PROBLEM FORMULATION IV

CONSIDER THE PROBLEM WHERE THE CONSTRUCTION COST OF THE ROAD IS GIVEN AS FOLLOWS:

COST OF CONSTRUCTION USING DESIGN j, $C_j(x_j)$

$$= \begin{cases} k_j + c_j x_j & x_j > 0 \\ 0 & x_j = 0 \end{cases}$$

where x_j is the length of the road constructed as per design j.

THE ABOVE COST STRUCTURE IMPLIES THAT EVERY METHOD HAS A FIXED COST (part of the cost which does not vary with quantity) AND A VARIABLE COST.

GIVEN THAT DIFFERENT DESIGN METHODS CAN BE EMPLOYED AT DIFFERENT SITES (WHICH ARE OF DIFFERENT LENGTHS) THE QUESTION IS HOW DO WE DETERMINE THE DESIGN METHODS TO BE EMPLOYED SO THAT COST IS MINIMIZED.

THE FIRST STEP IS TO REPRESENT THE TOTAL COST SUCH THAT THE TOTAL COST = SUM OF COSTS OF THE DESIGNS EMPLOYED. IN THE FOLLOWING WE TRY TO DO THIS.

FIRST ATTEMPT: MIN $Z = \sum_{\forall i} \sum_{\forall j} C_j(x_j^i)$

LET US SAY THERE TWO SITES AND THREE POSSIBLE DESIGN METHODS. FURTHER LET US SAY THAT AT SITE 1 DESIGN 2 IS USED AND AT SITE 2 DESIGN 3 IS USED. THEN THE COST SHOULD BE:

$$k_2 + c_2(x_2^1) + k_3 + c_3(x_3^2)$$

HOWEVER AS PER THE EXPRESSION OF Z THE COST IS

$$k_1 + k_2 + c_2(x_2^1) + k_3 + k_1 + k_2 + k_3 + c_3(x_3^2)$$

(note: x_1^1, x_3^1, x_1^2, x_2^2 are all zeros).

THIS SHOWS k_1, k_3, k_1 & k_2 ARE EXTRA. THEY ARE THERE BECAUSE THESE WERE INDEPENDENT OF LENGTH AND EVEN THOUGH A PARTICULAR DESIGN METHOD WAS NOT EMPLOYED (LIKE $x_1^1 = 0$) THE FIXED COST WAS ADDED.