Appendix E

Assignment 3

1. Consider that you are peeling off an adhesive plaster stuck onto a rigid base as shown in Fig. E.1(a).

   (a) From the general definition of the energy release rate

   $$\mathcal{G} = \frac{d}{dA}\left[U_{ext} - U_s\right] \tag{E.1}$$

   show that the $\mathcal{G}$ is given by

   $$\mathcal{G} = \frac{P_\infty^2}{2Eh} + P_\infty(1 - \cos\theta). \tag{E.2}$$

   (b) If we want to analyse a crack of the kind shown in Fig. E.1(b) it is useful to treat it in an isolated manner and subject it to a force P_o and a moment M_o at the left end so that these loads describe the interaction of the crack with the rest of the body.

   Now find the equivalent P_o and M_o for the peeling problem described in the first part of this question.

2. Take a circular contour around the sharp crack tip as shown in Fig. E.2. Using the expression for the J inegral show that in the case when LEFM holds, the J integral

in Mode I is given by

$$J = \frac{1-\nu^2}{E} K_I^2. \tag{E.3}$$

Note that we have derived this for $\mathcal{G}$ in the class. The above result is further proof of the fact that $\mathcal{G} = J$ as long as small scale yielding holds.

3. The J integral is defined as

$$J = \int_C [W\delta_{1i} - \sigma_{ij} u_{j,1}] n_i dC. \tag{E.4}$$

Convince yourself that this can be equivalently written as

$$J = \int_C [\sigma_{ij} - W\delta_{i1}] m_i q_1 dC, \tag{E.5}$$

where, q_1 is a function that is unity on Γ and vanishes on C_1 (see Fig. E.3). As shown in this figure, m_j is the outward normal so that on the crack faces $m_1 = 0$, and $m_2 = \pm 1$ and $m_j = -n_j$ on Γ.

Convert this form of J into an area integral over the shaded area shown in the Figure. This form of the J integral (as an area integral rather than a line integral) is found to be more reliable in numerical simulations.

4. Consider a LEFM problem C in which we have a mixed mode crack. So,

$$J = \frac{\beta K_I^2}{E} + \frac{\beta K_{II}^2}{E} \tag{E.6}$$

Consider that the problem C can be obtained by superposing two problems A and B. Show that

$$J^C = J^A + J^B + M^{A,B}, \tag{E.7}$$

where $M^{A,B}$ is given by

$$M^{A,B} = \int_\Gamma W^{A,B} dy - \left[T_i^A \frac{\partial u_i^B}{\partial x} + T_i^B \frac{\partial u_i^A}{\partial x} \right] d\Gamma. \tag{E.8}$$

And,

$$W^{A,B} = C_{ijkl} u_{i,j}^A u_{k,l}^B = C_{ijkl} u_{i,j}^B u_{k,l}^A. \tag{E.9}$$

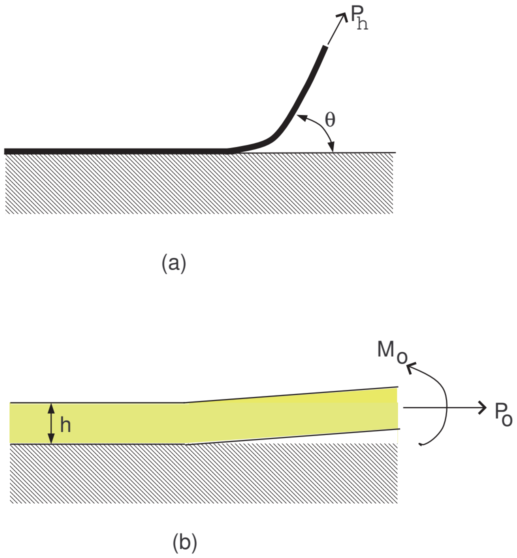

Figure E.1: Figures pertaining to Problem 1

Further, using the connection between J and $K_{I/II}$ show that

$$M^{A,B} = \frac{2\beta}{E}\left[K_I^A K_I^B + K_{II}^A K_{II}^B\right]. \tag{E.10}$$

The quantity $M^{A,B}$ is known as the interaction integral and can be used to derive stress intensity factors of mixed mode problems from stress intensity factors of problems with known solutions.

Find the values of $M^{A,B}$ when,

$$K_I^A = 1 \;\; K_{II}^A = 0 \;\text{ i.e. problem A is purely Mode I and} \tag{E.11}$$

$$K_I^A = 0 \;\; K_{II}^A = 1 \;\text{ i.e. problem A is purely Mode II.} \tag{E.12}$$

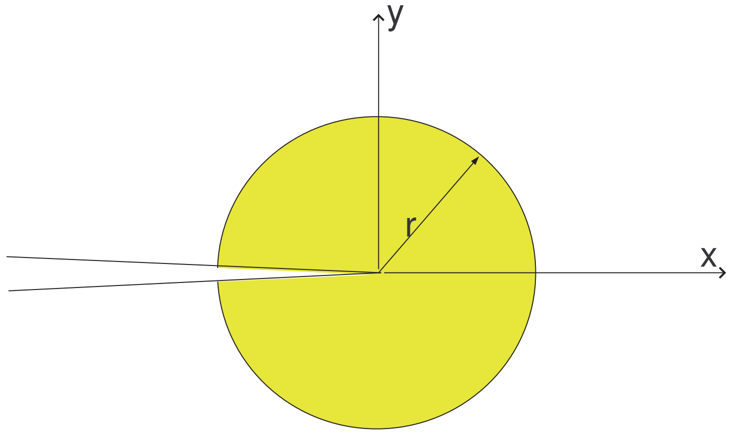

Figure E.2: Figure for Problem 2

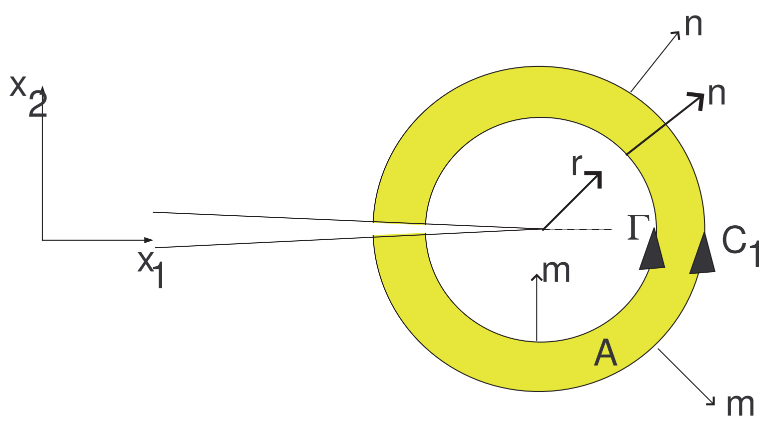

Figure E.3: Figure for Problem 3