+ := \{x \in X : (f_\nu(x))_{\nu\in\text{Cof}(\mathbb{D})}\}$ converging in X is a countable collection of isolated singletons for $\text{Cof}(\mathbb{D}) = \{\beta \in \mathbb{D}$ for some $\beta \succeq \alpha \in \mathbb{D}\}$ a cofinal subset of $\mathbb{D}$.

The significance of this ill-posed approach to chaos [6] in the discrete one-dimensional evolution of non-injective maps defined by the binary increasing $\uparrow$ and decreasing $\downarrow$ components of positive and negative slopes, has a direct significance to complexity, holism, and chanoxity in the sense that

(a) The multifunctional extension Multi(X) being a superspace of Map(X), elements $\Phi \in \text{Multi}(X) \notin \text{Map}(X)$ are in a special privileged position in the evolutionary stability of $F \in \text{Map}(X)$. Any $\Phi \notin \text{Map}(X)$ being a multi in both the forward and backward directions of increasing and decreasing times (iterates) – unlike F which is a multi only for backward times into the past through its "mind" – has the privilege of *symmetric* access to all its resources in both directions that would be a source of major embarrassment in X if this here and now were to be also multifunctionally ubiquitous. This spatial omnipresence throughout X – the signature of emergence in non-linear complex systems – bears comparison with quantum entanglement and non-locality.
(b) The collective macroscopic cooperation between Map(X) and its extension Multi(X) generates the equivalence classes in Map(X) through fixed points and periodic cycles of F. As all the points in a class are equivalent under F, a net or sequence converging to any one must necessarily converge to the others in the set. This implies that the cooperation between Map(X) and Multi(X) conspires to alter the topology of X to large equivalence classes whose Φ-images are the open sets in $\mathcal{R}(F)$, the inverse images of which in turn constitute the topology of X in $\mathcal{D}(F)$. This dispersion throughout the domain of F of initial localizations suggests increase in entropy and disorder with increasing chaoticity; complete chaos therefore corresponds to the state of maximum entropy of the second law of thermodynamics.
(c) This scenario indicates a strong case for a fresh look at some of the contentious issues of quantum mechanics like non-locality, entanglement and decoherence in the complex systems perspective. We attempt to do this in the following.

2.2. ChaNoXity: The dynamics of opposites

The basic undertaking in our approach to a formal theory of complex systems [7] consists in the establishment of a rigorous exposition of bi-directionality as the main tool in their evolution, capable of providing a rational understanding of why and how Nature apparently defeats the all-embracing outreach of the Second Law. This is achieved by identifying the multifunctional world Multi(X) as a "negative" $\mathfrak{W}$ of the real world W of Map(X) as follows.

Let W be a set and suppose that for every $w \in W$ there exists a negative element $\mathfrak{w} \in \mathfrak{W}$ with the property that

$$\mathfrak{W} \stackrel{\text{def}}{=} \left\{\mathfrak{w} : \{w\} \bigcup \{\mathfrak{w}\} = \emptyset\right\} \tag{1a}$$

defines the negative, or exclusion, set of W. This means that for every subset $A \subseteq W$ there is a complementary neg(ative)set $\mathfrak{A} \subseteq \mathfrak{W}$ associated with (generated by) A such that

$$A \bigcup \mathfrak{G} \triangleq A - G, \quad G \longleftrightarrow \mathfrak{G} \qquad \text{implies}$$
$$A \bigcup \mathfrak{A} = \emptyset. \tag{1b}$$

Hence a negset and its generating set act as relative discipliners of each other in restoring a measure of order in the evolving confusion, disquiet and tension, with the intuition of the set–negset pair "undoing", "controlling", or "stabilizing" each other. The complementing negelement is an unitive inverse of its generator, with $\emptyset$ the corresponding identity and G the physical manifestation of $\mathfrak{G}$. Thus if $r > s \in \mathbb{R}_+$, the physical manifestation of any $-s \in \mathfrak{R}_+ (\equiv \mathbb{R}_-)$ is the smaller element $r - s \in \mathbb{R}_+$.

Compared with the directed set $(\mathcal{P}(W), \subseteq)$, the direction of *increasing supersets* induced by $(\mathcal{P}(W), \supseteq)$ proves useful in generating a co-topology $\mathcal{U}_-$ on $(W, \mathcal{U}_+)$ as follows. Let $(w_0, w_1, w_2, \ldots)$ be a sequence in W converging to $w_* \in W$, and consider the backward arrow induced at w_* by the directed set $(\mathcal{P}(W), \supseteq)$ of increasing supersets at w_*. As the reverse sequence $(w_*, \ldots, w_{i+1}, w_i, w_{i-1}, \ldots)$ does not converge to w_0 unless it is eventually in every neighbourhood of this initial point, we define an additional *exclusion topology* $\mathcal{U}_-$ on $(W, \mathcal{U}_+)$, where the *w-exclusion topology* consists together with W, all the subsets $\mathcal{P}(W - \{w\})$ that *exclude* w. Since $\mathcal{N}_w = \{W\}$ and $\mathcal{N}_{v \neq w} = \{\{v\}\}$ are the neighbourhood systems at w and any $v \neq w$ in the w-exclusion topology, it follows that while every net must converge to the defining point of its own topology, only the eventually constant net $\{v, v, v, \ldots\}$ converges to a $v \neq w$. All directions with respect to w are consequently rendered equivalent; hence the directions of $\{1/n\}_{n=1}^{\infty}$ and $\{n\}_{n=1}^{\infty}$ are equivalent in $\mathbb{R}_+$ as they converge to 0 in its exclusion topology, and this basic property of the exclusion topology induces an opposing direction in W.

With respect to a sequence $(w_i)_{i \geq 0}$ in $(W, \mathcal{U}_+)$ converging to $w_* = \bigcap_{i \geq 0} \mathrm{Cl}(N_i) \in W$, let there exist an increasing sequence of negelements $(\mathfrak{w}_i)_{i \geq 0}$ of $\mathfrak{W}$ that converges to $\mathfrak{w}_*$ in the $\mathfrak{w}_*$-inclusion topology $\mathfrak{U}$ of $\mathfrak{W}$ generated by the $\mathfrak{W}$-images of the neighbourhood system $\mathcal{N}_{w*}$ of $(W, \mathcal{U}_+)$. Since the only manifestation of negsets in the observable world is their regulating property on W, the $\mathfrak{W}$-increasing sequence $(\mathfrak{w}_i)_{i \geq 0}$ converges to $\mathfrak{w}_*$ in $(\mathfrak{W}, \mathfrak{U})$ if and only if the sequence $(w_0, w_1, w_2, \ldots)$ converges to w_* in $(W, \mathcal{U}_+)$. Affinely translated to W, this means that the $\mathfrak{w}_*$-inclusion arrow in $(\mathfrak{W}, \mathfrak{U})$ induces an w_0-exclusion arrow in $(W, \mathcal{U}_+)$ generating an additional topology $\mathcal{U}_-$ in W that opposes the arrow converging to w_*. This *direction of increasing supersets of* $\{w_*\}$ *excluding* w_0 associated with $\mathcal{U}_-$ is to be compared with the *natural direction of decreasing subsets containing* w_* in $(W, \mathcal{U}_+)$. We take the reference natural direction in $W \cup \mathfrak{W}$ to be that of W pulling the inclusion sequence $(w_0, w_1, w_2, \ldots)$ to w_*; hence the decreasing subset direction in $\mathfrak{W}$ of the inclusion sequence $(\mathfrak{w}_0, \mathfrak{w}_1, \mathfrak{w}_2, \ldots, \mathfrak{w}_*)$ appears in W as an exclusion sequence converging to w_0 because any sequence in an exclusion space must necessarily converge to the defining element in its own topology. The left-hand side of Eq. (1b), read in the more familiar form $a + (-b) = a - b$ with $a, b \in \mathbb{R}_+$ and $-b := \mathfrak{b} \in \mathfrak{R}_+$, represents a "+" operation which in the actual bi-directional physical-world manifests on the right as a retraction in the "−" direction.

Although the backward sequence $(w_j)_{j=\ldots,i+1,i,i-1,\ldots}$ in $(W, \mathcal{U}_+)$ does not converge, the effect of $(\mathfrak{w}_i)_{i \geq 0}$ of $\mathfrak{W}$ on W is to regulate the evolution of the forward arrow $(w_i)_{i \geq 0}$ to an effective state of stasis of dynamical equilibrium, that becomes self-evident on considering for W and $\mathfrak{W}$ the sets of positive and negative reals, and for $w_*, \mathfrak{w}_*$ a positive number r and its negative $-r$. The existence of a negelement $\mathfrak{w}$ in $\mathfrak{W}$ for every $w \in W$ requires all forward arrows in W to have a matching forward arrow in $\mathfrak{W}$ that actually *appears backward when viewed in* W. It is this opposing complimentary effect of the apparently backward-$\mathfrak{W}$ sequences on W – responsible by (1b) for moderating the normal uni-directional evolution in W – that is effective in establishing a stasis of dynamical balance between the opposing forces generated in the composite of a compound system with its environment. Obviously, the evolutionary process ceases when the opposing influences in W due to itself and that of its moderator $\mathfrak{W}$ balance marking a state of dynamic equilibrium. In summary, the exclusion topology of large equivalence classes in Multi(X) successfully competes with the normal inclusion topology of Map(X) to generate a state of dynamic homeostatic equilibrium in W that permits out-of-equilibrium complex systems to exist despite the privileged omnipresence of the Second Law.[2]

An additional technical justification of the hypothesis of Eqs. (1a) and (1b) is provided by the inverse and direct limits, see Sengupta [7] for details related to the present application. The direct (or inductive) limit is a general method

[2] $f^{-i} f^{i}(x) = [x]$; hence $f^{-i} f^{i} = \mathbf{1}_{X/\sim}$.

of taking limits of a “directed families of objects”, while the inverse (or projective) limit allows “gluing together of several related objects”, the precise nature of which is specified by morphisms between them. Thus in terms of the family of continuous maps $\eta_{\alpha\beta} : X_\alpha \to X_\beta$ and $\pi^{\beta\alpha} : X^\beta \to X^\alpha$ connecting the family of spaces as indicated and satisfying $\eta_{\alpha\alpha}(x) = x \in X_\alpha$, $\eta_{\alpha\gamma} = \eta_{\beta\gamma} \circ \eta_{\alpha\beta}$ for all $\alpha \preceq \beta \preceq \gamma$ and $\pi^{\alpha\alpha}(x) = x \in X^\alpha$, $\pi^{\gamma\alpha} = \pi^{\beta\alpha} \circ \pi^{\gamma\beta}$ for all $\alpha \preceq \beta \preceq \gamma$. The basic content of these commutative diagrams is that the limits ${}_{\to}X$ and $X_{\leftarrow}$ are generated by opposing directional arrows whose existence follow from very general considerations; thus for example existence of the union of a family of nested sets entails the existence of their intersection, and conversely. In the context of Hilbert spaces these constructions generalize in the form rigged Hilbert space, useful in the correspondence of intrinsic irreversibility in terms of bi-directional chanoxity (Sengupta [8]).

A complex system can hence be represented as

$$\begin{gathered}\left.\begin{array}{r}\underline{\text{BACKWARD-DIRECT ARROW}}\\ \text{Analysis of } P\text{-contraction,}\\ \text{disorder, entropy increasing}\\ \text{bottom–up } \textit{emergence}\ {}_{\to}C\end{array}\right\} \bigoplus \left\{\begin{array}{l}\underline{\text{FORWARD-INVERSE ARROW}}\\ \text{Synthesis of } E\text{-expansion,}\\ \text{order, entropy decreasing top-}\\ \text{down } \textit{self-organization}\ C_{\leftarrow}\end{array}\right. \\ \Longleftrightarrow \\ \text{Synthetic cohabitation of opposites } \mathcal{C} = C_{\leftrightarrow},\end{gathered} \tag{2}$$

with $\oplus$ denoting a non-reductionist sum of the components of a top–down engine and its complimentary bottom–up pump.[3] A complex system behaves in an organized collective manner with properties that cannot be identified with any of the individual parts but arise from the structure as a whole: these systems cannot dismantle into their components without destroying itself. Analytic methods cannot simplify them as such techniques do not account for characteristics that belong to no single component but relate to the parts taken together, with all their interactions. This analytic base must be integrated into a synthetic whole with new perspectives that the properties of the individual parts fail to add up to. A complex system is therefore a dynamical, interactive, interdependent, hierarchical homeostasy of *P-emergent, disordering instability* competitively collaborating with adaptive *E-self-organized, ordering stability* generating a non-reductionist structure that is more than the sum of its constituent parts. We have attempted to provide, in the above context, an essentially thermodynamic analysis of the defining role of the (gravity) pump in generating complex structures in Nature.

To investigate further the question of equivalence between W and $\mathfrak{W}$, consider the reduction of the inverse-direct model as a coupled thermodynamic engine–pump system in which heat transfer between temperatures $T_h > T_c$, is reduced to a engine E-pump P combination operating respectively between temperatures $T_c < T = T_{\leftrightarrow} < T_h$, as shown in Fig. 1. We assume that a real complex adaptive system is distinguished by the full utilization of the fraction $W := (1-\iota)W_{\text{rev}} = (1-\iota)\eta_{\text{rev}}Q_h = (1-\iota)(1-T_c/T_h)Q_h$ of the work output of an imaginary reversible engine running between temperatures T_h and T_c, to self-generate the demonic pump P working in competitive collaboration with a reversible engine E, where the irreversibility

$$\iota \triangleq \frac{W_{\text{rev}} - W}{W_{\text{rev}}} \tag{3a}$$

accounts for that part ιW_{rev} of available energy (exergy) $W_{\text{rev}} = (U - U_0) + P_{\text{R}}(V - V_0) - \Sigma_{j=1}^{J}\mu_{j,0}(N_j - N_{j,0}) - T_{\text{R}}(S - S_0) \triangleq W + T_{\text{R}}\Delta S$ (with $_{\text{R}}$ the infinite reservoir with which the system attains equilibrium $_0$), that cannot be gainfully utilized but must be degraded in increasing the entropy of the universe. Hence

$$\iota = \left(\frac{T_{\text{R}}}{W_{\text{rev}}}\right) S$$

[3] The interpretation of this equation in the global context, everything else remaining the same, is that “hot” objects have higher entropy than “cold” ones, and when two bodies of different resources are brought in contact, entropy of the expanding hot body decreases while that of the contracting cold body increases with the entropy decrease in the former being more than compensated by its increases in the latter. This spontaneous flow of “heat” is associated with an overall entropy increase of the joint system that continues till the combined order is a minimum: globally, the expansion of the engine is distinguished by an overall effect of the pump!

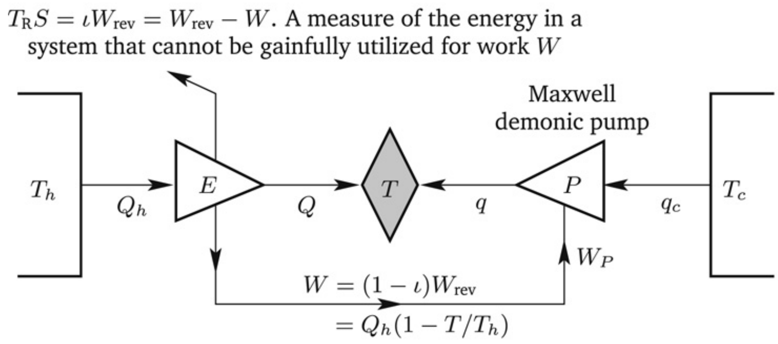

Fig. 1. Reduction of the dynamics of opposites to an equivalent engine–pump thermodynamic system. The fraction $W = (1-\iota)W_{rev}$ of the available maximum reversible work $W_{rev} = \eta_{rev} Q_h := (1 - T_c/T_h) Q_h$ of a reversible engine operating between $[T_c, T_h]$ is gainfully utilized internally to self-generate a heat pump P to inhibit, through gradient dissipation, the entropy that would otherwise be produced by the system. The induced pump P prevents the entire internal resource $T_h - T_c$ from diffusion at $\iota = 1$ by stimulating a temperature difference $T_h - T$ at some $\iota < 1$ that defines the homeostatic temperature $T < T_h$.

defines the effective entropy in terms of the parameter $\iota \in [0, 1]$ and available internal energy $W_{rev} = Q_h(T_h - T_c)/T_h$. The self-induced demonic pump effectively decreases the engine temperature-gradient $T_h - T_c$ to $T_h - T$, $T_c \le T < T_h$, thereby inducing a degree of dynamic stability to the system.

Let the irreversibility ι be computed on the basis of dynamic equilibrium through the definition of an equilibrium temperature $T(\iota)W_E := Q_h(1 - T/T_h) \triangleq W_P = Q_h(1-\iota)(1 - T_c/T_h)$ of the engine–pump system. Hence

$$\iota(T) = \frac{\Delta T}{T_h - T_c}, \quad \Delta T \triangleq T - T_c \tag{4}$$

where $T_h - T_c$ represents the internal energy of the system that is divided into the non-entropic $T_h - T$ free energy A internally utilized to generate the pump P, and a reduced $T - T_c$ manifestation of entropic dissipation by E, with the equilibrium temperature T serving to actually establish this looped recursive definition. The generated pump is a realization of the energy available for useful, non-entropic, work arising from reduction of the original entropic gradient ΔT_h to ΔT. The irreversibility $\iota(T)$ can be considered to have been adapted by the engine–pump system such that the induced instability due to P balances the imposed stabilizing effort of E to the best possible advantage of the system and its surroundings. Hence a measure of the energy in a system that cannot be utilized for work W but must necessarily be dumped to the environment is given by the "generalized entropy" expression

$$\begin{aligned} TS &= W_{rev} - W \\ &= U - A \end{aligned} \tag{5}$$

which the system does by adapting itself to a state that optimizes competitive collaboration for the greatest efficiency consistent with this competitiveness. This distinguishing feature of non-equilibrium dynamics as compared to the corresponding equilibrium case lies in the mobility of the defining temperature T: for the introverted self-adaptive systems, the dynamics organizes to the prevailing situation by best adjusting itself *internally* for maximum possible global advantage.

Define the equilibrium steady-state representing $X_{\leftrightarrow}$ of optimized $E{-}P$ adaptability between E and P to be given by

$$\alpha(T) = \left(\frac{T_h - T}{T_h}\right)\left(\frac{T}{T - T_c}\right) \tag{6}$$

in terms of the adaptability function $\alpha := \eta_E \zeta_P$ that represents an effective adaptive efficiency of the engine–pump system to the environment $(T_c,\ T_h)$ as the product of the efficiency of a reversible engine operating between $(T,\ T_h)$ and the coefficient of performance of a reversible pump in $(T_c,\ T)$. Fig. 2 representing this tension for the specific case of $T_h = 480\,\text{K}$ and $T_c = 300\,\text{K}$, shows that the engine–pump duality has the significant property of supporting

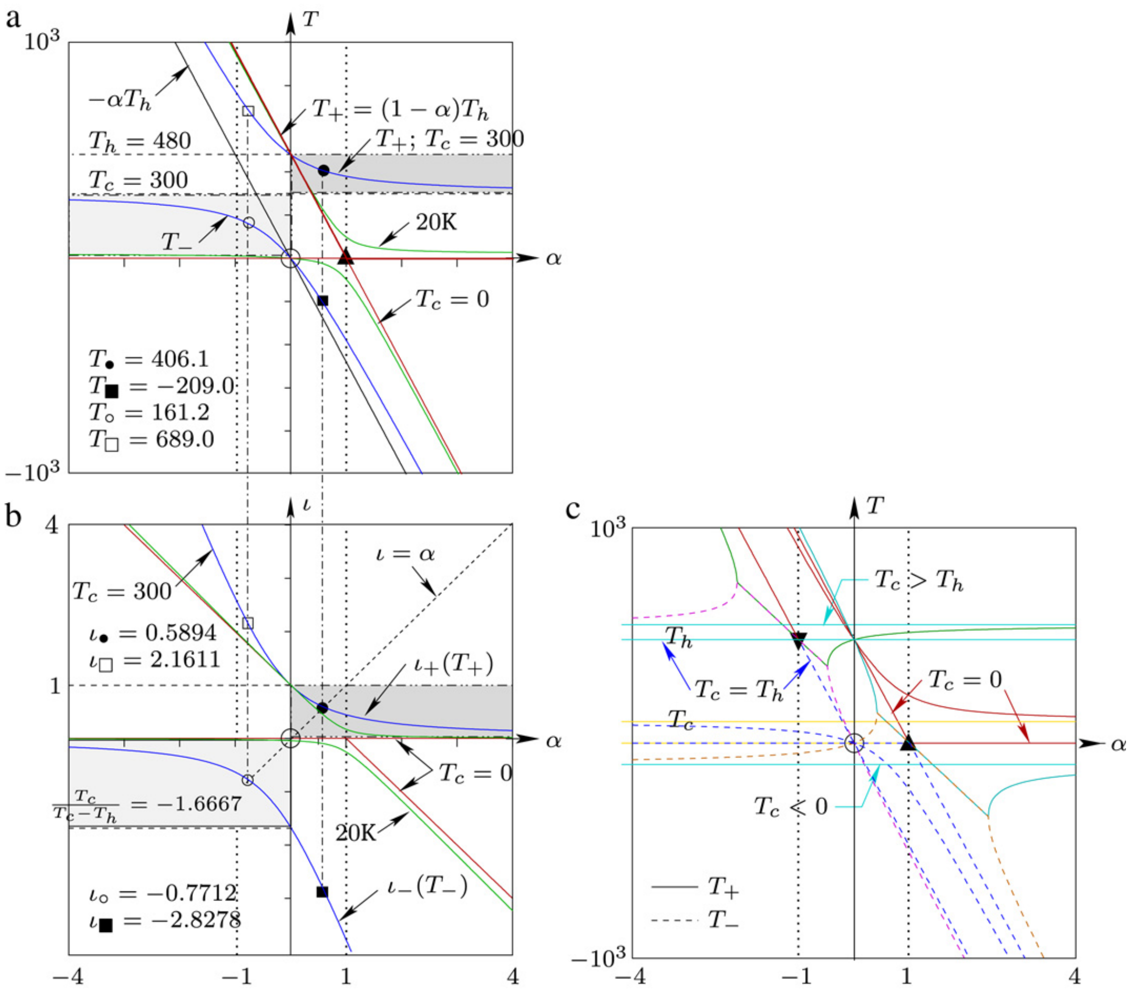

Fig. 2. The "Participatory Universe" for $T_h = 480\,\text{K}$, $T_c = 300\,\text{K}$. The real world evolutionary dynamics in W shown by the shaded regions defined by $\iota\alpha > 0$ for $T_+ \in (T_c, T_h)$ and $T_- \in (0, T_c]$ is partnered by the complementary neg-world effects of $\mathfrak{W}$of $\iota\alpha < 0$ for $T_+ \in [T_h, \infty)$ and $T_- \in (-\infty, 0]$. A significant property of the E–P system is that $\lim_{T\to T_c/-\infty} \alpha(T) \to \infty$ and $\lim_{T\to T_h/0} \alpha(T) \to 0$; hence displacement from the steady state T of a non-equilibrium system motivates opposition from either of the components, thereby inducing it back to a state of dynamical equilibrium.

more than one condition based on

$$T_\pm = \frac{1}{2}\left[(1-\alpha)T_h \pm \sqrt{(1-\alpha)^2 T_h^2 + 4\alpha T_c T_h}\right] \tag{7}$$

for any given value of α. Fig. 2(c) then allows identification of the real and negative worlds in terms of the irreversibility and adaptability product $\alpha\iota = T(T_h - T)/T_h(T_h - T_c)$ as

$$\frac{W,\ \alpha\iota > 0}{0 < T < T_h} \quad \begin{array}{l} 0 < T_c < T_h : \alpha > 0, \iota \in (0, 1],\ T_c < T_+ \le T_h, \\ \alpha < 0, \iota \in [\iota_c, 0),\ 0 \le T_- < T_c. \end{array} \tag{8a}$$

$$\frac{\mathfrak{W},\ \alpha\iota < 0}{T < 0,\ T > T_h} \begin{cases} T_c \le 0 : \ \alpha < 0,\ T_- \in (T_c, 0],\ T_+ \in [T_h, \infty), \\ \qquad\quad\ \alpha > 0,\ T_\pm \in (-\infty, T_c),\ T_- \in (-\infty, 0]. \\ T_h \le T_c : \ \alpha < 0,\ T_\pm \in (T_c, \infty),\ T_+ \in [T_h, \infty), \\ \qquad\quad\ \alpha > 0,\ T_+ \in [T_h, T_c),\ T_- \in (-\infty, 0] \end{cases} \tag{8b}$$

where $\iota_c := -T_c/(T_h - T_c)$. The real world, with α and ι of the same sign, attains order–disorder homeostasy in collaborative competition with the negworld identified by negative $\alpha\iota < 0$. The displayed difference in the temperature profile of W and $\mathfrak{W}$ is essentially a reflection of the multifunctional symmetry of the latter compared to the functional asymmetry of W; thus T_+ and T_- appear as a continuous curve in $\mathfrak{W}$ for $T_c < 0$, $T_c > T_h$ bifurcating as separate branches at these defining temperatures to give birth to W. The bi-directional expansive–compressive tuning of the dynamics of a non-equilibrium system as implied by Eq. (6), means that any displacement from its steady-state motivates opposition from one of its constituents inducing the system back to the state of dynamic equilibrium. The non-equilibrium system X can therefore be expected to oscillate about this equilibrium $X_\leftrightarrow$ by responding to changes in its environmental gradients. The temperatures (7) serve to parametrize the functional and multifunctional components of the universe: *complex systems by definition represent homeostasy between them with respect to convergence and hence their induced topologies*.

What is the most advantageous state of this emergent, self-organizing system? With the self-induced equilibrium $T_\leftrightarrow$ denoting an effective T_c and T_h for the engine and pump respectively, the adaptability α and irreversibility ι define the entropy $S_\leftrightarrow = \iota(W_{\text{rev}}/T_{\text{R}})$ that represents a dynamical balance of the expansive and compressive opposites generated within the system. The nature of these indices illustrated in Fig. 2(b) suggests that the balancing condition

$$\iota(T) = \alpha(T) \tag{9}$$

can be taken to define the most appropriate value of the equilibrium steady-state $T_\leftrightarrow \in [T_c,\ T_h]$.

A correspondence between the dynamics of the engine–pump system and the logistic map $\lambda x(1-x)$ – in which the direct iterates $f^i(x)$ correspond to the "pump" $\mathfrak{W}$ and the inverse iterates $f^{-i}(x)$ to the "engine" W – is summarized below, with the fixed points $x_{\text{fp}} = (0,\ (\lambda-1)/\lambda > 1/2)$ designated as $\bullet$ = stable and $\circ$ = unstable.

$$\begin{aligned}
&\alpha \geq 0,\ \iota \in (-\infty,\ \iota_c],\ T_- \in (-\infty, 0].\ \lambda \in \{(0,1],\ (1,2]\},\\
&x_{\text{fp}} = \{(\bullet, -),\ (\circ, -)\}: \text{Functional simple } \mathfrak{W},\ \alpha\iota < 0. \qquad \text{(IV)} && (10a)\\
&\alpha < 0,\ \iota \in (\iota_c,\ 0],\ T_- \in (0, T_c].\ \lambda \in (2,3],\ x_{\text{fp}} = (\circ, \bullet):\\
&\text{Functional simple } W,\ \alpha\iota > 0. \qquad \text{(III)} && (10b)\\
&\alpha \geq 0,\ \iota \in (0,1),\ T_+ \in (T_c, T_h).\ \lambda \in (3, \lambda_*),\ x_{\text{fp}} = (\circ, \bullet/\circ):\\
&\text{Multifunctional complex } W,\ \alpha\iota > 0. \qquad \text{(I)} && (10c)\\
&\alpha < 0,\ \iota \in [1,\infty),\ T_+ \in [T_h, \infty).\ \lambda \in [\lambda_*, \infty),\ x_{\text{fp}} = (\circ, \circ):\\
&\text{Multifunctional chaotic } \mathfrak{W},\ \alpha\iota < 0. \qquad \text{(II)} && (10d)
\end{aligned}$$

The complex region $\lambda \in (3, \lambda_*)$, $x_{\text{fp}} = (\circ, \bullet/\circ)$ corresponding to $T_+ \in (T_c, T_h)$ for $\iota \in (0,1)$, of region (10c) is the outward manifestation of the tension between the regions of (10b) and (10d): observe from Eq. (7) and Fig. 2 that at an environment $T_c = 0$ the two worlds merge at $\alpha = 1$ bifurcating as individual components for $T_c > 0$. Fig. 2 gives a graphical representation of these regions in quadrants IV, III, I and II respectively characterized by the positive/negative values of $\alpha\iota$.

Let us consider an example of (2) for the living human system maintained at a constant temperature of $T = 37°\text{C}$. The question we ask is: Why are we most comfortable at an environment of $T_c = 295$ K to 300 K but find temperatures approaching our stable equilibrium difficult to bear? What is responsible for this asymmetry? Our source of constant energy input T_h is the sun, the food we eat and the air we breathe. With the environment at a temperature $T_c < T$, we are effectively P-controlled, thereby inhibiting the expansive E effects responsible for minimum order. With increasing T_c, the level of discomfort increases because the contribution of E increases relative to P, with the second law gradually asserting itself, until at $T_c = T$ we are at the reversible value $\alpha = \infty$ having fully integrated with the environment. However, $\alpha = \infty \backsim -\infty$ as $\iota(\infty) = \iota(-\infty)$ and the system undergoes a transition to T_- operating in a completely different environment between 0 and T_c.

Although it is possible to establish an overall correspondence between the dynamics of discrete and continuous systems [7], a careful consideration reveals some notable fundamentally distinctive characteristics between the two, which ultimately depends on the greater number of space dimensions available to the differential system. This has the consequence that continuous time evolution governed by differential equations is well-defined and unique, unlike in the discrete case when ill-posedness and multifunctionality form its defining character with the system being severely restrained in its manifestation, not possessing a set of equivalent yet discernible possibilities to choose from. It is our premise that the "kitchen of Nature" functions in a one-dimensional iterative analogue, not just to take advantage of the multiplicities inherent therein, but more importantly to format its dynamical evolution in a hierarchical canopy in the process, so essential for the evolution of an interactive, non-trivial, complex structure. The three-dimensional serving space of the real world only provides a convenient and attractive presentation of Nature's produce of its uni-dimensional kitchen.

3. Quantum non-locality, complex holism and the "participatory universe"

Possibly the most primitive of the tensions between classical and quantum perspectives of Nature is the conflict between the Copenhagen interpretation pioneered by Bohr and Heisenberg that regards quantum mechanics to be intrinsically about awareness, observations, and measurements emanating from the unitary evolution of the Schrodinger equation with precious little to offer on what it really is *ontologically,* or what in fact it seeks to describe,[4] and the deterministic world-views as advanced for example by Bohm [1] and 't Hooft [10]. According to this doctrine, the wavefunction is simply an auxiliary mathematical tool devoid of any physical significance, whose only physical import lies in its ability to generate probabilities: it compactly represents our knowledge of the preparation and subsequent evolution of a physical system. Niels Bohr persisted that only experimental results lie in the purview of physical theories; any ontological questions are unscientific and must be eschewed. The $|\Psi\rangle$ function has only a "symbolic" significance in associating expectation values with dynamic variables and does not represent anything real; according to Bohr, who lays defining significance to it, the imaginary component in the state variable does not allow it to pictorially represent the real world. The quantum "object" defined by the state function $|\Psi\rangle$ is distinct from the classical "measuring device" and the association of these complementary classical concepts to the measurement process depends on the specific experimental "context" of the phenomena. These mutually exclusive experiments on the complete setup of the quantum and classical components provide an exclusively exhaustive objective knowledge of the system in terms of a complete set of orthogonal projectors $\{P_i\}$. The dynamics of the Schrodinger equation describes how the knowledge of the system changes as a function of time. In this section we interpret this global view of the quantum–classical coexistence in the context of our negworld-world paradigm.

Entanglement. Any $|\Psi\rangle_{SE} = \sum_{i,j} \alpha_{ij} |\phi_i\rangle \otimes |\psi_j\rangle \in \mathcal{H}_S \otimes \mathcal{H}_E$ that cannot be factored as a tensor product of vectors of its parts $\{|\phi_i\rangle\} \in \mathcal{H}_S$ and $\{|\psi_j\rangle\} \in \mathcal{H}_E$, that is $|\Psi\rangle_{SE} \neq |\phi\rangle \otimes |\psi\rangle$, is said to be ***entangled*** *(non-local);* $|\Psi\rangle_{SE}$ is ***unentangled*** *(separable)* if it is factorizable into its components.

Thus for physically separated S and E, a measurement outcome of $|0\rangle$ on S implies that any subsequent measurement on E in the same basis will always yield $|0\rangle$. If $|1\rangle$ occurs in S, then E will be guaranteed to return $|1\rangle$; hence system $|E\rangle$ has been altered by local random operations on $|S\rangle$. This non-local puzzle of entangled quantum states – the orthodox Copenhagen doctrine maintains that neither of the particles possess any definite position or momentum before they are measured – is resolved by bestowing quantum mechanics with non-local properties determined by Bell's inequality.

In the linear setting of quantum mechanics, multipartite systems modeled in 2^N-dimensional tensor products $\mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_N$ of two-dimensional spin states, correspond to the 2^N "dimensional space" of unstable fixed points of chanoxity, see Sengupta [7, page 336]. This formal equivalence illustrated in Fig. 3 while clearly demonstrating how holism emerges in 2^N-cycle complex systems, also focuses on the significant differences between complex holism and quantum non-locality that can eventually be traced to the constraints of linear superpositions. The converged holistic behaviour of the "entanglement" reflects the fact that the subsystems have combined non-linearly to form an emergent, self-organized system – denoted by the heavy 2^1 and 2^2 cycles in Fig. 3(a) and (b) – that cannot be decoupled without destroying the entire structure itself; in contrast with the quantum concept of entanglement and the notion of partial tracing for obtaining properties of individual components from the whole. Periodic cycles are the "eigenfunctions" of the *generalized non-linear eigenvalue equation* $f^{(p)}(x) = x$ with "eigenvalue" p; unlike the linear case, however, these composite cycles are not linearly superposed but appear as emergent, self-organized, holistic entities. ***In this respect complex holism represents a stronger form of "entanglement" than Bell's non-locality***.

A *thermodynamic analogy of entanglement* has been proposed (Horodecki et al. [5]) as a formal equivalence with Eq. (5). This allows us to define an irreversibility of entanglement and to treat it in the engine–pump perspective. However, this linear non-local resource cannot match the holistic outreach of non-linear complexity: the indicator of

[4] "It seems clear that quantum mechanics is fundamentally about atoms and electrons, quarks and strings, not those particular macroscopic regularities associated with what we call *measurements* of the properties of these things. But if these entities are not to be somehow identified with the wave function itself then where are they to be found in the quantum description?" Goldstein [3].

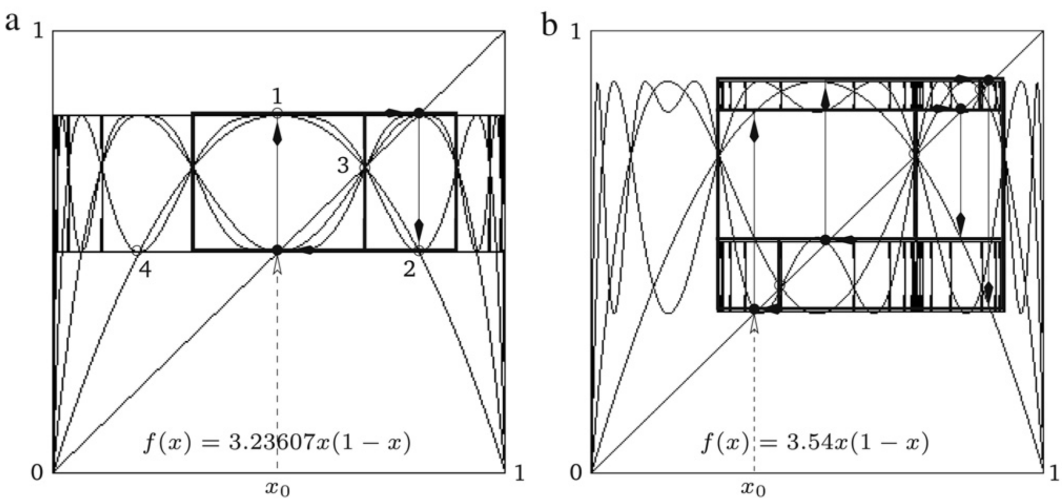

Fig. 3. "Entanglement" and "non-locality" in discrete dynamical systems. Panels (a) and (b) demonstrate the increasing complexity of evolution with increasing λ: the significant point is that the stable dynamics generated by the respective emergent 2^N-periodic cycle display "entanglement" of the L and R components that spreads out in space and time as the system self-organizes itself. Thus the parts surrender their individuality to the holism induced by the periodic cycles. The model $1 \to 2, 2 \to 1, 4 \to 1, 3 \to 3$ shows that "if information is allowed to dissipate we have to treat the equivalence classes of states as the basis of a quantum Hilbert space", 't Hooft [10], see Section 4.

entanglement r at most as large as the smaller of the dimensions of the entangling Hilbert spaces compares with the exponentially increasing 2^N degrees of freedom available to the emergent holistic patterns. With non-locality it is possible to predict with certainty the outcome of a separated distant partner through observations made on the other; it cannot self-generate patterns or structures, it cannot teleport.

The maximally ill-posed multifunctional limit of the time-irreversible logistic difference equation can be correlated with evolutions of the time reversible Schrodinger equation: non-locality and entanglement correspond to the graphically converged 2^N periodic cycles; qubits and "contextual objectivity" of quantum states endowed with objective reality only in the context of observed classical macro-experiments to non-injectivity of the positive–negative slope curve, compare Ref. [4] with the interpretation of the negworld in inducing competitively collaborating homeostasy between the two worlds as outlined above. Quantum non-locality is a natural consequence of quantum entanglement that assigns multipartite systems with definite properties at the expense of the individual constituents thereby rendering it impossible to reconstruct the state of a composite from knowledge of its parts.

The case for chanoxity is quite similar in the assignment of an effective power law $f(x) = x^{1-\chi}$ to a complex system. Most significantly, as our calculations in Sengupta [7] demonstrate, the dynamics of the logistic map undergoes a dramatic discontinuous transition from the monotonically increasing $0 < \chi \leq 1$ in $3 < \lambda \leq \lambda_*$ of region (10c) to a disjoint different world at $\chi = 0$ in the fully chaotic range $\lambda_* \leq \lambda$, (10d). This reduces the chaotic world to one of effective linear simplicity and suggests integration of quantum mechanics with chanoxity by identifying the average value $\langle x \rangle$ with the dimension of the resulting Hilbert space leading to the

Quantum Mechanics is an effective linear representation of the fully chaotic, maximally ill-posed negworld that manifests itself only through a bi-directional, contextually objective, inducement of the real world in adapting to the demands of the Second Law of Thermodynamics. The opposites of the (pump) preparation of the state and the subsequent (engine) measurement acting in collaboration defines the holistic contextual reality of the present.

The extreme values of $\chi = 0$ and $\chi = 1$ are especially interesting: the effective linearity of the former implies the zero evolution of $f(x) = x$ while maximal non-linearity of the latter generates maximal irreversible transformation in the system, $f(x) = 1$. In $3 < \lambda \leq \lambda_*$ therefore the correspondence $\chi = \iota$ between chanoxity and irreversibility allows complexity to be typically a manifestation of the irreversibility of a thermodynamic system in its surroundings.

4. Discussions and conclusion

For the interested reader, we recount here the approach of Gerard 't Hooft who considers quantum mechanics and gravity from the point of dissipative dynamical systems. The principal feature of the 't Hooft analysis of deterministic systems ('t Hooft [10]) is the identification of equivalence classes of non-invertible, non-injective maps of the type we consider to constitute a base of the quantum Hilbert spaces. As a specific example, he considers the equivalence classes $[1] = \{1\}$, $[2] = \{2, 4\}$, $[3] = \{3\}$ generated by the points 1, 2, 3, 4 of Fig. 3(a) under the mapping $1 \to 2 \to 1$, $3 \to 3$, $4 \to 1$ to define the two-dimensional quantum space spanned by $\{([1] + [2])/\sqrt{2}, [3]\}$. The significance of this deterministic construction of quantum mechanics is information loss through dissipation: the information content in the primordial ontological states $\{1, 2, 3, 4\}$ is dissipated in the smaller number of equivalence classes: "equivalence classes tend to form discrete quantum sets only if one allows information to dissipate". The stable 2^N limit cycles of our complex states of the referenced figure represent the discrete, finite set of equivalence classes of quantum states possessing the same distant future in the 't Hooft universe. In this setting, with black holes representing massive sets of equivalence classes in extreme situations of information loss, the smaller number of classes compared to the primordial states is thought to explain how the entropy of a black hole is proportional only to its area rather than to the volume that depends on the number of ontological states.

The binary two-spin system, a perfect analogue of unimodal maps with rising and falling branches, makes the correspondences sketched above appear naturally intuitive. The main issue between them however largely revolves around their very basic premises; while one is based in the linearity of superpositions and tensor products to generate non-locality, the other is a strongly non-linear discrete system of equivalence classes and ill-posedness increasingly evolving into the future. This antithetical difference in the structure of the systems – between smoothness, continuity, reversibility and regularity on the one hand and singularity, jump, irreversibility and discontinuity on the other – warrants the search for a possibly more general foundation of the simple, especially in view of the current interest in the complexity approach to Nature. Abandoning the severe restrictions imposed by linearity also helps toward a natural resolution of many of the long-standing paradoxes associated with it; thus, for example, as multiple degeneracy is a natural consequence of non-injective ill-posed induced equivalence classes, preparation-measurements now turn out to be paradox-free because experimental contextuality directly follows from the Axiom of Choice.

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