

A Sparse Reconstruction Method for Speech Source Localization using Partial Dictionaries over a Spherical Microphone Array

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Abstract

Sparse reconstruction methods have been used extensively for source localization over uniform linear arrays and circular arrays. In this paper a sparse reconstruction method for speech source localization using partial dictionaries over a spherical microphone array is proposed. The source localization method proposed in this work addresses two important research issues. It formulates the source localization problem in the spherical harmonics domain as a sparse reconstruction problem. Subsequently, a low complexity method to estimate the direction of arrival (DOA) of multiple sources is also proposed by using partial elevation angle dictionaries. The use of such dictionaries reduces the complexity of the search involved in the two dimensional DOA estimation. Source localization experiments are conducted at different SNRs and compared with conventional DOA estimation methods like MUSIC and MVDR. The experimental results obtained from the proposed method indicate a reasonable reduction in the localization error.

Index Terms: DOA estimation, Sparse reconstruction, Source localization, Spherical microphone array

1. Introduction

Spherical microphone arrays have become a topic of focus among researchers in the recent past [1, 2]. There are two main reasons for this. Firstly, array processing can be performed relatively easily in the spherical harmonics (SH) domain without any spatial ambiguity [3]. Secondly, various problems can be formulated in the SH domain which have similar structure in the spatial domain and hence existing results can be applied directly in SH domain.

Direction of arrival (DOA) estimation has proved to be one of the widely used applications of microphone array processing. There exist different methods for estimating the DOA including optimum beamforming, maximum likelihood methods [4], and subspace based methods. One of the most popular subspace based methods is the Multiple Signal Classification (MUSIC) [5] which uses the orthogonality of the noise subspace to the signal subspace to estimate the DOAs. SH-MUSIC has been proposed in [6] where the processing has been performed in the SH domain. In [7], the authors have proposed SH-MUSIC Group Delay (SH-MGD) for estimation of DOAs which performs better than the SH-MUSIC method and is able to resolve closely spaced sources as well. But both these methods require the information of the number of sources present in order to estimate the DOAs.

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Sparse reconstruction (SR) based methods have proved to be more robust than the subspace based methods, mainly due to the dictionary based searching that they employ. SR methods have been proposed for DOA estimation using uniform linear arrays (ULA) [8, 9] and circular arrays [10]. However, the application of SR based methods to the spherical arrays has not been exploited much. One of the reasons for this is the fact that the estimation of both the azimuth and the elevation increases the dictionary size, consequently it also increases the complexity. The work done in [11, 12, 13] is the only available literature on the concerned topic. In this work, we propose a novel method for source localization based on partial dictionaries and sparse reconstruction. This method hence provides a new perspective for DOA estimation using spherical microphone arrays.

The rest of the paper is organized as follows. Section 2 introduces the array processing in the SH domain and describes the proposed method. In Section 3, the proposed method is evaluated. Concluding remarks are given in Section 4.

2. Sparse Reconstruction Method for Speech Source Localization

Sparse reconstruction methods have been used extensively for source localization using linear and planar arrays. but its application to spherical arrays has not been studied to a great extent. In this section, we first introduce the signal model in the spherical harmonics domain. Subsequently, the proposed method for multi source localization using partial dictionaries is described.

2.1. Signal Model in Spherical Harmonics Domain

Consider a spherical microphone array with I microphones, radius r , and order N . An incident sound field with wave number k and consisting of L far field sources is assumed. The location of the l^{th} source is denoted by $\Psi_l = (\theta_l, \phi_l)$. The azimuthal angle $\phi \in [0^\circ, 360^\circ)$ is measured counter clock wise from the positive x axis and the elevation angle $\theta \in [0^\circ, 180^\circ)$ is measured down from the positive z axis.

The sound pressure, $\mathbf{p}(k) = [p_1(k), p_2(k), \dots, p_I(k)]^T$, recorded at the microphone array is given by

$$\mathbf{p}(k) = \mathbf{V}(k)\mathbf{s}(k) + \mathbf{n}(k) \quad (1)$$

where $\mathbf{V}(k)$ is the $I \times L$ steering matrix at wave number k , $\mathbf{s}(k)$ is the vector of signal amplitudes, $\mathbf{n}(k)$ is the zero mean uncorrelated Gaussian noise with covariance matrix $\sigma^2 \mathbf{I}$ and $(\cdot)^T$ represents the transpose. The steering matrix is defined in detail in [14].

Using the Spherical Fourier Transform (SFT) [15], Equa-

tion (1) can be written in spherical harmonics domain as [14]

$$\mathbf{p}_{nm}(k, r) = \mathbf{B}(kr) \mathbf{Y}^H(\Psi) \mathbf{s}(k) + \mathbf{n}_{nm}(k) \quad (2)$$

$$\mathbf{n}_{nm}(k) = \mathbf{Y}^H(\Phi) \mathbf{\Gamma} \mathbf{n}(k) \quad (3)$$

where $\mathbf{p}_{nm} = [p_{0,0}, p_{1,-1}, p_{1,0}, p_{1,1}, \dots, p_{N,N}]^T$ is the Fourier coefficients vector, $\mathbf{\Gamma}$ is the diagonal matrix containing the sampling weights [16], $\mathbf{B}(kr)$, $\mathbf{Y}(\Psi)$ and $\mathbf{Y}(\Phi)$ are defined later in this section. Here, Ψ and Φ denote the angular positions of the sources and microphones respectively. The dependence on k and r has been omitted for notational simplicity. Every p_{nm} is the SFT of the pressure received at the microphones, $p(k, r, \theta, \phi)$. The SFT is defined as [15]

$$p_{nm}(k, r) = \int_0^{2\pi} \int_0^\pi p(k, r, \theta, \phi) [Y_n^m(\theta, \phi)]^* \sin \theta d\theta d\phi \quad (4)$$

The spherical harmonic of order n and degree m is given by

$$Y_n^m(\theta, \phi) = \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}} P_n^m(\cos \theta) e^{jm\phi} \quad (5)$$

where P_n^m are the associated Legendre function and j is the unit imaginary number. Here n varies from 0 to N and m varies from 0 to n . For $m = -n, \dots, -1$, $Y_n^m(\theta, \phi) = (-1)^{|m|} |Y_n^{|m|}(\theta, \phi)|$. The matrix $\mathbf{Y}(\Psi)$ is given by

$$\mathbf{Y}^H(\Psi) = [\mathbf{y}_1^H, \mathbf{y}_2^H, \dots, \mathbf{y}_L^H] \quad (6)$$

$$\mathbf{y}_l = [Y_0^0(\Psi_l), Y_1^{-1}(\Psi_l), Y_1^0(\Psi_l), \dots, Y_N^N(\Psi_l)] \quad (7)$$

The matrix $\mathbf{Y}(\Phi)$ is defined similarly but with Ψ replaced by Φ which corresponds to the angular positions of the microphones on the spherical array.

The diagonal matrix $\mathbf{B}(kr)$ is $(N+1)^2 \times (N+1)^2$ and is given by

$$\mathbf{B}(kr) = \text{diag}(b_0(kr), b_1(kr), b_1(kr), b_1(kr), \dots, b_N(kr)) \quad (8)$$

where $b_n(kr)$ is the mode strength and it is defined as

$$b_n(kr) = \begin{cases} 4\pi j^n j_n(kr) & \text{for open sphere} \\ 4\pi j^n [j_n(kr) - \frac{j'_n(kr)}{h'_n(kr)} h_n(kr)] & \text{for rigid sphere} \end{cases} \quad (9)$$

where j_n and h_n denote the spherical Bessel and Hankel functions respectively, j'_n and h'_n are their derivatives. Figure 1 shows the mode strength plot for an open sphere as a function of kr and n . Equation (2) can be multiplied by $\mathbf{B}^{-1}(kr)$ on the left to get the following model

$$\mathbf{a}_{nm}(k) = \mathbf{Y}^H(\Psi) \mathbf{s}(k) + \mathbf{z}_{nm}(k) \quad (10)$$

$$\mathbf{z}_{nm}(k) = \mathbf{B}^{-1}(kr) \mathbf{n}_{nm}(k) \quad (11)$$

It is to be noted that computation of $\mathbf{B}^{-1}(kr)$ is not always possible in the case of an open sphere [14] and hence the rigid sphere is used in this work. This model can now be represented in the following standard form to apply the sparse reconstruction method,

$$\mathbf{x}(k) = \mathbf{A}(\Psi) \mathbf{s}(k) + \mathbf{w}(k) \quad (12)$$

where $\mathbf{x}(k)$ is the observation vector, $\mathbf{A}(\Psi) = \mathbf{Y}^H(\Psi)$ is the steering matrix, and $\mathbf{w}(k) = \mathbf{z}_{nm}(k)$ is the noise vector.

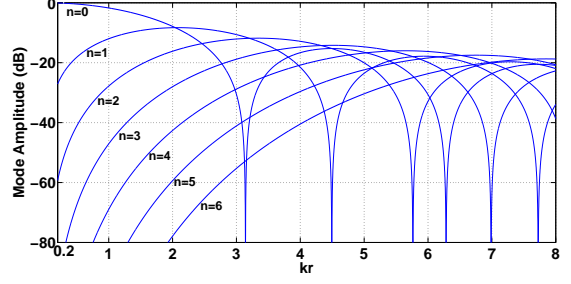


Figure 1: Mode amplitude b_n plot for open sphere as a function of kr and n

2.2. Development of the Co-Array Framework for Source Localization

The model in Equation (12) can be used to estimate the source locations efficiently by transforming the problem into the co-array domain [17]. As $\mathbf{s}(k)$ and $\mathbf{n}(k)$ are uncorrelated, the array covariance matrix can be described by

$$\mathbf{R}(k) = E\{\mathbf{x}(k)\mathbf{x}^H(k)\} = \mathbf{A}(\Psi)\mathbf{R}_s(k)\mathbf{A}^H(\Psi) + \mathbf{R}_w(k) \quad (13)$$

where $E\{\cdot\}$ is the expectation operator. For uncorrelated sources, the source covariance matrix $\mathbf{R}_s(k) = E\{\mathbf{s}(k)\mathbf{s}^H(k)\} = \text{diag}(\sigma_s(k))$ with $\sigma_s(k) = [\sigma_1^2(k), \sigma_2^2(k), \dots, \sigma_L^2(k)]^T$. The noise covariance matrix $\mathbf{R}_w(k) = \sigma^2 \mathbf{E}(k)\mathbf{E}(k)^H$, where $\mathbf{E}(k)$ is defined as

$$\mathbf{E}(k) = \mathbf{B}^{-1}(kr) \mathbf{Y}^H(\Phi) \mathbf{\Gamma} \quad (14)$$

To make use of all the information, in case of wide band signals, frequency smoothing can be applied to Equation (13). The smoothed array covariance matrix can be computed as

$$\tilde{\mathbf{R}} = \frac{1}{K} \sum_{k=f_1}^{f_K} \mathbf{R}(k) \quad (15)$$

where f_1, \dots, f_K denote the K wave numbers over which the smoothing is applied. As the matrix $\mathbf{A}(\Psi)$ is frequency independent, the model now becomes,

$$\tilde{\mathbf{R}} = \mathbf{A}(\Psi) \tilde{\mathbf{R}}_s \mathbf{A}^H(\Psi) + \tilde{\mathbf{R}}_w \quad (16)$$

where $\tilde{\mathbf{R}}_s$ and $\tilde{\mathbf{R}}_w$ are defined as

$$\tilde{\mathbf{R}}_s = \frac{1}{K} \sum_{k=f_1}^{f_K} \text{diag}(\sigma_s(k)) \quad (17)$$

$$\tilde{\mathbf{R}}_w = \sigma^2 \frac{1}{K} \sum_{k=f_1}^{f_K} \mathbf{E}(k)\mathbf{E}(k)^H \quad (18)$$

Vectorization operator can now be applied to Equation (16) in order to transform the problem to co-array domain

$$\begin{aligned} \mathbf{v} &= \text{vec}(\tilde{\mathbf{R}}) = [\mathbf{A}^*(\Psi) \odot \mathbf{A}(\Psi)] \sigma_p + [\mathbf{E}^* \odot \mathbf{E}] \sigma_n \\ &= \mathbf{D} \sigma_p + [\mathbf{E}^* \odot \mathbf{E}] \sigma_n \end{aligned} \quad (19)$$

where \odot is the Khatri-Rao product [17], σ_n is $L \times 1$ vector with all entries as σ^2 , and \mathbf{D} is the new steering matrix. The

matrix \mathbf{E} and vector $\boldsymbol{\sigma}_p$ are

$$\mathbf{E} = \frac{1}{K} \sum_{k=f_1}^{f_K} \mathbf{E}(k) \mathbf{E}(k)^H \quad (20)$$

$$\boldsymbol{\sigma}_p = \frac{1}{K} \sum_{k=f_1}^{f_K} \boldsymbol{\sigma}_s(k) \quad (21)$$

The steering matrix \mathbf{D} is described by

$$\mathbf{D} = [\mathbf{y}_1^T \otimes \mathbf{y}_1^H, \dots, \mathbf{y}_L^T \otimes \mathbf{y}_L^H] \quad (22)$$

where \otimes denotes the Kronecker product. An overcomplete dictionary, $\tilde{\mathbf{D}}$, of all possible elevation and azimuth steering vectors is constructed as

$$\tilde{\mathbf{D}} = [\mathbf{y}^T(\tilde{\theta}_1, \tilde{\phi}_1) \otimes \mathbf{y}^H(\tilde{\theta}_1, \tilde{\phi}_1), \dots, \mathbf{y}^T(\tilde{\theta}_1, \tilde{\phi}_{c_2}) \otimes \mathbf{y}^H(\tilde{\theta}_1, \tilde{\phi}_{c_2}), \dots, \mathbf{y}^T(\tilde{\theta}_{c_1}, \tilde{\phi}_1) \otimes \mathbf{y}^H(\tilde{\theta}_{c_1}, \tilde{\phi}_1), \dots, \mathbf{y}^T(\tilde{\theta}_{c_1}, \tilde{\phi}_{c_2}) \otimes \mathbf{y}^H(\tilde{\theta}_{c_1}, \tilde{\phi}_{c_2})]$$

where c_1 and c_2 denote the possible values of the elevation and azimuth respectively. The angles $\tilde{\theta}$ and $\tilde{\phi}$ are look up elevations and azimuths respectively. The model in Equation 19 can now be represented as

$$\mathbf{v} = \tilde{\mathbf{D}}\mathbf{u} + [\mathbf{E}^* \odot \mathbf{E}]\boldsymbol{\sigma}_n \quad (23)$$

where \mathbf{u} is the L -sparse vector whose non zero elements represent the source powers σ_i^2 . Hence by locating the non zero elements in \mathbf{u} , the DOA of the sources can be estimated by solving the following convex minimization problem

$$\begin{aligned} \min_{\mathbf{u}} \|\mathbf{v} - \tilde{\mathbf{D}}\mathbf{u} - [\mathbf{E}^* \odot \mathbf{E}]\boldsymbol{\sigma}_n\|_2^2 + \lambda \|\mathbf{u}\|_1 \\ \text{subject to } \mathbf{u} \geq \mathbf{0} \end{aligned} \quad (24)$$

where the parameter $\lambda \geq 0$ is determined empirically in this work and $\boldsymbol{\sigma}_n$ can be estimated from the eigenvalue decomposition of $\tilde{\mathbf{R}}$.

The size of the dictionary used in Equation (24) is $(N + 1)^4 \times (c_1 c_2)$. Hence it is very time consuming to optimize Equation (24). This is probably the reason that SR based methods have not been used for joint elevation-azimuth estimation. We propose a method which optimizes Equation (24) over partial dictionaries instead of one huge dictionary.

2.3. Development of Partial Dictionaries based Method for Multi Source Localization

Consider c_1 number of dictionaries and each such dictionary consists of the steering vectors corresponding to all possible azimuthal angles at a fixed elevation. Mathematically,

$$\tilde{\mathbf{D}}_i = \tilde{\mathbf{D}} \times \mathbf{J}_i \quad (25)$$

where $\tilde{\mathbf{D}}_i$ is the partial dictionary corresponding to elevation $\tilde{\theta}_i$, and \mathbf{J}_i is the selection matrix used to extract the columns of the dictionary $\tilde{\mathbf{D}}$ corresponding to the elevation $\tilde{\theta}_i$. Hence instead of using a big dictionary, we now use c_1 dictionaries, each of size $(N + 1)^4 \times c_2$. The optimization problem is reduced to,

$$\begin{aligned} \min_{\mathbf{u}_i} \|\mathbf{v} - \tilde{\mathbf{D}}_i \mathbf{u}_i - [\mathbf{E}^* \odot \mathbf{E}]\boldsymbol{\sigma}_n\|_2^2 + \lambda \|\mathbf{u}_i\|_1 \\ \text{subject to } \mathbf{u}_i \geq \mathbf{0} \end{aligned} \quad (26)$$

where the optimization is performed over all $i = 1, 2, \dots, c_1$. As all the optimizations are independent of each other, they can be performed in parallel thereby reducing the complexity.

Let \mathbf{u}_i^{opt} be the solution to Equation (26) and define the error corresponding to the i^{th} partial dictionary as follows,

$$\mathbf{e}(i) = \|\mathbf{v} - \tilde{\mathbf{D}}_i \mathbf{u}_i^{opt} - [\mathbf{E}^* \odot \mathbf{E}]\boldsymbol{\sigma}_n\|_2^2 + \lambda \|\mathbf{u}_i^{opt}\|_1 \quad (27)$$

To estimate the elevation angles of the sources, we plot the inverse error, $1/\mathbf{e}(i)$, achieved in Equation (27) for each i versus the elevation angles. The maxima in the plot are the estimates of the elevation angle. The sparse vectors \mathbf{u}_i^{opt} corresponding to the estimated elevations are then used to estimate the azimuthal angles. It is to be noted that the proposed method does not require the information about the number of sources present. The complete algorithm for source localization is described in Algorithm 1.

Algorithm 1 Algorithm for DOA estimation using sparse reconstruction and partial dictionaries

- 1: Process the data received at the microphones to get the model of Equation (12)
 - 2: Compute the array covariance matrix \forall the K wave numbers considered as
 - 3: **for** $k = f_1, \dots, f_K$ **do**
 - 4: $\mathbf{R}(k) = \mathbf{A}\mathbf{R}_s(k)\mathbf{A}^H + \mathbf{R}_w(k)$
 - 5: **end for**
 - 6: Compute the smoothed array covariance matrix $\tilde{\mathbf{R}}$ as $\tilde{\mathbf{R}} = \frac{1}{K} \sum_{k=f_1}^{f_K} \mathbf{R}(k)$
 - 7: Apply vectorization operator to $\tilde{\mathbf{R}}$ to get Equation (19)
 - 8: Create the partial dictionaries $\tilde{\mathbf{D}}_i \forall i = 1, \dots, c_1$
 - 9: Compute the sparse vectors, \mathbf{u}_i^{opt} , \forall partial dictionaries using Equation (26)
 - 10: Compute the errors corresponding to all partial dictionaries by using Equation (27)
 - 11: Get the elevations of the sources by locating the maxima in the plot of $1/\mathbf{e}(i)$
 - 12: Get the azimuth of the sources by locating the non-zero coefficients in the sparse vectors \mathbf{u}_i^{opt} corresponding to the elevations in step (11)
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2.4. Localization of Multiple Sources

Figure 2 shows the results of the localization performed for the three source scenario. The sources are uncorrelated and

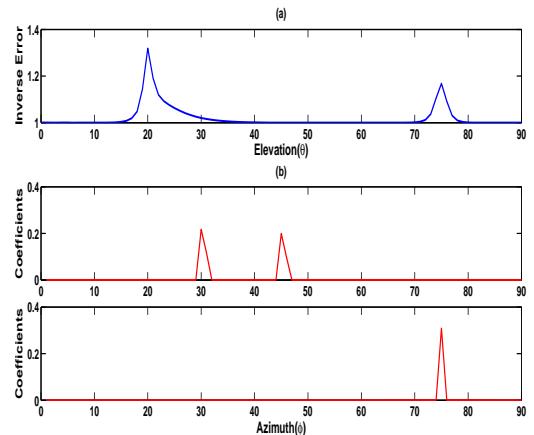


Figure 2: Results of source localization experiment (a) Elevation estimation plot (b) Azimuth estimation plots for the above estimated elevations.

of equal powers. The angular positions of the sources are $\Psi_1 = (20^\circ, 30^\circ)$, $\Psi_2 = (20^\circ, 45^\circ)$, and $\Psi_3 = (75^\circ, 75^\circ)$ respectively. The signal to noise ratio (SNR) is kept at 10dB. The order of the spherical array is $N = 4$. Figure 2(a) illustrates the plot of inverse error versus elevation angle. The two maxima located at 20° and 75° correspond to the elevations of the sources. The optimal sparse vectors for the dictionaries corresponding to the estimated elevations are then plotted in Figure 2(b). The first plot shows azimuths at an elevation of 20° and second plot shows azimuth at an elevation of 75° . As the results indicate, the proposed method is able to localize sources efficiently.

3. Performance Evaluation

From hereon the proposed method is called the SRPD (Sparse Reconstruction using Partial Dictionaries) method. Experiments were performed on various data sets to evaluate the SRPD method of source localization. First experiment calculates the probability of resolution for a two source scenario to show the statistical significance of the proposed method. Second experiment shows the robustness of the SRPD method compared to the existing ones.

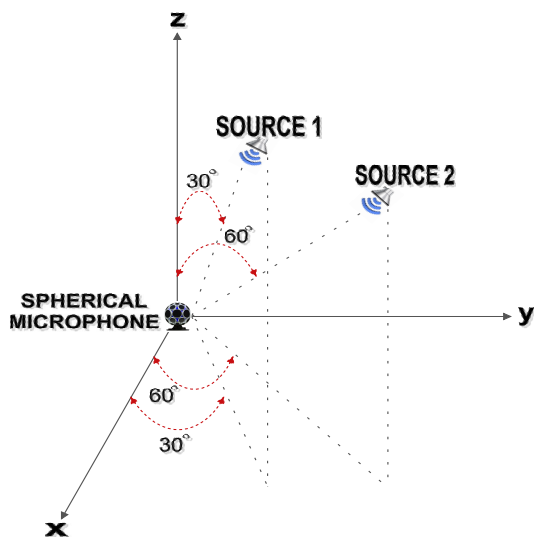


Figure 3: Source and microphone setup for conducting the localization experiments

3.1. Experimental Conditions

A rigid Eigen mike microphone array [18] is used for the simulations which consists of 32 microphones. The radius of the sphere is 4.2 cm. The order of the microphone array is $N = 4$. The dictionaries used for the experiments are created using a resolution of 1° for both the elevation and azimuth. Convex optimization package CVX [19] is used for solving the Equation (26). Two uncorrelated speech sources are used in the experiments. The angular positions of the sources are $\Psi_1 = (30^\circ, 30^\circ)$ and $\Psi_2 = (60^\circ, 60^\circ)$ respectively. The sensor noise is considered to be white and uncorrelated Gaussian random variable. Figure 3 depicts the experimental setup used to conduct the following experiments.

3.2. Experiments on Probability of Resolution for DOA Estimation

To evaluate the performance of the proposed method, resolution probabilities are calculated and compared with the SH-MVDR [14] and SH-MUSIC [6] methods. The confidence interval used is 2° wide. The results are given as a bar plot in Figure 4 at three different SNRs. The results were obtained over 400 independent iterations. The height of the bars correspond to the probability of resolution. As the results indicate, the SRPD method is able to resolve the sources with higher probability as compared to the other two methods. It is to be noted that both SRPD and SH-MUSIC tend to a similar performance at high SNRs.

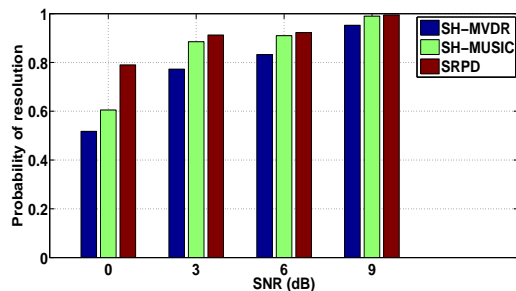


Figure 4: Comparison of probability of resolution for various methods at different SNRs

3.3. Experiments on RMSE Comparison

The RMSE values are computed for the SRPD method and compared with the SH-MUSIC and SH-MVDR methods. The values are computed over 400 independent iterations. Figure 5 shows the results for the three different methods. The plots indicate that the SH-MUSIC and SRPD outperform the SH-MVDR method at low SNRs while the methods tend to have a similar performance at high SNRs.

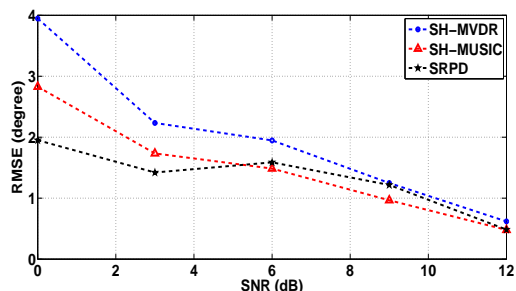


Figure 5: RMSE plot for the three methods against SNR

4. Conclusion

In this work, a new method is proposed for source localization using sparse reconstruction over spherical arrays. The method splits a huge dictionary into several small dictionaries to perform the localization task. Hence this method reduces the computational complexity. The experiments conducted on various data sets also confirm the efficacy of the proposed method and serve as a motivation to conduct further research into the topic. Effect of correlation among the sources and reverberation in the environment will also be taken into consideration in the future work.

5. References

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