MTH102N
ASSIGNMENT–LA 4

(1) Determine whether the following sets of vectors is linearly independent or not
   (a) $\{(1,0,2,1),(1,3,2,1),(4,1,2,2)\}$ in $\mathbb{R}^4$.
   (b) $\{(1,2,6),(-1,3,4),(-1,-4,-2)\}$ in $\mathbb{R}^3$.
   (c) $\{u+v, v+w, w+u\}$ in a vector space V given that $\{u,v,w\}$ is linearly independent.

(2) Let $\{w_1, w_2, \ldots, w_n\}$ be a basis of the finite dimensional vector space V. Let v be any non zero vector in V. Show that there exists w_i such that if we replace w_i by v then we still have a basis.

(3) Find the dimension of the following vector spaces
   (a) $\{A : A \text{ is } n\times n \text{ real upper} - \text{traingular matrices}\}$.
   (b) $\{A : A \text{ is } \ n\times n \text{ real symmetric matrices}\}$
   (c) $\{A : \ A \ \text{ is } n\times n \text{ real matrices with tr } A = 0\}$
   (d) $\{A : \ A \ \text{ is } n\times n \text{ real matrices with } \ A + A^t = 0\}$

(4) Let $P_n(\mathbb{R})$ = The vector space of polynomials with real coefficients and degree less or equal to n. Show that the set $\{x+1, x^2+x-1, x^2-x+1\}$ is a basis for $P_2(\mathbb{R})$. Hence determine the coordinates of the following elements: $2x-1,\ 1+x^2,\ x^2+5x-1$ with respect to the above basis.

(5) Describe all possible ways in which two planes (passing through origin) in $\mathbb{R}^3$ could intersect.

(6) Let W be a subspace of V
   (a) Show that there is a subspace U of V such that $W\cap U = \{0\}$ and $U + W = V$.
   (b) Show that there is no subspace U such that $U\cap W = \{0\}$ and $\dim U + \dim W > \dim V$.

(7) Let $W_1 = L\{(1,1,0),(-1,1,0)\}$ and $W_2 = L\{(1,0,2),(-1,0,4)\}$. Show that $W_1+W_2 = \mathbb{R}^3$. Give an example of a vector $v\in\mathbb{R}^3$ such that v can be written in two different ways in the form $v = v_1 + v_2$, where $v_1\in W_1, v_2\in W_2$.

(8) Determine which of the following are linear transformations $T : V \longrightarrow W$, where the vector spaces V, W are given:
   (a) $V = W = \mathbb{R}^3$; $T(x,y,z) = (2x+y, z, |x|)$

(b) $V = W = M_2(\mathbb{R})$, the space of all 2×2 real matrices; (i) $T(A) = A^t$, (ii) $T(A) = I + A$, (iii) $T(A) = A^2$, (iv) $T(A) = BAB^{-1}$, where B is some fixed 2×2 matrix.

(9) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear transformation such that $T(1,0,0) = (1,0,0)$, $T(1,1,0) = (1,1,1)$ and $T(1,1,1) = (1,1,0)$. Find (a) $T(x,y,z)$ (b) $ker(T)$ (c) $R(T)$. Also show that $T^3 = T$.

(10) Find a linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ with (a) $R(T) = L(\{(1,1,1)\})$ (b) $R(T) = L(\{(1,2,3),(1,3,2)\})$.

(11) Let $T : \mathbb{C} \longrightarrow \mathbb{C}$ be the map $T(z) = \overline{z}$. Show that T is $\mathbb{R}$-linear but not $\mathbb{C}$-linear.

(12) Find all linear transformations from $\mathbb{R}^n \longrightarrow \mathbb{R}$.

(13) Let V, W be vector spaces and let $L(V,W)$ be the vector space of all linear transformations from V to W. Show that $\dim L(V,W) = \dim V . \dim W$.

(14) let $T : V \longrightarrow W$ and $S : W \longrightarrow U$ be linear transformations. Show that the map $S \circ T : V \longrightarrow U$ is a linear transformation.

(15) Show that a linear transformation is one-one if and only if null-space of T is $\{0\}$.