ME 721 Theory of Plasticity Homework 2

Problem 1: axisymmetric problem, velocity components

$$v_r = -r(Az+B) \qquad v_\theta = 0 \qquad v_z = A\left(\frac{r^2}{2}+z^2\right)+2Bz.$$

A, B constants.

a) $\dot{\epsilon}_{rr} = \dfrac{\partial v_r}{\partial r} = -(Az+B)$

$\dot{\epsilon}_{\theta\theta} = \dfrac{1}{r}\dfrac{\partial v_\theta}{\partial \theta} + \dfrac{v_r}{r} = 0 + [-(Az+B)] = -(Az+B)$

$\dot{\epsilon}_{zz} = \dfrac{\partial v_z}{\partial z} = 2Az+2B = 2(Az+B)$

$\dot{\epsilon}_{r\theta} = \dot{\epsilon}_{\theta z} = 0$

$\dot{\epsilon}_{zr} = \dfrac{1}{2}\left(\dfrac{\partial v_z}{\partial r} + \dfrac{\partial v_r}{\partial z}\right) = \dfrac{1}{2}(Ar - Ar) = 0.$

b). volumetric strain rate $\dot{\epsilon}_v = \dot{\epsilon}_{rr} + \dot{\epsilon}_{\theta\theta} + \dot{\epsilon}_{zz}$

$= -(Az+B) - (Az+B) + 2(Az+B)$

$= -2(Az+B) + 2(Az+B)$

$= 0.$

c) $d\underset{\sim}{u} = \underset{\sim}{v}\,dt$

Now $du_r = v_r\,dt$; $du_\theta = v_\theta\,dt$ $\quad du_z = v_z\,dt.$

$\therefore\ du_r = -r(Az+B)\,dt.$; $du_\theta = 0.dt = 0$; $du_z = \left[A\left(\frac{r^2}{2}+z^2\right)+2Bz\right] \times dt.$

d). The incremental strain displacement relation in cyl. coor.

$d\epsilon_{rr} = \dfrac{\partial\, du_r}{\partial r} = -(Az+B)\,dt$

$d\epsilon_{\theta\theta} = \dfrac{1}{r}\dfrac{\partial\, du_\theta}{\partial\theta} + \dfrac{du_r}{r} = 0 - (Az+B)\,dt$

$d\epsilon_{zz} = \dfrac{\partial(du_z)}{\partial z} = 2(Az+B)\,dt$

$d\epsilon_{r\theta} = d\epsilon_{\theta z} = 0$

$d\epsilon_{zr} = \cancel{\frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right)} = \dfrac{1}{2}\left(\dfrac{\partial(du_z)}{\partial r} + \dfrac{\partial(du_r)}{\partial z}\right) = \frac{1}{2}(Ar - Ar)\,dt = 0$

$\therefore \quad d\epsilon_{rr} = -(Az+B)\,dt.$

$d\epsilon_{\theta\theta} = -(Az+B)\,dt.$

$d\epsilon_{zz} = 2(Az+B)\,dt.$

$d\epsilon_{r\theta} = d\epsilon_{\theta z} = 0.$

$d\epsilon_{zr} = 0.$

e) Incremental volumetric strain $d\epsilon_v = d\epsilon_{rr} + d\epsilon_{\theta\theta} + d\epsilon_{zz}$

$$= -(Az+B)dt - (Az+B)dt + 2(Az+B)dt.$$

$$= 0.$$