EXAMPLE ON LAGRANGEAN

Maximize $g(x) = 3x_1x_2^2$

s.t. $h(x) = x_1 + x_2 = 3$

Naive way

$$x_1 = 3 - x_2$$

$$\therefore \quad g(x) = 3(3 - x_2)x_2^2 = 9x_2^2 - 3x_2^3$$

$$\frac{dg}{dx_2} = 18x_2 - 9x_2^2 = 0$$

$$= 9x_2(2 - x_2) = 0; \quad x_2^* = 2, \; x_1^* = 1$$

AND $g(x^*) = 12$

LAGRANGEAN

$$\mathcal{L} = 3x_1x_2^2 - \lambda(x_1 + x_2 - 3)$$

① $\dfrac{\partial \mathcal{L}}{\partial x_1} = 3x_2^2 - \lambda = 0$

② $\dfrac{\partial \mathcal{L}}{\partial x_2} = 6x_1x_2 - \lambda = 0$

③ $\dfrac{\partial \mathcal{L}}{\partial \lambda} = x_1 + x_2 - 3 = 0$

FROM ① & ②

$3x_2^2 = 6x_1x_2$

or $x_2 = 2x_1$

Using ③

$x_1^* = 1, \; x_2^* = 2, \; g(x^*) = 12$

$\lambda^* = 12$

LETS SEE WHAT HAPPENS IF THE CONSTRAINT $x_1 + x_2 = 3$ IS CHANGED TO $x_1 + x_2 = 3.1$

① & ② remain unchanged. Hence $x_2 = 2x_1$

From the new constraint $x_1 + 2x_1 = 3.1$

Hence $x_1^* = \dfrac{3.1}{3}$; $x_2^* = \dfrac{6.2}{3}$

$$g(x^*) = 3 \times \frac{3.1}{3} \times \frac{(6.2)^2}{3^2} = 13.2$$

NOTE $13.2 = 12 \times 0.1 + 12$

New optimum: 13.2

Earlier value of λ^*: 12

Increase in RHS of constraint: 0.1

Earlier optimum: 12

λ^* GIVES THE SENSITIVITY OF OPTIMUM VALUE TO CHANGES IN RHS OF CONSTRAINT