# Assignment 2

1. Consider a bimaterial interface where the top part $S^+$ $(y > 0)$ is occupied by a material with elastic constants $\mu_1$ and $\nu_1$ (Shear modulus and Poisson's ratio respectively) while the lower part $S^-$ $(y < 0)$ is occupied by a material with $\mu_2$ and $\nu_2$. K-M potentials $\psi_1,\ \chi_1$ and $\psi_2,\ \chi_2$ are defined for the upper and lower half planes, pertaining to the two materials.

   Prove that, considering continuity of stresses and displacements across the perfect boundary between the two materials show that

$$
\begin{aligned}
(\mu_1 + \mu_2\kappa_1)\psi_1'(z) &= \mu_1\theta(z) + \phi'(z) \ \text{ for } z \in S^+ \\
(\mu_2 + \mu_1\kappa_2\psi_2'(z) &= \mu_2\theta(z) + \phi'(z) \ \text{ for } z \in S^-
\end{aligned}
$$

   where, the functions $\theta(z)$ and $\phi(z)$ are analytic all over the plane and are defined as:

$$
\begin{aligned}
\theta(z) &= \psi_1(z) - z\bar{\psi}_2''(\bar{z}) - \bar{\chi}_2''(\bar{z}) - \bar{\psi}_2'(\bar{z}) \ \text{ on } S^+ \\
\theta(z) &= \psi_2(z) - z\bar{\psi}_1''(\bar{z}) - \bar{\chi}_1''(\bar{z}) - \bar{\psi}_1'(\bar{z}) \ \text{ on } S^-
\end{aligned}
$$

   and

$$
\begin{aligned}
\phi(z) &= \mu_2\kappa_1\psi_1(z) + \mu_1 z\bar{\psi}_2'(\bar{z}) + \mu_1\bar{\chi}_1'(\bar{z}) \ \text{ on } S^+ \\
\phi(z) &= \mu_1\kappa_2\psi_2(z) + \mu_2 z\bar{\psi}_1'(\bar{z}) + \mu_2\bar{\chi}_2'(\bar{z}) \ \text{ on } S^-
\end{aligned}
$$

2. Using the Westergaard functions (see Problem 7 of Assignment 1) prove that the displacements $u$ and $v$ in case of a plane stress Mode I crack can be found by

$$
\begin{aligned}
2\mu u &= \frac{1-\nu}{1+\nu}\mathrm{Re}\phi - y\mathrm{Im}\psi' \\
2\mu v &= \frac{1-\nu}{1+\nu}\mathrm{Re}\phi + y\mathrm{Im}\psi'.
\end{aligned}
$$

3. Find the stress intensity factors for a Griffith crack of length $2a$ inclined at an angle $\alpha$ to the x-axis.