Additional Problems

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The solution to these problems are available along with the solution manual.

Chapter 2 : Channel flow

Problem 1. Water with viscosity, $\mu = 1 \times 10^{-3}$ kg/m.s flows in a capillary of 8 μm diameter and 5 cm long under pressure gradient, $\Delta P = 0.005$ Pa. The capillary is hydrophobic having slip length equal to 100 nm. Calculate the flow rate through the capillary in zepto-liter/sec.

Problem 2. Two parallel plates with momentum accommodation coefficient, σ_v are separated by distance, h. The bottom plate is stationary and the top plate is dragged at constant velocity, U. The mean free path of air between two plates, $\lambda = 10\,\mu m$, $h = 100\,\mu m$, $\sigma_u = 0.7$ and $U = 1\,mm/sec$.
(a) What is the flow rate per unit width of the plate in $\mathrm{m^3/sec}$?
(b) What is the slip velocity at the lower wall in μm/sec ?
(c) What is the slip velocity at the upper wall in μm/sec ?

Problem 3. A fluctuating pressure of $\Delta P = \Delta P_0(1 + \cos wt)$ is applied across a flexible silicon tube of diameter, d = 1 mm and length, L= 10 cm. The hydraulic capacitance of the channel, $C_{hyd} = 0.245 \times 10^{-9}\,\mathrm{m^3/Pa}$. Calculate the average flow rate of water from the capillary at actuation frequency, f= 100 Hz, and 0.1 Hz. The $\Delta P_0 = 10,000$ Pascal. $\mu_{water} = 1 \times 10^{-3}$Pa.s

Problem 4. Air flow in a rectangular cross section capillary of height, h = 1 mm, width, w = 10 mm and length, L = 10 mm at an average flow rate of $15.11 \times 10^{-8}\,\mathrm{m^3/s}$. Air flows in a second capillary of height , h =100 μm, width, w = 2 mm and length, L =10 mm at the same flow rate as that of the first capillary. Which capillary will satisfy the negligible inertia force or translation invariance criteria of the velocity profile ? Justify your answer. The viscosity of air is equal to $15.11 \times 10^{-6}\,\mathrm{m^2/s}$

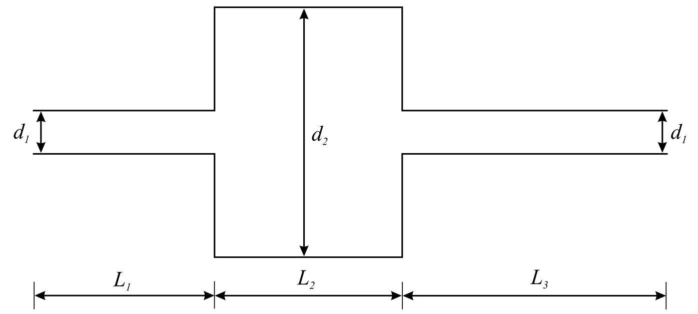
Problem 5. Calculate the pressure drop for the piping circuit below with three different cases of pipe length and size. The working fluid is air with viscosity $\nu = 15.11 \times 10^{-6}\,\mathrm{m^2/s}$.

Case 1:
d_1 = 2 mm, L_1 = 5 cm, d_2 = 6 mm, L_2= 5 cm and L_3 = 10 cm, $Q = 2.4 \times 10^{-6}\,\mathrm{m^3/s}$.

Case 2:
d_1 = 500 μm, L_1 = 5 cm, d_2 = 1.5 mm, L_2= 5 cm and L_3 = 10 cm, $Q = 2.4 \times 10^{-8}\,\mathrm{m^3/s}$.

Case 3:
d_1 = 500 μm, L_1 = 5 cm, d_2 = 1.5 mm, L_2= 5 cm and L_3 = 10 cm, $Q = 0.6 \times 10^{-8}\,\mathrm{m^3/s}$.

Problem 6. An air bubble of 2 mm long and 100 μm diameter entrapped in a micro-capillary of 100 μm diameter and 10 mm long is suddenly exposed to an over-pressure of 10 pascal. Calculate the rise in air bubble pressure after time elapsed, t= 1 μsec, 5 μsec and 10 μsec. The initial pressure is atmosphereic, $P_a = 1.01 \times 10^5$ Pa. The viscosity of air, $\mu_a = 1.81 \times 10^{-5}$ Pa-s.



Chapter 3 :Transport Laws

Problem 1. A steady flow of an incompressible fluid with constant property (μ, σ) at low Reynolds number takes place inside a large aspect ratio ($L/R \gg 1$) micro tube of radius, R with pressure gradient, $\frac{dP}{dx}$ imposed along the tube. The Knudsen number ($Kn = \lambda/2R$) is in between 10^{-3} and 10^{-1} indicating the slip flow regime.

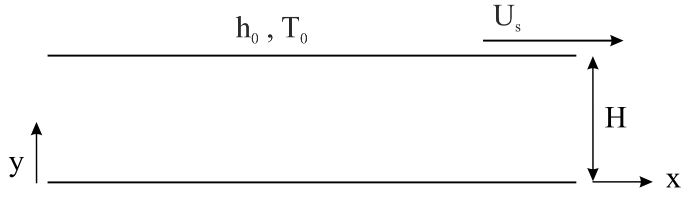
(a) Determine the velocity distribution ($u(r)$) using 1st order slip flow boundary condition.
(b) Caculate the ratio of the mass flow rate with slip flow boundary condition and without the slip flow boundary condition $\left(\frac{\dot{m}_{slip}}{\dot{m}_{no-slip}}\right)$ at Kn = 0.1 and 0.001. Use momentum accommodation coefficient, $\sigma_v = 0.8$.

Problem 2. Micro-couette flow of air takes place due to movement of a flat plate located at separation distance, h = 500 μm from the bottom wall with velocity, U = 5 m/sec. The momentum accommodation coefficient between wall and air, $\sigma_v = 0.7$. The mean free path of air, $\lambda = 50\,\mu$m and viscosity of air, $\mu = 20 \times 10^{-6}$ Pa-s.

(a) Calculate the slip velocity in m/s at the bottom wall.
(b) Calculate the slip length in micron at the bottom wall.
(c) Calculate the shear stress at the bottom wall.
(d) What will be the shear stress at the bottom wall using no-slip boundary condition ? Compare with the shear stress results in (c) and discuss your results.

Problem 3. Water ($\rho = 1000\,\mathrm{kg/m^2}$, $\mu = 8.9 \times 10^{-4}$ Pa.s) flows through a 10 cm long micro channel having equilateral triangular cross section of side, $a = 50\,\mu$m under the pressure difference of 100 Pa. What is the flow rate in through the micro channel ?

Problem 4. A micro air bearing is approximated as a Couette flow with separation distance, H and velcoity of the top surface, U_s as shown in the figure below.



(a) Calculate the velocity of the fluid at the top surface when U_s = 1 m/s, H = 500 μm, σ_v = 0.5, Kn = 0.1.

(b) Calculate the slip velocity at the bottom surface.

(c) Calculate the flow rate per unit length in m^2/s for the condition given in part (a).

Chapter 4 : Diffusion, Dispersion and Mixing

Problem 1. For a dilute gas (Schmidt no (Sc) = 1) flowing at a free stream velocity, U over a reacting flat plate with C_w being the solute concentration at the wall and C_0 the solute concentration at the edge of the diffusion boundary layer, show that mass transfer coefficient (C_{diff}) is 0.5 times the skin friction coeffiecient (C_f).

Problem 2. **(1)** A single species solute is supplied at a constant flux i_w^*(mol/m^2 − s from the wall of a long cylindrical tube of radius $'a'$. Liquid with zero initial solute concentration enters the inlet of the tube. The velocity profile is fully developed at the entrance of the tube and is given by

$$u = 2\bar{u}\left(1 - \frac{r^2}{a^2}\right)$$

Where, 'r' is the radial coordinate measured from the tube centre and $\bar{u}$ is the mean fluid velocity. Prove that the fully developed concentration profile is given by

$$c(x,r) = \frac{2j_w^* x}{\bar{u}a} + \frac{j_w^* a}{4D}\left[4\left(\frac{r}{a}\right)^2 - \left(\frac{r}{a}\right)^4\right]$$

where, 'x' is the distance from the origin of the axial coordinate, where the concentration profile becomes fully developed and 'D' is the diffusion coefficient.

(2) Find out the Sherwood number for the above problem defined by:

$$Sh = \frac{2aj_w^*}{D(C_w - C_m)}$$

where, C_w is the concentration at the wall and C_m is the bulk average or mixing-cup concentration defined by:

$$C_m = \frac{1}{\pi a^2 \bar{u}} \int_0^a uC \cdot 2\pi r dr$$

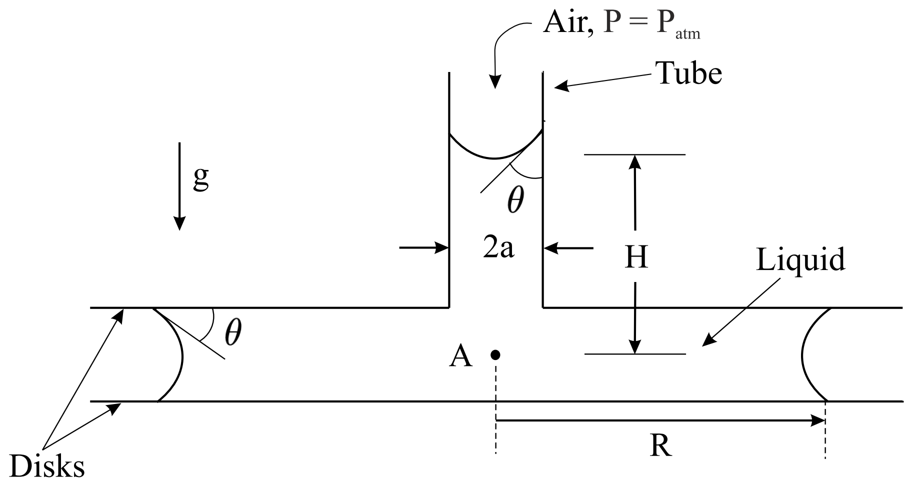
Chapter 5 : Surface Tension Dominated Flows

Problem 1. A capillary pump supplies biochemical fluid from two reservoirs to the measuring site through two micro-channels. One reservoir is located at 5 mm and another reservoir is located at 15 mm from the measuring site. The height of the micro channel is 20 μm and the width is 200 μm. The channel is made of PMMA and the surface tension between the liquid and vapor interface is 72.9 mJ/m^2. The contact angle at the liquid-solid-air contact line is 73.7 deg. The viscosity of the fluid is 10^{-3} N-s/m^2.
(a) Calculate the arrival time for both the liquid advancing from the source reservoirs to the measuring site.
(b) Calculate the arrival time if the walls of the capillary pump are changed to platinum where the contact angle between liquid-air-solid is equal to 40 deg.
Note: This problem demonstrates how the design of biochip for biochemical analysis can be controlled by channel configuration, its geometry and material of the channel.

Problem 2. A volume 'V' of a liquid of density, 'ρ' and surface tension, 'σ' is contained between two parallel, concentric circular disks that are oriented horizontally and separated from each other by a distance, 'a'. A thin circular capillary tube of radius, a is connected to the upper disk at its centre and oriented vertically. The liquid level in the tube measured upward from the center of the space between the disk (a/2) to the meniscus is 'H' (the meniscus height itself is taken to be small). The distance the liquid extends radially outwards between the disks, measured from the centerline of the capillary to the meniscus formed between the disks is equal to radius, 'R' such that $R > a$ and $H > a$. The static contact angle is denoted by θ and the system is open to the atmosphere.

Assuming $\sigma/\rho g a^2 \gg 1$, obtain an implicit relation for H as a function of V, a, σ, ρ, g as :

$$\left(H - \frac{a}{2}\right) a^2 + \frac{a}{H^2}\left(\frac{\sigma}{\rho g}\right)^2 = \frac{V}{H}$$



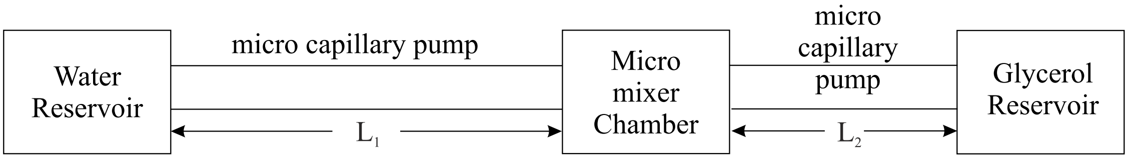
Problem 3. A container holds liquid (viscosity, $\mu = 0.001\,Pa.s$, Density, $\rho = 998\,kg/m^3$, surface tension, $\sigma = 75\,mN/m$) of initial length, $h = 150\,\mu m$. A temperature gradient ($dT/dx = 100\,k/m$) is imposed on the side wall of the container i.e. one side wall of the container is heated with respect to the other. The variation of the surface tension with respect to the temperature of the fluid is gievn by $d\sigma/dT = 0.00015\,N/m - K$. Calculate the appropriate non-dimensional numbers for verification of pure thermo-capillary motion of the fluid inside the container and discuss the implications.

Problem 4. A capillary pump consisting of a circular capillary of radius , $a = 20\,\mu m$ and length , $L = 1\,cm$ connects between two reservoirs of radius, $R = 3\,mm$. A buffer solution (viscosity, $\mu = 1$ mPa-s, surface tension, $\sigma = 0.0712$ N/m, conatct angle, $\theta = 60^0$ is trnasported from the reservoir to another. Calculate the total time required for the buffer solution to travel from one reservoir to another.

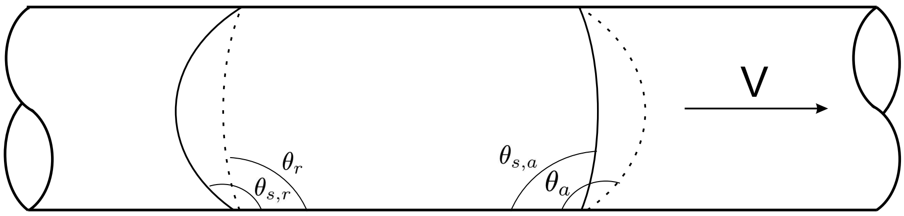
Problem 5. A microcapillary pump arrangement needs to be fabricated for pumping glycerol and water from a reservoir to a micromixer as shown below. The miro-channel used for pumping has height equal to 10 micron and width equal to 200 micron. The distance of the micromixter channel from the water reservoir, $L_1 = 1\,cm$. What should be the length, L_2 of the glycerol channel from the micromixer such that both water and glycerol reaches the micromixer at the smae time. Use the following data:

μ (glycerol) = 1.412 Pa-s, θ (glycerol) = 70 ^{0}
σ (glycerol) = 63.0 mN/m, μ (water) = 8.9×10^{-4} Pa-s

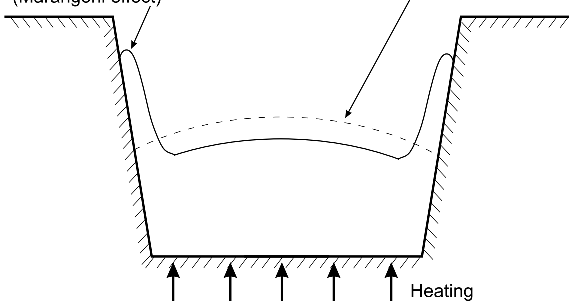
θ (water) $= 85\,^0$, $\sigma_{water} = 73.0$ mN/m



Problem 6. A static liquid plug inside a horizontal micro capillary has static contact angle, $\theta_{s,a} = \theta_{s,r} = 120^0$ as per figure below. The viscosity of the liquid, $\mu = 1\,mPa - sec$. The surface tension, σ_{lg} is equal to 0.0712 N/m. The capillary is given an orientation leading to a constant velocity of the liquid plug i.e. $V = 6.53\,m/s$. Calculate the contact angle of the advancing (θ_a) and receding (θ_r) font of the liquid plug. Assume the validity of Tanner's law with constant, A= 30.



Problem 7. Consider a microwell for DNA analysis having radius of 200 micron as shown below. A sample is introduced into the microwell resulting in height of the sample equal to 100 micron. the center of the well has temeprature of 25 ^{0}C and the side wall has temperature of 20 ^{0}C corresponding to surface tension of 72.1 mN/m and 72.9 mN/m respectively. What will be the height of the sample at the wall ? Density of the sample solution is equal to 950 kg/m^3.



Problem 8. (a) Flow of water takes place inside a capillary of 50 mm length and 100 μm dia at a flow rate of $10^{-9}\,m^3/sec$. Calculate the pressure drop across the capillary.

(b) If 50 air bubbles of 100 μm diameter gets entrapped inside the capillary, find out the pressure drop for the same water flow rate through the capillary. Assume the total length of both water plug and air bubbles remains same i.e. 50 mm.

Use the following data:

The static contact angle of air water interface inside the capillary (θ_s): 60^0 deg.
Viscosity of air (μ_a): 1.85×10^{-5} Pa-s
Viscosity of water (μ_w): 8.90×10^{-4} Pa-s
Surface tension between air-water (σ): 72.8×10^{-3} N/m

Tanner,s law constant: 30

Problem 9. A reservoir ($2mm \times 2mm \times 2mm$) containing blood sample is connected to two other similar reservoirs containg water and ethanol by micro capillary of 50 μm height and 500 μm width.

The length of the micro capillary connecting the blood sample reservoir and water sample reservoir is equal to 10 mm. What should be the length of the capillary connecting the blood sample reservoir and the ethanol reservoir such that both ethanol and water reaches the blood sample reservoir at the same time after initiating the capillary flow?

Viscosity of ethanol: $1.074 \times 10^{-3} Pa - s$
Viscosity of water: $8.90 \times 10^{-4} Pa - s$
$\sigma_{water-air} = 72.8 \times 10^{-3} N/m$
$\sigma_{ethanol-air} = 21.4 \times 10^{-3} N/m$
Contact angle for water-air interface : 85^0
Contact angle for ethanol-air interface: 30^0

Chapter 6 : Charged Species Flow

Problem 1. (a) An electro-osmotic micro pump is made of cylindrical channel with radius 'a' and length 'L'. The flow is induced by an external potential difference 'ΔV'. Calculate the electro-osmotic flow rate, Q (nano-liter/sec) assuming small Debye layer ($\lambda_D/a < 1$) and using the folowing data:

$\xi = 0.1V,\ a = 10\mu m,\ l = 100\mu m,\ \mu = 0.001 Pa - sec$

ϵ(permittivity) $= 0.78 \times 10^{-9} coul^2/N - m^2, \Delta V = 1V$

(b) What is the back pressure needed at the end of the channel (micro-pump) to exactly cancel the electro osmotic flow ?

(c) The solution in (a) will indicate that the electro-osmotic flow rate is not very high. One approach to enhance the flow rate is to construct a large number of narrow channels in parallel coupling. The usual pumping rate requirement in micro fluidics is about 1.0 liter/sec. How many micro channels need to be connected in parallel for the pumping requirement of 0.1 μ-liter/sec ?

Problem 2. An electro osmotic micro pump consists of 1000 number of micro -tubes (diameter (d) = 10 μm, length (l) = 100 μ m) connected in parallel and pumps against a pressure difference of 10.0 Pa. Assuming small Debye layer with respect to the tube diameter, determine the flow rate (nano-liter/sec) through the pump. Use the following data:
Zetal potential (ξ) = 0.1 volt.
Applied electrical potential difference (ΔV) = 1.0 volt.
Viscosity of fluid (μ) = 0.001 Pa-sec
Permeability (ϵ) = $0.78 \times 10^{-9} coul^2/N - m^2$

Problem 3. Calculate the electrophoretic velocity of sodium and potassium ion under an external electric field of 1 volt/cm using the following data:
μ (viscosity of fluid) = 1 mPa-s
μ_{ion} (ioninc mobility of sodium) = $5.19 \times 10^{-8} m^2 - V/s$
μ_{ion} (ioninc mobility of potassium) = $7.62 \times 10^{-8} m^2 - V/s$
Based on the above calculation, which ion will be sensed first by the CCD camera of a capillary electrophoresis system?

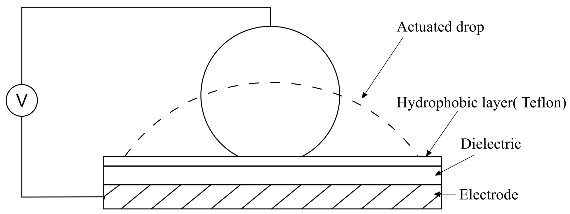
Problem 4. Consider an electro-osmotic flow inside a glass capillary flowing against a pressure difference of 0.2 KPa with the following data: D(capillary dia) 10 μm, ξ (zeta potential of capillary wall) = 0.1 volt, L (Length of capillary) = 500 μ m, μ (Viscosity of fluid) = 1.0 mPa-s, ϵ (permittivity) $= 0.78 \times 10^{-9} coul^2/N - m^2$, external electrical potential difference (V) = 20.0 volt. What is the velocity of fluid at the center of the capillary ?

Problem 5. Calculate the electrical conductivity (in S/mol-m) of NaCl solution with concentration, $C = 10^{-4}\, mol/m^3$.

Use the following data:
Faraday's constant: $9.65 \times 10^4 coul/mol$
Mobility of Na^+ ion : $5.19 \times 10^{-8} m^2/V - s$
Mobility of Cl^- ion : $7.91 \times 10^{-8} m^2/V - s$

Problem 6. The water with 0.3 % by weight of NaCl concentration has electrical conductivity equal to $5.69 \times 10^{-3} S/cm$. What is the NaCl concentartion of water with electrical conductivity equal to $18.9 \times 10^{-3} S/cm$?

Problem 7. A liquid droplet (permittivity, $\epsilon = 500 \times 10^{-10} CV^{-1}m^{-1}$) is placed on an insulator surface (dielectric) of thickness, d = 0.5 mm as shown in Figure below. The hydrophobic teflon coating layer has negligible thickness. The surface tension of the drop arrangement is given as $\sigma_{sl} = 0.029 N/m$, $\sigma_{sg} = 0.01 N/m$, $\sigma_{lg} = 0.0712 N/m$. Here, subscript s, g and l stands for solid, gas and liquid respectively. A voltage, V of 40 volt is applied across the droplet. Calculate the contact angle before and after the actuation.

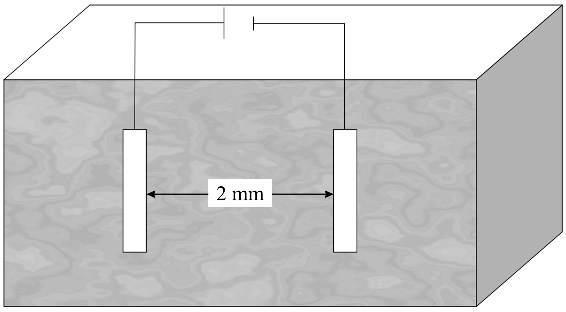


Problem 8. In a capillary electrophoresis system, a photodiode is mounted on the top of the micro channel at 10 mm from the entrance. The solution is exposed to the electric field of 100 volt/m. At what time after actuation of the electrode will a 5 5 μm diameter particle be observed by the photodiode? Assume the electric double layer thickness to be much smaller than the diameter of the particle.

Use the following data:
Viscosity of fluid (μ) $= 1 \times 10^{-3} Pa - s$
Permeability (ϵ) $= 0.78 \times 10^{-9} coul^2/N - m^2$ Zetal potential (ξ) = 0.1 volt.

Problem 9. Two electrodes are immersed in an electrolyte solution with separation distance of 2 mm between them. The permittivity of the electrolyte is equal to $0.78 \times 10^{-9} coul^2/N - m^2$ and conductivity is equal to 10^4 S/m. The Debye layer length, λ_D is equal to 10 nm for DC applied voltage across the electrode. If AC voltage is applied across the electrode instead of DC, calculate the maximum frequency after which the applied electric voltage will not allow the formation of Debye layer.

Problem 10. A glass sphere of 200 micron diameter floating inside a capillary filled with water has a double layer thickness of 20 nm. An electric field of 100 volt/m is applied across the capillary. What is the electrophoretic velocity of the glass sphere?



Viscosity of water (μ_w): $8.90 \times 10^{-4} pa - s$.
Electric permittivity of water (ε): $0.78 \times 10^{-9}\, coul^2/N - m^2$
Zeta potential ($zeta$): 20 mV

Problem 11. An electro-osmotic micro pump is fabricated by stacking 400 number of micro capillary inside the first half of a capillary. The capillary is 4 cm long and its diameter is equal to 2 mm. The diameter of the micro capillary is equal to 100 μm. A 5 volt potential difference is applied across the first half of the capillary and 5 volt potential difference is applied across the second half of the capillary in opposite direction such that the net potential difference across the capillary is equal to zero.

What is value of Q_{eo} and P_{eo} if the potential of the channel wall is equal to 25 mV?

Viscosity of water (μ_w): $8.90 \times 10^{-4} pa - s$.
Electrical permittivity of water (ϵ): $0.78 \times 10^{-9}\, coul^2/N - m^2$

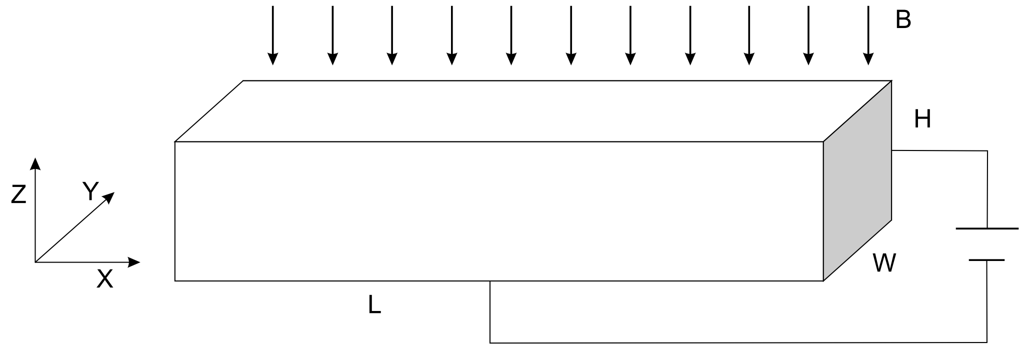
Chapter 7 : Magnetism and Microfluidics

Problem 1. A magnetic field flow fractionation (MFFF) unit consisting of a square channel of $500\mu m \times 500\mu m$ cross section is used to separate magnetic beads of different sizes. Magnetic beads of 1 micron and 10 micron diameter submerged in water flow inside the MFFF unit at a flow rate of 0.1 m^3/s . The 1 μm sized beads get deposited at a distance of 5 mm from the entrance of the channel. Calculate the distance at which the 10 μm particle will be deposited. The magnetic susceptibility of water and bead is equal to -9.035×10^{-6} and 3500 respectively.

Problem 2. A DC MHD micro pump uses a rectangular channel of length, L with large aspect ratio (Width, W $\gg$ Height, H). The current flow through the planar electrode is equal to 0.1 amp. The pump has to work against a pressure head (ΔP) equal to 3000 pascal. The magnetic flux density, B is equal to 1.0 tesla (kg/amp-sec^2).
Calculate:
(i) the velocity of the fluid in m/sec at the center (L/2, W/2, H/2) of the channel.
(ii) the total flow rate (m^3/s).
The viscosity of the pumping liquid is equal to 6×10^{-4} pa-s. The channel dimension is given as: L = 50 mm, W = 1000 μm and H = 100 micron.

Problem 3. Water flows inside a micro channel of 100 μm height (h) and 1000 μm width (w) at Raynolds number based on channel height equal to 0.5. Mixture of iron oxide and platinum particles is released at the top entrance region of the micro channel (x=0, y = h) with negligible initial velocity. An external magnetic field is applied perpendicular to the flow direction. Diameter of both iron oxide and platinum particles is equal to 2 μm. If iron oxide particles are deposited at 1 cm from the entrance of the channel, calculated the distance at which the platinum particles will be deposited. Neglect effect of gravity.

Magnetic susceptibility of iron oxide: 720×10^5
Magnetic susceptibility of platinum : 26×10^5
Magnetic susceptibility of water: -9.05×10^{-6}



Chapter 8 : Microscale Conduction

Problem 1. (a) A thin film of thickness, h = 1 μm, made from silicon initially at atmospheric temperature, T_0 is exposed to a higher temperature, T_w. Write down the governing equation and boundary condition for determination of the instantaneous temperature distribution inside the thin film. The mean free time, $\tau = 10^{-14}$ sec and thermal diffusivity, $\alpha = 95 \times 10^{-6}\, m^2/s$ for silicon.

(b) Write down the governing equation when the thickness of the film is equal to 0.005 μm.

Problem 2. The second speed of sound in liquid helium at 1.8 K is equal to 1.8 m/s. The density, thermal conductivity and specific heat at 1.8 K temperature is equal to 0.145 gm/cm^3, 26.402 W/cm-K and 2.81 J/gm-K respectively. calculate the relaxation time of liquid helium at 1.8 K.

Problem 3. A gas trubine combustion chamber is coated with zirconia of 50 nm thickness. the gas temperature inside the combustion chamber is equal to 1600 ^{0}C. The heat loss from the combustion chamber wall is $10^6 kW/m^2$. the combustion chamber wall is made of steel and 20 mm thick. the thermal conductivity of steel and zirconia is respectively equal to 19 W/m-K and 1.0 W/m-k. Calculate the maximum temperature of the combustion chamber wall assuming validity 0f 1D conduction. Assume the mean free path of path of phonon in zircnia is equal to 0.02 micron.

Problem 4. (a) A Bismuth thin film of 1 μm thickness is exposed to a heat pulse. The thermal conductivity and diffusivity of Bismuth is respectively equal to 12.49 w/m-k and 8.55 mm^2/s. The relaxation time of Bismuth is equal to 4.7×10^{-13} second. The mean free path of conduction in Bismuth is equal to 3 μm.

(i) What is the thermal wave speed in Bismuth ?

(ii) Will Fourier modeling approach be valid for heat conduction in Bismuth?

Chapter 9 : Microscale Convection

Problem 1. Consider a couette flow with convective heat transfer from the top moving surface. The relevant data for the problem are as follows:

$U = 1\, m/s,\ \rho = 1.15\, kg/m^3,\ C_p = 1.005\, KJ/kg - K$

$h_0 = 0.5\, W/m^2 - K,\ T_\infty = 20^0 C,\ k = 1\, W/m - k$

$\beta_u = 0.5,\ \beta_T = 2.0,\ Kn(\lambda/H) = 0.1$

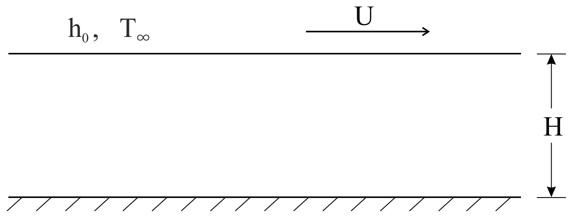
$H = 100\, \mu m,\ \mu = 1.98 \times 10^{-5}\, Pa - s$

Velocity distribution between the two plates is given as:

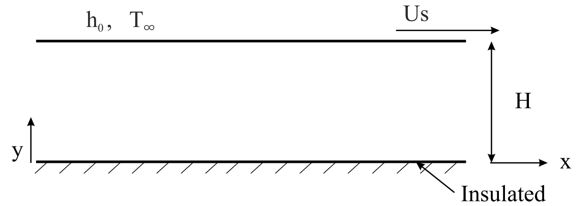
$$\frac{u}{U} = \frac{1}{1 + 2\beta_u Kn}\left(\frac{y}{H} + \beta_u Kn\right)$$

(a) Derive the expression for temp distribution.

(b) What is the surface temperature of the top plate for the above condition ?



Problem 2. A micro air bearing is approximated as a Couette flow with separation distance, H and velocity of the top surface, U_s as shown in the figure below.



(a) Calculate the temperature of the top plate, T_s. Assume the dominance of viscous dissipation and convection heat transfer taking place from the top plate with convective heat transfer coeffiecient, h_0. The bottom plate is insulated.

(b) Calculate the temperature of air adjacent to the top plate, at y=H and bottom plate at y=0.

Use the following data:
$U_s = 10\,m/s,\ H = 100\,\mu m,\ \sigma_v = 0.5,\ Kn = 0.1,\ \sigma_T = 0.5,\ k_{air} = 10^{-2}\,W/m-k,\ h_0 = 12.7\,W/m^2-k,\ T_{ambient} = 30^0C,\ \nu = 1.4,\ Pr = 0.7,\ \mu_{air} = 1.81 \times 10^{-5}\,pa-s.$

Problem 3. Derive the expression for transitional diameter for transition from macro-channel to micro-channel two phase flow as:

$$D_{trans} = \frac{160}{9}\frac{(\sigma\rho_f - 3\mu_f G^2)}{G^2}$$

Note that the drag coefficient on a sphere is given as:

$$C_D = \frac{24}{Re_{trans}}\left(1 + \frac{3}{160}Re_{trans}\right)$$

(b) Calculate the transitional diameter for two phase flow of water at mass velocity, G=500 kg/m^2-s. the viscosity of water, $\mu_f = 2.83 \times 10^{-4}\,kg/m-s$, the surface tension of water air interface, $\sigma = 73\,mN/m$ and the density of water, $\rho_f = 1000\,Kg/m^3$.

Chapter 10 : Microfabrication

Problem 1. A spin coating process can be modeled as the balance between viscous force ($\mu\frac{\partial^2 v}{\partial z^2}$) and centrifugal force ($\rho w^2 r$) in (r, θ, z) coordinate, where ρ is the density and μ is the viscosity of fluid being spin coated. Develop an expression for the velocity profile (V) during the spin coating process as a function of ρ, μ w, r, h and z, where h is the height of the spin coated surface.

Chapter 13 :Heat Pipe

Problem 1. A heat pipe operates under the capillary limit with the following specification:
Liquid pressure drop $= 5.45 \times 10^{-3} L_{\text{eff}} q \, N/m^2$
Vapor pressure drop $= 2.45 \times 10^{-3} L_{\text{eff}} q \, N/m^2$
Length of evaporator section = 0.1 m
Length of adiabatic section = 0.3 m
Length of condenser section = 0.1 m
Heat pipe inclination = 15 ^{0}
Capillary radius = 6.4×10^{-5} m
Vapor core diameter = 2.01×10^{-2} m
Tube inner diameter = 2.21×10^{-2} m
Surface tension coefficient = $5.84 \times 10^{-2} m$
Density of liquid = 961 kg/m^3
Here, q is the heat transfer rate in watt.

(i) What is the maximum heat transfer (q) in the heat pipe if the evaporator section is above the condenser ?
(ii) What is the maximum heat transfer (q) in the heat pipe if the evaporator section is below the condenser ?

Problem 2. The avergae capillary pressure of a heat pipe (width = 30 μm and depth = 80 μm) is equal to 6×10^3 Pa. The surface tension between the liquid and vapor is 75×10^{-3} N/m. can this be considered as a micro heat pipe ? Justify your answer.

Problem 3. For a heat pipe with the following specification, determine the capilalry limit.
Liquid pressure drop $= 5.45 \times 10^{-3} L_{eff} q \, N/m^2$
Vapor pressure drop $= 2.45 \times 10^{-3} L_{eff} q \, N/m^2$
Length of evaporator section = 0.1 m
Length of adiabatic section = 0.3 m
Length of cindenser section = 0.1 m
Heat pipe inclination = 15 ^{0}
Capillary radius = 6.4×10^{-5} m
Vapor core diameter = 2.01×10^{-2} m
Tube inner diameter = 2.21×10^{-2} m
Surface tension coefficient = $5.84 \times 10^{-2} m$
Density of liquid = 961 kg/m^3
Here, q is the heat transfer rate in watt.

What is the maximum heat transfer achievable for the heat pipe.

Problem 4. (a) A heat pipe of diameter, 0.6 cm dissipates heat at a rate of 180 W with a temperature difference of 3 ^{0}C across the heat pipe. The length of the evaporator section, L_e = 5 cm, the length of the condenser section, L_c = 5 cm and the length of the adiabatic section, L_a = 25 cm.

(i) Calculate the effective thermal conductivity of the heat pipe.
(ii) Calculate the diameter of a copper rod with same total length as that of heat pipe, which will dissipate the same amount of heat by axial conduction mode as that of heat pipe. Note that the thermal conductivity of copper is equal to 386 W/m-^{0}C.

(b) Calculate the maximum capillary pressure in kPa of a heat pipe with wick having cylinder pore of diameter equal to 200 μm. The water is the working fluid having surface tension of the wick material equal to 73 mN/m. The contact angle of the wick material with water is equal to 70 ^{0}.

Problem 5. Determine the boiling limitation, entrainment limitation and sonic limit of heat pipe using water as working fluid at 373.15 K with the following specification.

Tube outer dia = $2.54 \times 10^{-2} \, m$
Tube innner dia = $2.21 \times 10^{-2} \, m$
Vapor core dia = $2.01 \times 10^{-2} \, m$
Heat pipe inclination = 0 rad

End condenser length = 0.1 m
Adiabatic length = 0.3 m
End evaporator length = 0.1 m
Nucleation radius = $2.54 \times 10^{-7}\,m$
Effective thermal conductivity = 1.61 w/m-k
Surface tension coefficient of water = $5.84 \times 10^{-2} N/m$
Capillary radius = 6.4×10^{-5}
Vapor specific heat ratio = 1.33
Hydraullic radius of the wick structure near the wick-vapor interface = $3.225 \times 10^{-5}\,m$

Problem 6. The four major heat transport limit for a heat pipe is :
$Q_{c,max}$(capillary limit) = $153w$
$Q_{s,max}$(Sonic limit) = $1.28 \times 10^5 w$
$Q_{e,max}$(Entrainment limit) = $1013w$
$Q_{b,max}$(Boiling limit) = $65w$

What is the maximum heat transfer possible for this heat pipe?