EXAMPLE USING K-T CONDITION

MAXIMIZE $g(x) = x_1 + 2x_2$

SUBJECT TO: $h_1(x) = (x_1-1)^2 + (x_2-2)^2 \leq 5$

$h_2(x) = x_1 \leq 4$

$$\mathcal{L} = x_1 + x_2 \cdot 2 - \lambda_1\left((x_1-1)^2 + (x_2-2)^2 + s_1^2 - 5\right) - \lambda_2\left(x_1 + s_2^2 - 4\right)$$

USING K-T CONDITIONS

$$0 = \frac{\partial \mathcal{L}}{\partial x_1} \Rightarrow 1 - 2\lambda_1(x_1-1) - \lambda_2 = 0 \quad \cdots\cdots \text{(A)}$$

$$0 = \frac{\partial \mathcal{L}}{\partial x_2} \Rightarrow 2 - 2\lambda_1(x_2-2) = 0 \quad \cdots\cdots \text{(B)}$$

$$0 = \frac{\partial \mathcal{L}}{\partial \lambda_1} \Rightarrow (x_1-1)^2 + (x_2-2)^2 + s_1^2 - 5 = 0 \quad \cdots\cdots \text{(C)}$$

$$0 = \frac{\partial \mathcal{L}}{\partial \lambda_2} \Rightarrow x_1 + s_2^2 - 4 = 0 \quad \cdots\cdots \text{(D)}$$

$$0 = \frac{\partial \mathcal{L}}{\partial s_1} \Rightarrow \lambda_1 s_1 = 0 \quad \cdots\cdots \text{(E)}$$

$$0 = \frac{\partial \mathcal{L}}{\partial s_2} \Rightarrow \lambda_2 s_2 = 0 \quad \cdots\cdots \text{(F)}$$

$$\lambda_1, \lambda_2 \geq 0 \quad \cdots\cdots \text{(G)}$$

If $s_2 = 0$ $(\lambda_2 \neq 0) \Rightarrow x_1 = 4$ (USING D) $\Rightarrow (4-1)^2 + (x_2-2)^2 + s_1^2 \neq 5$

$\lambda_2 = 0 \Leftarrow s_2 \neq 0 \Leftarrow$ Violation of C

FROM A or B $\lambda_1 \neq 0 \Rightarrow s_1 = 0 \Rightarrow (x_1-1)^2 + (x_2-2)^2 = 5 \quad \cdots\cdots$ (H)

From A and B and $\lambda_2 = 0$ and $\lambda_1 \neq 0$ we get $\left\{ \begin{array}{l} x_2 - 2 = 2(x_1-1) \\ x_2 = 2x_1 \end{array} \right. \cdots$ (I)

Using (H) and (I) $(x_1-1)^2 = 1 \Rightarrow x_1 = 0$ or 2.

From (A) we get $\lambda_1 = -\frac{1}{2}$ or $+\frac{1}{2}$

Since $\lambda_1 \geq 0$, hence $\lambda_1 = +\frac{1}{2} \Rightarrow x_1^* = 2$, $x_2^* = 4$

$\Rightarrow g^*(x) = 10$

Note if we used $\lambda_1 = -\frac{1}{2} \Rightarrow x_1 = 0, x_2 = 0 \Rightarrow g(x) = 0$

OBVIOUOSLY THATS NOT OPTIMAL