

$$\int_A v_x^2 dA = \int_A (V + v')^2 dA = V^2 A + \int_A v'^2 dA \geq V^2 A$$

(since the areal average of deviations vanishes)