**MTH102N**
**ASSIGNMENT–C5**

(1) Give examples of entire functions $f$ such that $f(z^2) = [f(z)]^2$ for all $z \in \mathbb{C}$. Determine all such entire functions.

(2) Find:

(a) Taylor series of the function $f(z) = \dfrac{1}{z^2}$ in powers of $(z-1)$.

(b) Laurent series of the function $f(z) = \dfrac{1}{z^2}$ for $\{z : |z-1| > 1\}$.

(3) (a) Find Laurent series of the function $f(z) = \dfrac{6z+8}{(2z+3)(4z+5)}$ in the regions

$$\{z \in \mathbb{C} : \frac{5}{4} < |z| < \frac{3}{2}\}, \quad \{z \in \mathbb{C} : |z| < \frac{5}{4}\}, \quad \{z \in \mathbb{C} : |z| > \frac{3}{2}\}.$$

(b) Find Laurent series of the function $f(z) = \dfrac{1}{z^3 - z^4}$ in the regions

$$\{z \in \mathbb{C} : 0 < |z| < 1\}, \quad \{z \in \mathbb{C} : |z| > 1\}.$$

(4) Find the Laurent series of the function $f(z) = \exp(z + \frac{1}{z})$ around 0. Hence show that $\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} e^{2\cos\theta}\cos n\theta d\theta = \sum_{j=0}^{\infty} \frac{1}{(n+j)!j!}$.

(5) Is there a polynomial $P(z)$ such that $P(z)e^{1/z}$ is an entire function? Justify your answer!

(6) Which of the following singularities are removable/pole:

$$a)\ \frac{\sin z}{z^2 - \pi^2},\ z = \pi \quad b)\ \frac{\sin z}{(z-\pi)^2},\ z = \pi \quad c)\ \frac{z\cos z}{1 - \sin z},\ z = \pi/2$$