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AXIAL HEAT CONDUCTION IN THE CONTEXT OF DEVELOPING FLOWS IN MICROCHANNELS

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ABSTRACT

For practical microchannel applications involving convective heat transfer, the flows are usually, not only laminar, they are also simultaneously developing in nature. Moreover, flat plate substrates with microchannels engraved/ machined or etched on them are emerging as one of the most popular flow geometries. Not analyzing such situations as conjugate heat transfer problems with multi-dimensional effects, often leads to erroneous estimation of heat transfer coefficients. In this context, we report three dimensional numerical simulations of simultaneously developing internal convective flow through a square microchannel (side = 400 μm), treating the substrate thickness, flow Reynolds number and the thermal conductivity of the substrate and the fluid, in the conjugate formulation. Constant heat flux is applied at the bottom of the substrate, away from the fluid-solid interface, as in real-time situations. The parametric study reveals that depending on geometry considerations, flow parameters and thermo-physical properties of fluid-solid combination, conjugate heat transfer effects must be accounted for, to correctly estimate the local Nusselt number.

Keywords: Conjugate heat transfer; Microchannels; Axial heat conduction; Simultaneously developing flow.

INTRODUCTION

Conventionally, for fully developed internal convective flows, all other flow parameters remaining the same, constant heat flux at the fluid-solid interface of the channel/duct leads to maximum heat transfer coefficient. In real-time applications, most situations lead to conjugate nature of heat transfer wherein the exact heat flux distribution (or exact wall temperature distribution) at the fluid-solid interface depends on many factors such as dimensions of both the solid and the fluid domain, thermal properties of solid and fluid involved, and flow characteristics of working fluid. In reality, for many practical applications, the boundary condition(s) is(are) applied at a certain finite distance from the fluid-solid interface; this leads to a distortion of the boundary condition which the fluid actually experiences at the interface vis-à-vis the true or desired boundary condition applied at the channel walls. This distortion gets amplified in microchannel systems and may affect the accurate estimation of heat transfer coefficient. Temperature gradients in the substrate wall, due to difference in wall temperature at the outlet and inlet sections, result in heat flow in the substrate (the nature of this heat transfer may be three-dimensional) which essentially manifests as a conjugate problem. This phenomenon has attracted many researchers

because of widespread implications on the estimation of species transport coefficients in various fields of engineering applications.

NOMENCLATURE

A_{sf}	ratio of cross sectional area of solid substrate to channel (A_s/A_f)
c_p	specific heat, J/kg-K
D_h	hydraulic diameter, m
k	thermal conductivity, W/m-K
k_{sf}	solid-to-fluid thermal conductivity ratio (k_s/k_f)
L	length, m
Nu	Nusselt number (hD_h/k_f)
Pr	Prandtl Number ($C_p\mu/k_f$)
q'	actual average heat flux at the channel walls, W/m ²
\bar{q}'	heat flux applied at the bottom of the substrate, W/m ²
Re	Reynolds Number ($\rho u D_h/\mu$)
T	temperature, K
z^*	non-dimensional axial distance along channel length (-)
Greek Symbol	
δ	thickness, m
ω	width, m
μ	dynamic viscosity, Pa-s
ρ	density, kg/m ³
Θ	non-dimensional temperature (-)
ϕ	non-dimensional heat flux (q'/\bar{q}')
δ_{sf}	ratio of substrate thickness to channel height (δ_s/δ_f)
Subscript	
f	fluid
i	inlet condition
o	outlet condition
s	solid
w	wall surface
z	axial length along the channel

LITERATURE REVIEW

The study of axial conduction in solid substrates exposed to convective flows is not new. To understand the parameters affecting axial conduction, a parameter called “conduction parameter” was used as early as in 1964, by Bahnke and Howard [1], and later on by Peterson [2]. Petukhov [3] studied axial conduction in the wall of a circular tube and used a parameter which is function of thermal conductivity ratio of the flowing liquid and the solid substrate, and the tube thickness to radius ratio. Others who followed similar studies are Faghri and Sparrow [4], Cotton and Jackson [5], and Chiou et al. [6].

With the emergence of devices employing microscale convective flows, the study of conjugate heat transfer has again gained momentum due to its relative importance to such small geometries as compared to heat transfer applications employing conventional sized channels. Maranzana et al. [7] introduced the concept of axial conduction number (M), defined as the ratio of the conductive heat flux to the convective counterpart. For macro-channels, the value of M is usually very low because of lower ratio of solid to fluid thickness, higher flow velocity and longer channel lengths. In contrast for microchannels, the ratio of solid to fluid thickness is very high, flow velocity is usually rather low and the overall device lengths are also relatively much smaller. Based on preliminary analysis, Maranzana et al. [7] stated that axial conduction cannot be neglected if parameter $M > 0.01$. The axial conduction number proposed by Maranzana et al. [7] is based on the assumption that the axial temperature difference between the inlet and the outlet location in solid as well as in fluid domain is equal. In reality, this assumption is not usually valid. To correct the shortcoming with this assumption, Li et al. [8] and Zhang et al. [9] considered individual temperature differences between the inlet and the outlet location in the solid as well as the fluid domain, respectively.

Zhang et al. [9] in their study on conjugate heat transfer in thick circular tubes found that the criterion for neglecting axial conduction as proposed by Maranzana et al. [7] may not be always valid. They concluded that the criteria for judging the effect of axial wall conduction may vary on case to case basis, depending on the boundary conditions and geometrical parameters.

Although a number of numerical [8,10-15] as well as experimental studies [8,10,16-19] have been performed, an explicit parameter for discerning the effect of axial conduction on the heat transport coefficient in microchannel flows, under a given set of geometry and boundary conditions, is still not available. Secondly, most of the studies deal with circular micro tubes. It is also worthwhile to note that most flows in microchannel heat transfer applications will be simultaneously developing in nature. The types of inlet headers which are usually provided do not allow the hydrodynamic development of flow. Further, looking into the popularity of using rectangular/square microchannels fabricated on flat substrates, and the frequent occurrence of simultaneously developing laminar flows in such systems, we report three-dimensional numerical heat transfer studies (on the commercial platform FLUENT® [20]) to study the effect of axial heat conduction along the solid substrate in such situations, subjected to a uniform wall heat flux condition (H1) at the bottom of the substrate. Peripherally averaged local heat flux, wall temperature along the channel and average bulk fluid temperature are numerically calculated as functions of the dimensionless axial distance, substrate to channel thickness ratio, and thermal conductivity ratio of solid substrate to the working fluid. The conjugate effects are analyzed for various flow and geometry configurations.

NUMERICAL ANALYSIS

The problem has been investigated by considering that the heat transfer and fluid flow takes place at steady state, the flow is laminar, incompressible with constant thermo-physical properties and the loss of heat either by radiation or by means of natural convection is negligible.

The width (ω_f), height (δ_f), and length (L) of the channel in the computational model are kept constant at 0.4 mm, 0.4 mm, and 120 mm respectively. Thus, the hydraulic diameter (D_h) of the channel is 0.4 mm. While the width of the substrate ($\omega_f + 2\omega_s$) is kept constant, the thickness of the substrate (δ_s) is varied to understand the effect of substrate thickness on the conjugate heat transfer behavior. This is because, as the thickness of the substrate increases, the boundary on which constant heat flux is applied moves away from the desired surface on which we envisage a constant heat flux, i.e. the fluid-solid interface.

Figure 1. shows the schematic diagram of the microchannel cross-section considered in this study; in actual computations, only one half of this section, along the vertical plane of symmetry, was meshed. Constant heat flux is applied at bottom of the substrate while all other outer surfaces are insulated, as shown in Fig. 1. Coupled equations for conservation of mass, momentum and energy are solved on this domain using commercial platform FLUENT®. The standard scheme was used for pressure discretization. The SIMPLE algorithm was used for velocity-pressure coupling in the multi-grid solution procedure. The momentum and energy equations were solved using the second order upwind scheme. The working fluid (chosen to be water) enters the microchannels with a slug velocity profile at an inlet temperature of 300K. Thus, the flow is hydrodynamically as well as thermally developing, i.e. simultaneously developing in nature.

The entire domain was meshed using hexahedral elements and the grid sensitivity was checked before the size of the grid

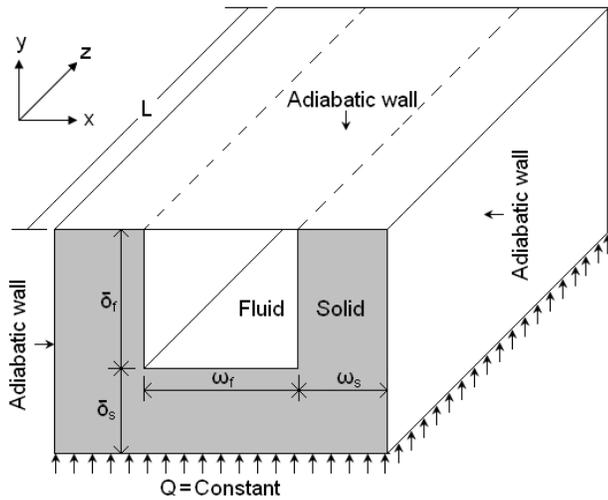


FIGURE 1. MICROCHANNEL WITH CONDUCTIVE WALLS

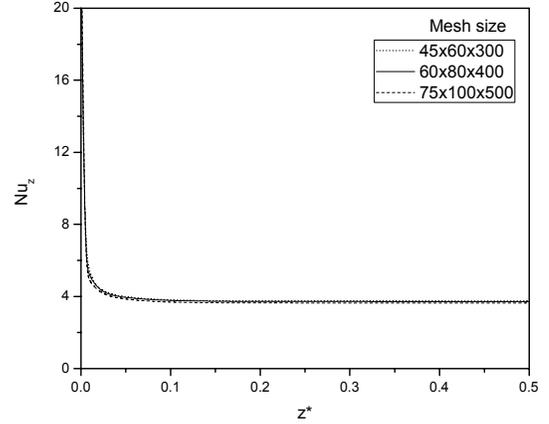


FIGURE 2. VARIATION OF LOCAL NUSSLETT NUMBER ALONG THE CHANNEL AXIS FOR DIFFERENT GRIDS

was decided for present case under consideration. As an example, local Nusselt number obtained for a substrate with $\delta_{sf} = 1$ for three mesh sizes of $45 \times 60 \times 300$, $60 \times 80 \times 400$ and $75 \times 100 \times 500$ (for half of the geometry as in Fig. 1.), for $Re = 100$ as shown in Fig. 2. The peripheral averaged local Nusselt number at the channel outlet (fully developed flow regime) changed by 1.06% from the mesh size of $45 \times 60 \times 300$ to $60 \times 80 \times 400$, and changed by less than 1% upon further refinement to mesh size of $75 \times 100 \times 500$. Since no appreciable change is observed as we moved from first to the third mesh, the intermediate grid ($60 \times 80 \times 400$) was chosen. For this grid size, the actual physical average spacing was 0.01 mm along the width and height (x and y axis) and 0.3 mm along the channel length (z axis). Finer meshing was used in the channel entrance and the boundary layers.

DATA REDUCTION

The thickness ratio (δ_{sf}) is defined as $\delta_{sf} = \delta_s / \delta_f$ where δ_s and δ_f are thickness of substrate and fluid respectively, as shown in Fig. 1. The conductivity ratio (k_{sf}) is defined as the ratio of thermal conductivity of solid (k_s) to that of fluid (k_f).

The axial coordinate z is non-dimensionalized as follows:

$$z^* = \frac{z}{Re Pr D_h} \quad (1)$$

The applied heat flux at the bottom of the substrate is defined as follows:

$$\bar{q}' = \frac{Q}{(2 \cdot \delta_f + \omega_f)L} \quad (2)$$

where Q is the heat input to the bottom of the substrate.

The non-dimensional local heat flux at the fluid-solid interface is given by:

$$\phi = q'_z / \bar{q}' \quad (3)$$

where q'_z is the peripheral averaged local heat flux transferred at the fluid-solid interface at different axial locations along the channel (only three heating sides included; the adiabatic top

wall is not included for calculating the peripheral average).

The dimensionless bulk fluid and channel wall temperatures are given by:

$$\Theta_f = \frac{\bar{T}_f - \bar{T}_{fi}}{\bar{T}_{fo} - \bar{T}_{fi}} \quad (4)$$

$$\Theta_w = \frac{\bar{T}_w - \bar{T}_{fi}}{\bar{T}_{fo} - \bar{T}_{fi}} \quad (5)$$

where \bar{T}_{fi} and \bar{T}_{fo} are the average bulk fluid temperature values at the channel cross section location $z = 0$ (inlet), and $z = L$ (outlet), respectively; \bar{T}_f is the average bulk fluid temperature at any location and \bar{T}_w is the peripheral average wall temperature at the same location (here too, only three heating sides are included to calculate the peripheral average wall temperature). The local Nusselt number is then given by:

$$Nu_z = \frac{h \cdot D_h}{k_f} \quad (6)$$

where $h = \frac{q'_z}{(\bar{T}_w - \bar{T}_f)}$ (7)

RESULTS AND DISCUSSION

Keeping ω_f , δ_f and ω_s constant, the thickness of the substrate is varied such that $\delta_{sf} = 2, 8$ and 16 , respectively. In addition, for each δ_{sf} , simulations have been performed for $k_{sf} = 26$ and 653 (which corresponds to the properties of stainless steel ($k \sim 16$ W/mK) and copper ($k \sim 400$ W/mK), respectively). The working fluid for all simulations is water at an inlet temperature of 300 K. Two flow situations, i.e., $Re = 200$ and 500 have been simulated. Thus, a total of twelve cases have been considered to study the conjugate nature of the problem. The axial variation of average bulk fluid temperature, peripheral average local wall temperature and the peripheral average local heat flux at the fluid-solid interface, are the main parameters of interest which allow us to discern the extent of axial conduction on the local Nusselt number, for a given flow and geometry configuration. As has been noted earlier, the heat flux is applied at certain finite distance away from the channel wall; in addition, the bottom surface on which constant heat flux has been applied is parallel to one conducting channel wall, i.e. the bottom fluid-solid interface, and perpendicular to other two side walls of the channel (see Fig. 1.), the top surface enclosing the fluid channel is kept insulated, as noted earlier.

To find the actual heat flux experienced at the three walls of the channel experiencing conjugate heat transfer, the axial variation of ϕ at the wall-fluid interface is presented in Fig. 3., for all the twelve cases under study. The fact that the actual heat flux experienced at the fluid-solid interface is approximately constant along the channel length at low conductivity ratio (k_{sf}), irrespective of the thickness ratio (δ_{sf}) and flow rate (Re) is exemplified in Fig. 3. For $k_{sf} = 635$, the

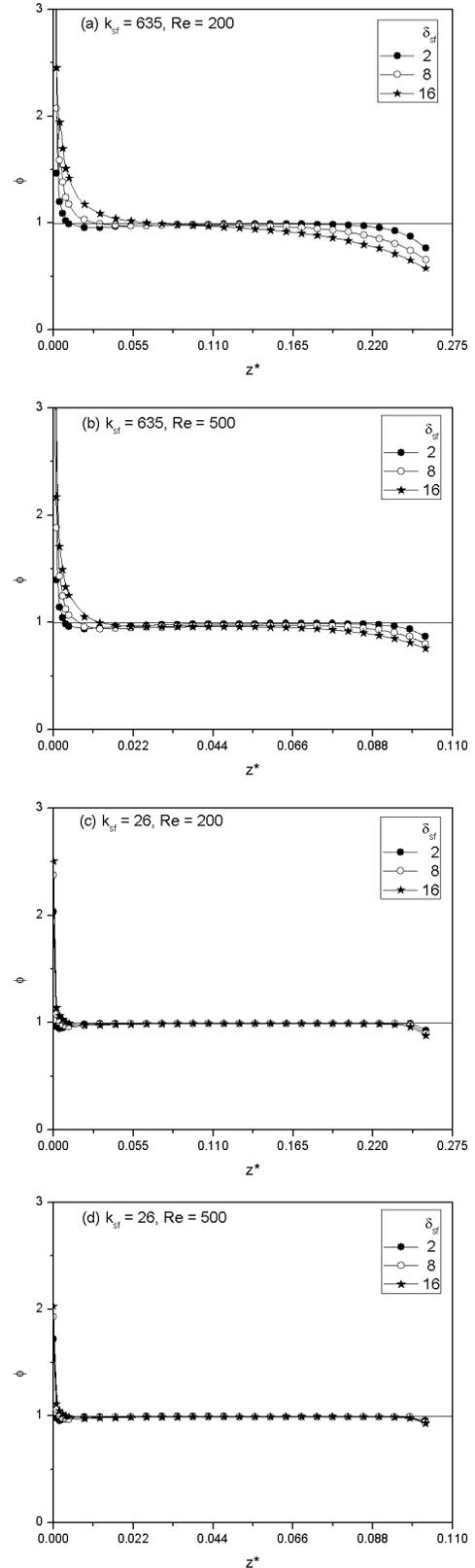


FIGURE 3. VARIATION OF DIMENSIONLESS LOCAL SURFACE HEAT FLUX ALONG THE CHANNEL

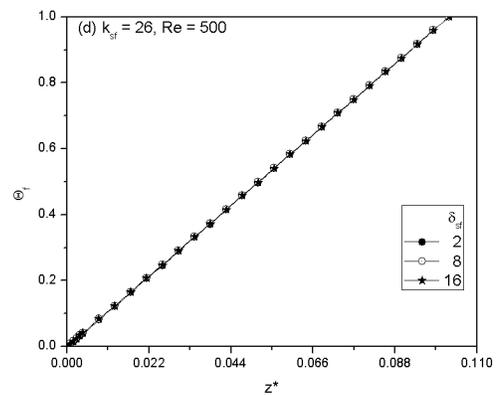
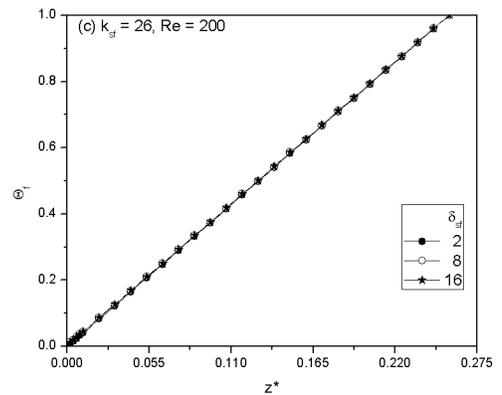
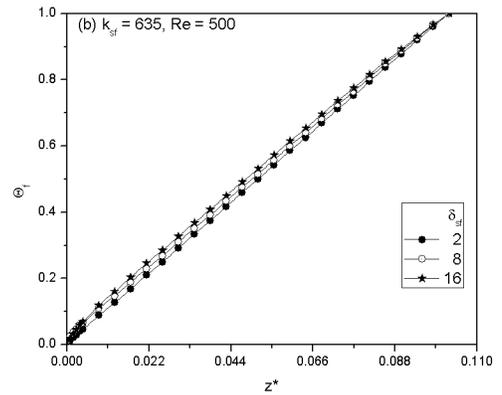
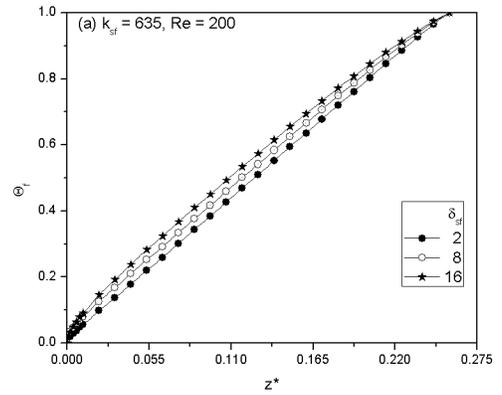
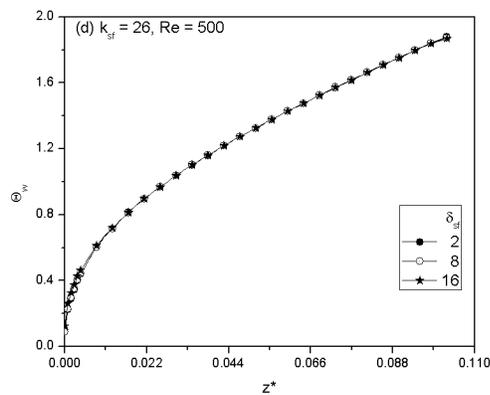
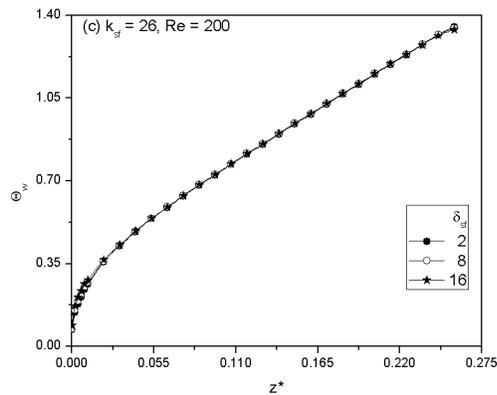
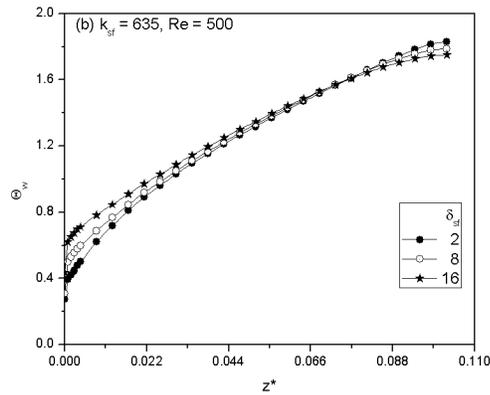
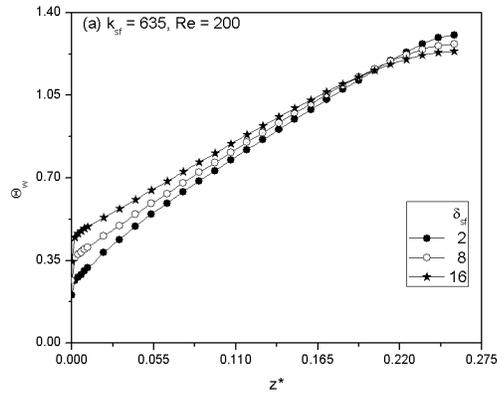


FIGURE 4. VARIATION OF DIMENSIONLESS LOCAL WALL TEMPERATURE ALONG THE CHANNEL

FIGURE 5. VARIATION OF DIMENSIONLESS BULK FLUID TEMPERATURE ALONG THE CHANNEL

peripheral average flux experienced at the fluid-solid interface is definitely not constant and substantially distorts from the constant value \bar{q}' applied at the bottom of the substrate. The distortion increases with increasing δ_{sf} . Increasing the thermal conductivity of the substrate and its thickness, essentially decreases the effective diffusive thermal resistance which the substrate offers to the heat flow through it, for a given $\bar{T}_w|_{z=0}$ and $\bar{T}_w|_{z=L}$. This highlights the importance of knowing the true heat flux distribution at the interface for accurate estimation of local and average Nusselt numbers under such operating conditions.

Next, the axial variation of channel wall temperature is explored. The axial variation of Θ_w is depicted in Fig. 4a-4d, for all the 12 cases considered. The corresponding variation of Θ_f for these cases is depicted in Fig. 5a-5d.

For low substrate conductivity system ($k_{sf} = 26$), the variation of Θ_w and corresponding Θ_f practically coincide with the ‘classical’ behavior of constant heat flux systems with no conjugate effects. The variation of Θ_f is practically linear, as expected, irrespective of change in thickness of the substrate. Variation of Θ_w also becomes nearly linear after the initial faster rate of increase due to high heat transfer coefficient in the developing region. The qualitative nature of axial variation of both, Θ_w and Θ_f , is not sensitive to the flow Re. For a given Re, after the flow fully develops, the difference between the wall and the fluid temperature becomes constant, which is expected as the heat flux experienced by the fluid at the fluid-solid boundary is not very different from the constant flux applied at the bottom substrate wall, as was discussed earlier in Fig. 3c-d.

When the conductivity ratio is higher ($k_{sf} = 635$), corresponding to a substrate of higher thermal conductivity, the axial variation of wall temperature starts getting affected by the axial conduction in the substrate. With increasing thickness ratio (δ_{sf}), the distortion in the axial variation of wall temperature increases as compared to the classical variation, in conjunction with the alteration in the heat flux variation, as discussed earlier in Fig. 3a, 3b. As can be noted, increasing axial conduction in the substrate with increasing substrate thickness, or reduced flow Re, makes $\bar{T}_w|_{z=0}$ to increase while $\bar{T}_w|_{z=L}$ starts decreasing. On similar lines, the variation of fluid temperature also does not remain linear as the heat flux received by the fluid per unit axial distance it travels in the channel is not uniform and axial heat conduction is substantial. Thus, an asymptotic situation can be imagined wherein as $k_{sf} \rightarrow \infty$, the constant heat flux applied at a distance from the fluid-solid interface, gets manifested as a constant temperature boundary condition to the fluid, resulting in lower effective Nu.

The direct implication of the axial variation of heat flux on the fluid-solid interface (as discussed in Fig. 3.) and the axial variation of wall and fluid temperatures, respectively (as discussed in Fig. 4 and 5.) is witnessed in the behavior of the local Nusselt number along the channel, as shown in Fig. 6a-d.

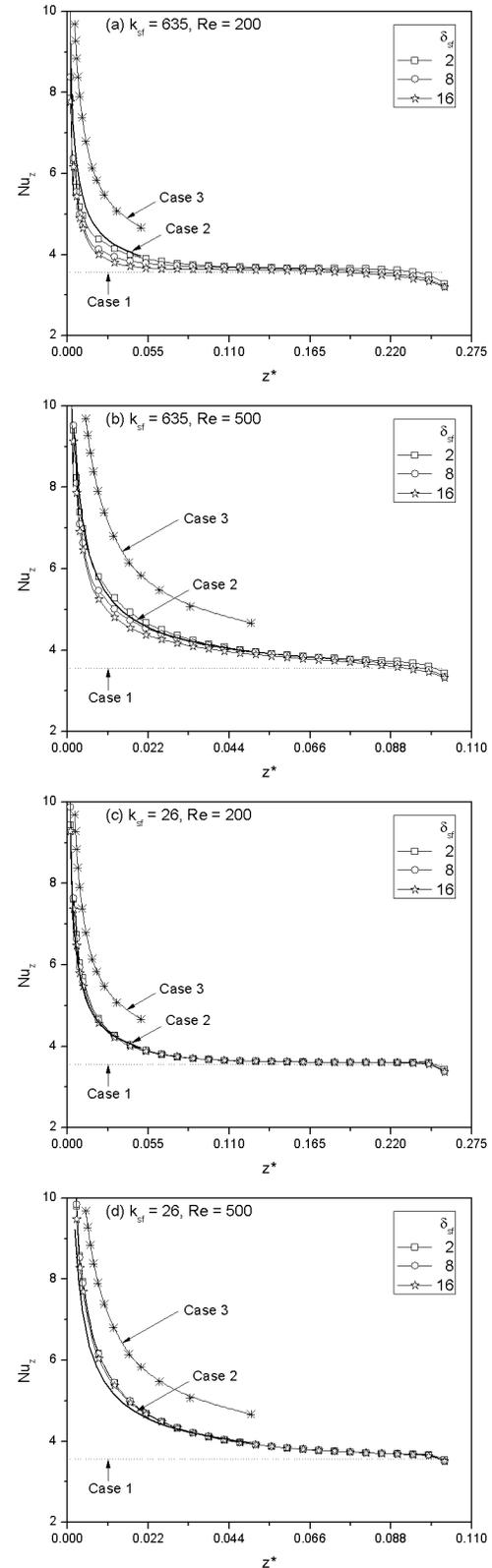


FIGURE 6. VARIATION OF LOCAL NUSSLETT NUMBER ALONG THE CHANNEL

For benchmarking, the figure also depicts Nu_z estimates for three reference cases, which are not exactly as the present case under study, nevertheless are similar in some respects:

- Case 1: Laminar, fully developed flow in square channel subjected to constant heat flux condition from three sides, the top side being adiabatic, without any conjugate effects being considered. Under such conditions for this case, Nu_z is constant and is equal to 3.556 [21].
- Case 2: Correlation developed by Lee and Garimella [22], for hydrodynamically fully developed but thermally developing flows (This correlation is valid in the thermally developing zone only).
- Case 3: Data for simultaneously developing laminar flow in a square channel for a fluid having $Pr = 0.7$, as reported in Shah and London [21]. For simultaneously developing flows, dependency of Nusselt number on Prandtl number is much stronger [21]. For the present study, since $Pr = 5.85$, Nu_z is always smaller than this reference case having $Pr = 0.7$, as expected for a simultaneously developing flow.

At lower conductivity ratio (k_{sf}), the variation of Nu_z does not show the effect of conjugate heat transfer. After the flow is fully developed, the results match with Case 1. Also, when $Re = 200$ and k_{sf} is low, i.e. 26, the correlation of Case 2 is able to predict the data reasonably well; this is because the hydrodynamic flow development length is very small. As the effective diffusive thermal resistance of the substrate goes on decreasing, it reduces the local Nusselt number. Another way of arguing this effect is to note that, decreasing thermal resistance of the substrate vis-à-vis the convective heat transfer in the liquid, actually makes the fluid-solid temperature more 'isothermal'. Thus, a constant heat flux boundary condition applied at a distance away from the fluid-solid interface is 'seen' by the convective fluid as tending towards an 'isothermal' boundary condition at the interface. This, in turn, reduces the local Nusselt number. Complementary observations have been reported by Zhang et al. [9], who have studied a constant temperature boundary condition on the outside surface of a thick circular tube, which distorts towards a constant flux condition at the fluid-solid interface, i.e., at the inner radius of the tube, as conjugate effects start dominating.

SUMMARY AND CONCLUSIONS

Most practical situations involving convective heat transfer in micro-devices employ laminar flows which are mostly simultaneously developing in nature. In addition, many situations demand flow channels on a flat surface and the application of heat (either constant heat flux or constant temperature). As the ratio of fluid flow area to the total cross section of the substrate is rather low for mini-micro devices, the possibility of conjugate heat transfer cannot be neglected.

In this work we have presented three-dimensional numerical heat transfer simulation of a square microchannel (0.4 mm x 0.4 mm) formed on a rectangular parallelepiped shaped substrate. The substrate geometry and thermo-physical properties of the solid substrate and the flowing liquid are part

of the conjugate heat transfer formulation with simultaneously developing flow conditions at different flow Re . The study clearly reveals that:

- (i) The thermal conductivity ratio of the solid substrate to the working fluid is an important parameter which determines the extent of the axial conduction in the substrate.
- (ii) For a given thermal conductivity of the substrate material, thicker substrates lead to a reduction in thermal resistance and therefore an increase in the axial back-conduction. Therefore, least possible thickness (so that the ratio of fluid flow area to the substrate cross-section is as high as possible) should be employed for increasing the Nusselt number in the channel.
- (iii) Increasing flow Re increases the flow development length and reduces the residence time of the fluid inside the channel. Also, the outlet temperature of the fluid (and therefore the wall temperature at the channel outlet) reduces with increasing flow Re . This, in turn, reduces the axial back-conduction.
- (iv) The local Nusselt number which ought to have been realized under true constant heat flux boundary condition without any conjugate effects gets reduced due to axial conduction in the substrate. The axial fluid temperature no longer remains linear. The difference between the wall temperature and the fluid temperature do not remain constant in the fully developed section of the channel.
- (v) Unless true distribution of temperature at the fluid-solid interface, true bulk fluid temperature and the heat flux is known, the estimates of Nusselt number can be misleading. This requires that the system should be thoroughly analyzed for conjugate effects.
- (vi) All other factors remaining the same, thin substrates made of low conducting materials, experiencing high flow rates provide a better solution in terms of minimizing the effect of axial conduction in the substrate.

In this work we have only investigated one Prandtl number. The Prandtl number of the fluid will also play a role in the conjugate system, e.g., low Prandtl number fluids will behave quite differently as axial conduction in the fluid domain will also become important, although this effect is generally not encountered in common fluids for heat transfer application. Parametric study of Prandtl number variation needs to be investigated in future for conjugate microchannel systems so that a unified single discerning parameter can emerge which will explicitly quantify the extent of the conjugate nature of heat transfer in a given system with simultaneously developing flows.

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