Effect of Prandtl number on internal convective heat transfer in laminar single-phase pulsating flows

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Abstract. In this paper, heat transfer during simultaneously developing single-phase laminar flow inside a pipe has been investigated numerically. A two dimensional axisymmetric model of a circular mini-channel is used, with a constant wall temperature boundary condition. In the present simulations, flow Reynolds number (*Re*) and amplitude ratio (*A*) are kept constant to 200 and 0.2, respectively. The variable quantities are imposed pulsating frequency (*f*) or non-dimensional frequency (Womersley number, *Wo*) and Prandtl number (*Pr*). Study is conducted for three different imposed flow frequencies of 5 Hz, 15 Hz and 25 Hz respectively while Prandtl number values are varied from 2.43 to 3404. These flow parameters can be grouped together to form different time scales which are important to explain the heat transfer behavior in laminar single-phase pulsating flow. It is observed that for the range of frequency (*f*) and Prandtl number (*Pr*) used in present work, change in heat transfer is marginal which is of no practical significance. Enhancement of heat transfer coefficient due to such periodic pulsatile internal flows, over and above the non-pulsatile regular flow conditions, is questionable, and at best, rather limited.

1 Introduction

The occurrence of single-phase pulsating flow can be encountered in many engineering systems, such as bio-fluidic devices, cooling systems for miniaturized electronic circuits, reciprocating pumps, etc. The hydrodynamic and thermal boundary layers of flow are expected to get disturbed due to externally imposed pulsations. Effect of such disturbances on the thermal performance of the flow has been an issue of debate in the existing open literature due to involvement of different parameters and competing time scales. A lot of studies on the fluid flow and heat transfer in pulsating flows have been performed in the literature with different parameters like pulsating frequency (*f*), pulsating amplitude ratio (*A*), mean flow Reynolds number (*Re*) and Prandtl number (*Pr*). While there are coherent conclusions for the hydrodynamics of such flows, contradictory results are found regarding the thermal performance of single-phase pulsating flow. Hence, revisiting the problem of heat transfer during laminar single-phase pulsating flow in circular mini-channels is necessary. The explicit effect of pulsations on local and average heat transfer in internal convection is an interesting problem *per se* due to several complexities involved related to different time scales.

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Various time scales are important for explaining the mechanism of momentum and heat exchange in pulsating flows. They are (i) imposed time scale by external pulsations (*1/f, f* is the frequency of imposed time scale) (b) timescale for momentum diffusivity (D^2/ν, ν is the kinematic viscosity) (c) time scale for thermal diffusivity (D^2/α, α is the thermal diffusivity) and (d) convective time scale (L_h/U_{av}, L_h is the heated length) which represents the residence time of a fluid particle inside the heated channel. Two non-dimensional numbers which can possibly reflect these competing time scales are $Wo^2 = D^2\cdot(\omega/\nu) = 2\pi\cdot St\cdot Re$ and $Wo^2\cdot Pr = D^2\cdot(\omega/\alpha)$; former being the ratio of time scale of momentum diffusivity to the externally imposed time scale and the latter representing the ratio of time scale of thermal diffusivity to the externally imposed time scale. In the absence of any geometrical disturbances, transverse exchange of momentum and energy is only possible by diffusion in the laminar flow condition. In general, for medium to high Peclet number ($Pe = Re\cdot Pr$) situations, diffusion in the axial direction is not influential as compared to convection but the degree of transverse exchange of species can contribute in augmenting/deteriorating the overall performance. In such situation, effect of Pr is an important parameter to study.

Several researchers have been working towards finding the insight of pulsatile flows from early decades of last century. In 1929, Richardson and Tyler [1] measured the velocity profile of the pulsating flow in tubes of different cross sections and sizes and found the 'annular effect'. They observed that the maximum velocity occurs near the wall and not at the center of the pipe. Womersley [2] obtained the drag in pulsating pipe flow due to pressure gradient. In 1956, Uchida [3] obtained an analytical solution of the velocity profile for the pulsating flow in a tube and confirmed the 'annular effect'. In this solution, phase-lag between velocity and pressure gradient was also observed and it was found that lag increases with frequency and asymptotically reaches to 90° after a threshold frequency.

Faghri et al. [4] discussed about the heat transfer in pulsating flow through a circular pipe with constant heat flux boundary condition. By theoretical analysis, expression for Nusselt number was found for low frequency pulsatile flows. It was shown that increasing the amplitude of oscillations augmented the heat transfer while increasing Pr had an adverse effect on augmentation. Seigel [5] argued that, for forced convection in slow laminar flow in a channel, oscillations reduced the heat transfer coefficient. Cho and Hyun [6], numerically investigated the effect of pulsation in a pipe using laminar boundary layer equations. They observed that at the fully developed downstream region, Nu may increase or decrease depending on the frequency parameter. It was also pointed out that deviation of heat transfer coefficient became prominent for low Prandtl number ($Pr << 1$) fluids. Kim et al. [7] studied thermally developing but hydrodynamically fully developed channel flow with isothermal channel wall. They showed that flow pulsations hardly affected the thermal behavior. Guo and Sung [8] have observed that for small amplitudes, heat transfer gets augmented within a band of operating frequency but at higher amplitudes, heat transfer is always augmented. Hemida et al. [9] observed that pulsations produce little change in heat transfer, this being always negative and also mentioned that for low Pr fluid this decrease is relatively less. This small change is limited to the thermally developing region only. Yu et al. [10] and Chattopadhyay et al. [11] observed no change in time-averaged *Nu* due to pulsations. Hence, the literature survey demonstrates that in a single-phase laminar pulsating flow (a) heat transfer behavior does not converge to coherent conclusions; the results are not only sparse but inconsistent and sometimes contradictory (b) as mentioned earlier, *Pr* may govern heat transfer behavior.

It is noteworthy that to maintain the same *Wo* (for example, 2.5) we need to have a flow pulsating at 0.5 Hz for *Pr* = 2.43 and at 1150 Hz for *Pr* = 3404. Maintaining the sinusoidal waveform at such high frequencies is difficult in real experimental situation. So, we have taken *f* as the parameter instead of *Wo*. Therefore, in this work, numerical investigation of heat transfer during the simultaneously developing laminar single-phase pulsating flow has been undertaken for constant wall temperature within a wide range of *Pr* (= 2.43 - 3404) and operating frequency *f* (5 Hz - 25 Hz). The flow Reynolds number (*Re*) and amplitude ratio (*A*) are kept constant to 200 and 0.2.

2 Mathematical Formulation and Numerical Scheme

2.1 Governing Equations

The flow and heat transfer are governed by N-S and energy equations. The non-dimensional generalized form of governing equations is presented below with applied boundary conditions. For an unsteady, laminar axisymmetric flow, the governing equations can be written as:

Continuity Equation: $$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

Momentum Equation: $$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v} \tag{2}$$

Energy Equation: $$\rho c_p\left(\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T\right) = k_f\nabla^2 T\;; \tag{3}$$

The non-dimensional quantities are defined as follows:

$$u^* = \frac{u}{u_{av}}; v^* = \frac{v}{v_{av}}; T^* = \frac{T-T_o}{T_w-T_o}; r^* = \frac{r}{D}; x^* = \frac{x}{D}; x_t^* = \frac{x}{Re.Pr.D}; p^* = \frac{p-p_{atm}}{\rho u_{av}^2}; t^* = \frac{t.u_{av}}{D}$$

Boundary Conditions:

Inlet: $$u_{in}^* = \frac{u_{in}}{u_{av}} = 1 + A\sin\left(\frac{Wo^2}{Re}t\right)\;;\; v^* = 0; T^* = 0 \tag{4}$$

Outlet: $$p^* = 0 \tag{5}$$

Wall: T_w = constant; No-slip at wall (6)

Initial conditions: $u_{in}^* = 1$; $p^* = 0$; $T^* = 0$

Different Nusselt numbers are defined as:

$$Nu = \frac{q'' D_h}{(T_w - T_b)k_f}; Nu_{ta} = \frac{1}{\theta}\int_0^\theta Nu(t)dt; Nu_r = \frac{Nu_{unsteady}}{Nu_{steady}}\;;\; \theta \text{ is time period} \tag{7}$$

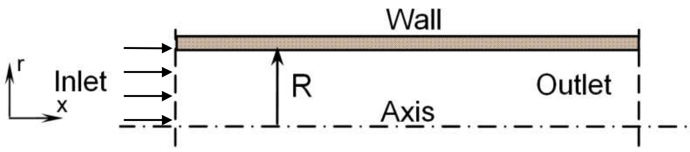

Fig. 1. Schematic of the computational domain

Figure 1 shows the computational domain for the present simulations. The diameter and length of the channel are kept as 1 mm and 100 mm, respectively. The flow is simultaneously developing with constant wall temperature boundary condition.

2.2 Numerical Scheme

The governing equations are solved by using a commercial CFD software package ANSYS® Fluent-15.0. Second-order upwind scheme is used to discretize the convective terms of the momentum and energy equatoins. The pressure-velocity coupling is handled using the SIMPLE algorithm. Implicit integration method is used for temporal discretization. The time varying velocity profile is imposed at the inlet using a User Defined Function. A grid independence study (Fig. 2b) revealed that a non-uniform (finer mesh near the wall) mesh having 25×500 cell distribution is optimum. An appropriate time step ($\Delta t = \theta/80$) was chosen such that it ensured a converged solution without costing excessive computational time. The convergence criteria were set as 10^{-5} for momentum and 10^{-8} for energy equation.

3 Result and Discussion

Before discussing the main results, it is necessary to validate and benchmark the numerical scheme with published data. Figure 2 (a) shows the fluctuating component of velocity profile obtained from present simulations (Wo = 10) along with analytical solution of Uchida [3] which clearly demonstrates the annular effect. Figure 2 (b) shows the fluctuating component of velocity profile at different phases which shows that at some phases the fluctuating component is out of phase, i.e., the direction of average velocity and fluctuating velocity are different. Figure 2 (c) shows the instantaneous velocity profile at different locations during the development of flow. It demonstrates that as the flow reaches to fully developed zone, velocity profile approximately becomes parabolic; 'annular effect' is observed only in the fluctuating velocity profiles. Figure 2 (d) shows axial variation of Nusselt number obtained from present simulation for the laminar steady flow for different grid points for Re = 200 and Pr = 2.4. The calculations have been done at three different grids to test the grid independency and it is found quite satisfactory. This figure also shows the value of Nu = 3.66 for constant wall temperature case and it is seen from the present simulations that as the flow develops hydrodynamically as well as thermally, Nu approaches to the value of 3.66.

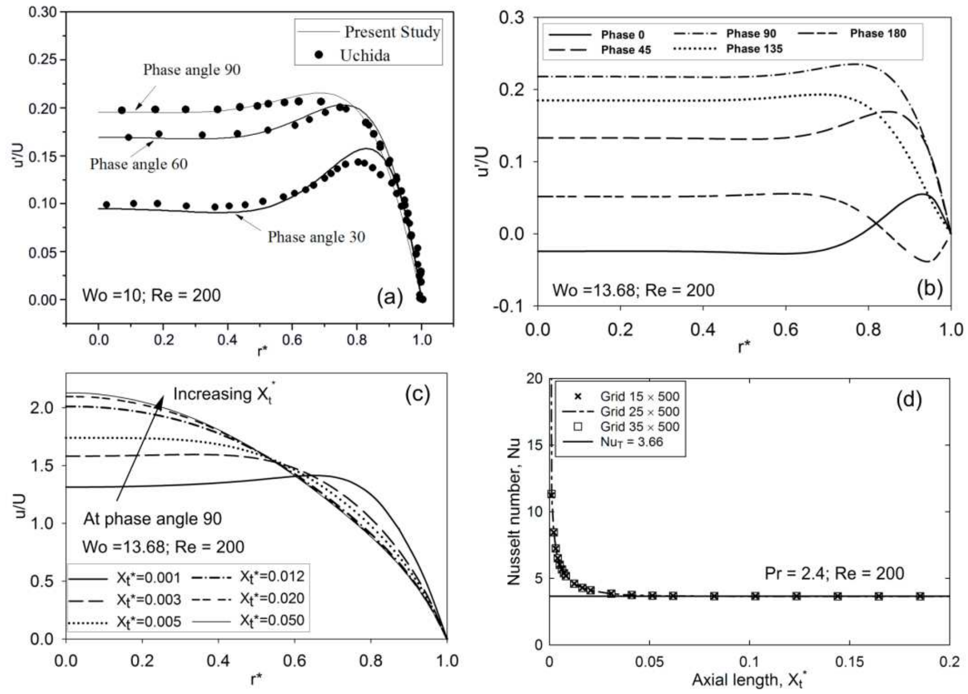

Fig. 2. (a) Fluctuating component of velocity profile obtained from present simulations and Uchida's solution for *Wo* = 10 (b) Fluctuating component of velocity profile at different phase for *Wo* = 13.68 in fully developed zone. (c) Velocity profile at various locations along the pipe during flow development. (d) Validation and grid independence of heat transfer.

3.1 Effect of Frequency

Figure 3 (a, b) shows the effect of frequency on the relative time-averaged *Nu* for two different *Pr*. First of all, it is clear from the results that for all imposed frequencies change in $(Nu_r)_{ta}$ is very marginal (practically insignificant). However, in that band of change, it can be seen that for a fixed value of *Pr*, whenever the imposed time scale is lower than the convective time scale (various time scales are shown in Table 1), $(Nu_r)_{ta}$ stabilizes in the very early part of the channel length while, when these time scales are comparable, variation in $(Nu_r)_{ta}$ prevails for greater length of the channel.

Table 1. Time scales for different prandtl numbers and frequencies

<table>
<tr><th>Pr</th><th>f</th><th>Wo</th><th>1/f</th><th>D^2/υ</th><th>D^2/α</th><th>L_h/U_{av}</th><td rowspan="7">Time Scales:
1/f: Imposed time scale
D^2/υ: Momentum diffusion time scale
D^2/α: Thermal diffusion time scale
L_h/U_{av}: Convective time scale</td></tr>
<tr><td rowspan="3">18.38</td><td>5</td><td>1.84</td><td>0.2</td><td>0.43</td><td>7.91</td><td>0.21</td></tr>
<tr><td>15</td><td>3.18</td><td>0.067</td><td>0.43</td><td>7.91</td><td>0.21</td></tr>
<tr><td>25</td><td>4.11</td><td>0.04</td><td>0.43</td><td>7.91</td><td>0.21</td></tr>
<tr><td rowspan="3">2.43</td><td>5</td><td>7.9</td><td>0.2</td><td>7.94</td><td>19.29</td><td>3.97</td></tr>
<tr><td>15</td><td>13.68</td><td>0.067</td><td>7.94</td><td>19.29</td><td>3.97</td></tr>
<tr><td>25</td><td>17.66</td><td>0.04</td><td>7.94</td><td>19.29</td><td>3.97</td></tr>
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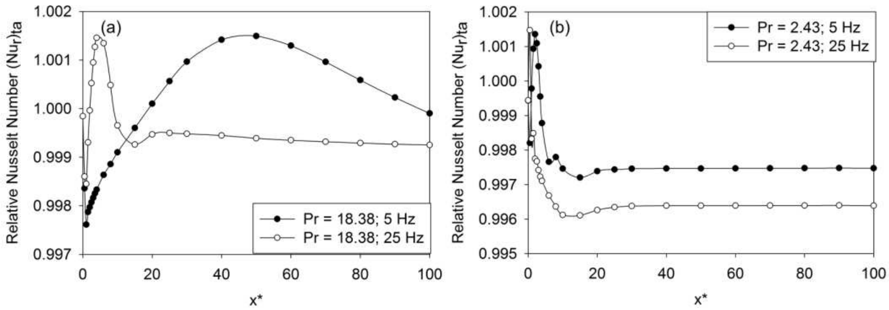

Fig. 3. Effect of frequency on the time-averaged Relative Nu for (a) Pr = 18.38, (b) Pr = 2.43.

3.2 Effect of Prandtl Number

Earlier, it has been seen that pulsation frequency does not exhibit the desired enhancment in the heat transfer as expected; rather, it detriorates the heat transfer sometimes. It is also outlined that in the absence of any other mechanism of transverse heat exchange, thermal diffusion will play a significant role in the enhancemnt of heat transfer by imposing the pulsation. In such a situation, *Pr* can be an effective parameter to change the heat transfer behaviour in single-phase pulsating flow. Hence, in this section, effect of *Pr* is observed on the *Nu* keeping *Re* and pulsation amplitude (*A*) fixed. Figure 4 (a, b) shows the axial variation of Nu_r at different phases of imposed pulsating velocity. It shows that Nu_r is almost symmetric about the steady-state *Nu* value, i.e., during some phases heat transfer is higher than the steady-state value while during the other phases, heat transfer is symmetrically opposite. Figure 4 (c) shows the temporal variation of Nu_r for different *Pr* at 5 Hz. It shows that Nu_r oscillates sinusoidally with time and with change in *Pr*, variation of amplitude of oscillation is not monotonous. This can be explained on the basis of different competing time scales involved. In an ideal situation (see Fig 4 (g)), the change in amplitude of oscillations is monotonous with *Pr* keeping momentum diffusivity fixed, i.e., *Wo* and *Re* are fixed. Thus, among all the time scales, thermal diffusion time scale is the only variable. For this case heat transfer decreases with increasing *Pr* (or decreasing thermal diffusivity). The fluid having lower thermal diffusion time scale would have better mixing. The thermal inertia of the fluid having higher thermal diffusion time scale would have higher thermal inertia and hence, poorer response to imposed fluctuations. But, in the situation where more than one time scales come into play, like in the present study, a non-monotonous variation in the amplitude of Nu_r is observed. Phase lag in Nu_r is also observed with change in *Pr*. It is understood from the fact that for the same *Re*, fluids with higher *Pr* would have lower convective time scale (L_h/U_{av}), i.e., higher the *Pr*, higher the momentum diffusivity and hence higher velocity for same *Re*. In such scenario, heat transfer is primarily dominated by convection which will eventually become in-phase with fluctuating velocity.

Fig. 5 (d-f) show the axial variation of $(Nu_r)_{ta}$ with different *Pr* for frequencies of 5 Hz, 15 Hz and 25 Hz, respectively. Qualitatively, the plots are similar for a specific *Pr*. For the chosen range of *Pr*, the thermal diffusivity decreases in the order 10.38, 18.38, 106.6 and 2.43 and 3.27 which resulted in fluctuations of heat transfer being propagated more in the entrance length with increasing thermal diffusivity. It can be seen that as the flow becomes thermally developed, all the curves become axially constant. The overall change in the heat transfer is insignificant from the practical viewpoint. In that very marginal change, $(Nu_r)_{ta}$ decreases with decreasing *Pr*. Figure 4 (h) compares the results of the present simulations with the results available in the literature.

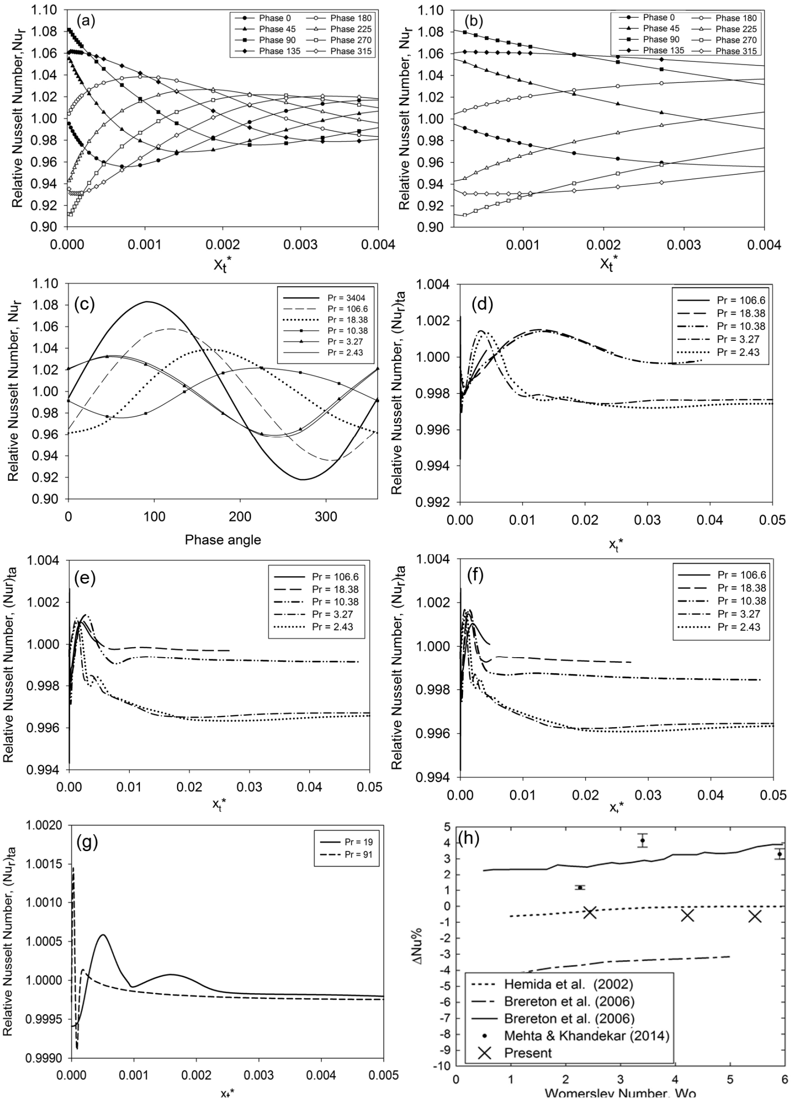

Fig. 4. Relative Nusselt number at different phases (a) Pr = 106.6 (b) Pr = 18.38 at 5 Hz.(c) Temporal variation of relative *Nu* for different *Pr* at 5 Hz; Axial variation of time-averaged relative *Nu* for different *Pr* at (d) 5 Hz (e) 15 Hz (f) 25 Hz; (g) Axial variation of time-averaged relative *Nu* for different *Pr* at same *Wo*. (h) Comparison of present simulations with literature.

4 Conclusions

Based on the results of the preceding sections, it can be stated that single-phase laminar pulsating flow does not show any significant enhancement as compared to the steady flow. Explicit effect of *Pr* on the heat transfer for pulsating flow has been studied. There is negligible effect on the relative time-averaged Nusselt number. In the range of frequencies studied, $(Nu_r)_{ta}$ decreases with decreasing *Pr* for any fixed frequency. In most of the cases, disturbance is observed in a very narrow entrance length of the channel. These disturbances propagate to greater lengths for fluids with higher thermal diffusivity. Mechanism of heat transfer in the pulsating flow in real situation is not straight forward, various time scales govern the flow and heat transfer phenomenon.

5 References

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