Answers to Problems

2.1 (a) $\sim 10^6$ m/s (b) ~ 42 m/s (c) $\sim m/M$

3.1 (a) $\frac{5}{3} C_k\, n(\vec{r})^{2/3} + v_{IE}(\vec{r}) + \int \frac{n(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} d^3r' - \left(\frac{3}{\pi}\right)^{1/3} n(\vec{r})^{1/3} = \mu$

(b) $\frac{5}{3} C_k\, n(\vec{r})^{2/3} + v_{IE}(\vec{r}) + \int \frac{n(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} d^3r' - \left(\frac{3}{\pi}\right)^{1/3} n(\vec{r})^{1/3}$

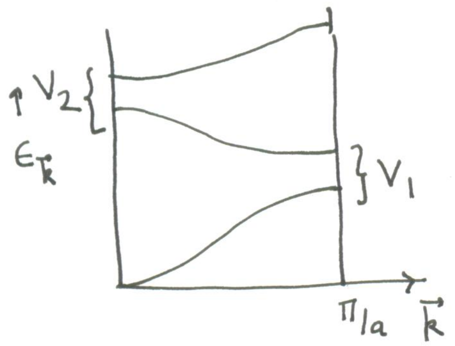
$+ \frac{1}{8} \frac{|\nabla n(\vec{r})|^2}{n^2(\vec{r})} - \frac{1}{4} \frac{\nabla^2 n(\vec{r})}{n(\vec{r})} = \mu$

3.3 (b) Assume uniform density

$E_x \sim -0.458$ Ha (exact value $=0$)

(c) $E = \sim -0.3$ Ha (exact value $= -0.5$ Ha)

5.5



5.6 (a) V_0 (b) 0 (c) Since the band gap is zero at $(\pi/a, 0)$, it is always a metal for divalent atoms.

5.7 (a) 3 lowest levels are 0, $\frac{\hbar^2}{2m}\left(\frac{2\pi}{b}\right)^2$ and $\frac{\hbar^2}{2m}\left(\frac{2\pi}{a}\right)^2$ at $\vec{k}=(0,0)$

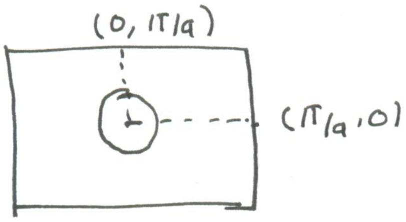
3 lowest levels at $(\frac{\pi}{a}, 0)$ are $\frac{\hbar^2}{2m}\frac{\pi^2}{a^2} - V_1$, $\frac{\hbar^2}{2m}\frac{\pi^2}{a^2} + V_1$

and $\frac{\hbar^2}{2m}\left(\frac{\pi^2}{a} + \left(\frac{2\pi}{b}\right)^2\right)$

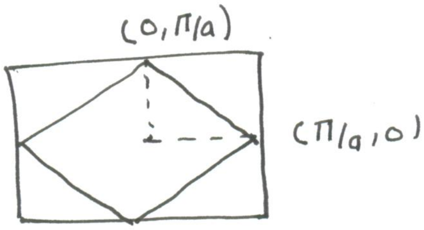
(b) yes when $2V_1 \geq \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}$

5.10 (a) $\epsilon_{\vec{k}} = 2t\,(2 - \cos k_x a - \cos k_y a)$

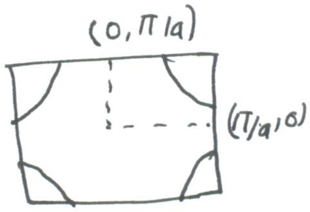
(b) (i)



(ii)



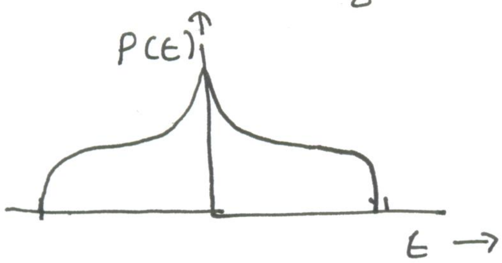
(iii)



5.11 $$\epsilon_{\vec{k}} = \epsilon_0 + 2t\left(\cos k_x a + \cos\left(\frac{k_x a}{2} + \frac{\sqrt{3}}{2}k_y a\right) + \cos\left(\frac{k_x a}{2} - \frac{\sqrt{3}}{2}k_y a\right)\right)$$

5.12 $$P(\epsilon) = \frac{2L}{N\pi a}\,\frac{1}{\sqrt{4t^2 - \epsilon^2}}$$

5.13 $$P(\epsilon) = \frac{1}{\pi N}\int_0^\infty dx\,\cos(x\epsilon)\,J_0^2(2tx)$$



5.14 $$P(\epsilon) = \frac{1}{\pi N}\int_0^\infty dx\,\cos(x\epsilon)\,J_0^3(2tx)$$

Plot is given in Fig 5.9 in the book.

5.15 $k_N \sim 3.3 \times 10^9\ m^{-1}$

12.2 α_{mn} can be found by zeroes of J_m.

For example $\alpha_{01} \simeq 2.4$ $\alpha_{11} \simeq 3.8$ etc

Then

$$\epsilon_{mn} = \frac{\hbar^2 k_z^2}{2m_e} + \frac{\hbar^2 \alpha_{mn}^2}{2m_e R^2}$$

12.3

$$P(\epsilon) = \frac{m_e}{\pi \hbar^2} \sum_{mn} \frac{1}{\left[\frac{2m_e}{\hbar^2}(\epsilon - \epsilon^0_{mn})\right]^{1/2}}$$

14.6

$$P(\epsilon) = \frac{A\,\epsilon}{\pi \alpha^2}, \quad \text{linear in } \epsilon$$

15.4

$$\epsilon = \frac{p^2}{2m} \pm c p$$

if c is large, linear behaviour will dominate.

15.5

$$\epsilon = \frac{p^2}{2m} \pm c|p|$$

For large c, linear behaviour will dominate.