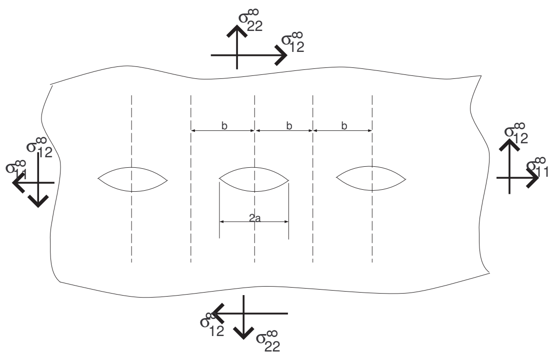

Figure 4.1: Infinite array of collinear cracks spaced $2b$ apart and lengths $2a$.

far-field (see, Fig. 4.3) stresses σ but without a crack (P). This can be looked at as a cracked plate under uniaxial tension but with loads applied on the crack faces to keep them closed. That problem again can be broken down into a problem A involving an unloaded crack under remote tension and the problem B of a crack with the faces forced shut! So,

$$K_I^P = K_I^A + K_I^B. \tag{4.47}$$

But since $K_I^P = 0$,

$$K_I^A = -K_I^B = \sigma\sqrt{\pi a}. \tag{4.48}$$

Reversing the sign of the σ term (for internally pressurised crack) we get that

$$K_I^A = p\sqrt{\pi a}. \tag{4.49}$$

4.5 A few practical applications

4.5.1 Leak before break

The concept of 'leak before break' is routinely applied to pressure vessel designs. The underlying idea is that any crack or flaw that develops in the component will grow to a

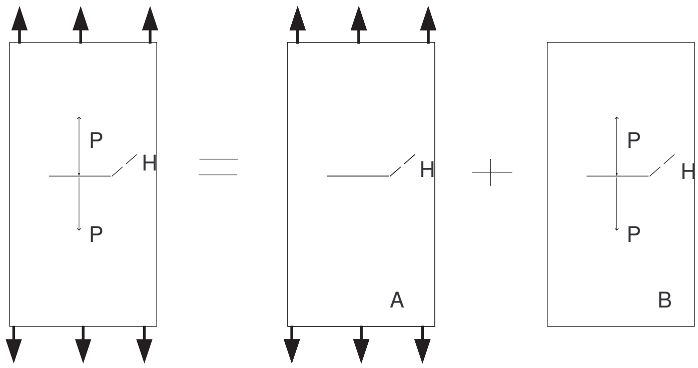

Figure 4.2: Superposition principle for SIFs.

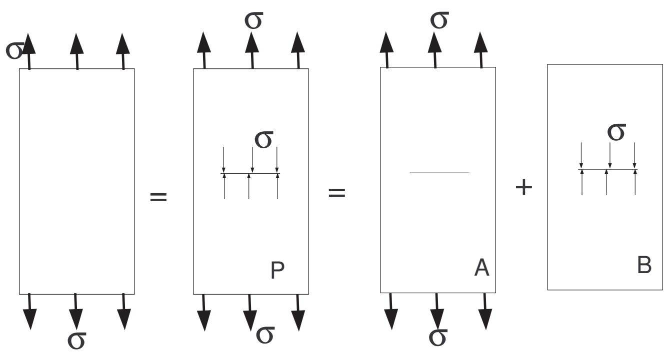

Figure 4.3: Superposition principle applied to a crack loaded with uniform pressure.

'through-wall' configuration and can be detectable by monitoring systems before reaching a size that can cause total component rupture.

Using stress intensity factor tables we will try to see what this concept implies in terms of pressure vessel design. Let us assume a simplified scenario where a crack of length a and width $2c$ ($c >> a$) exists in the vessel as shown in Fig. 4.4. Assume further that linear elastic fracture mechanics holds in the case considered. Now, if the radius of the vessel is large compared to the surface length c of the crack, the crack can be considered to be a plane strain edge crack and the stress intensity factor from the table supplied can be used.

$$K_I^r = \sigma^\infty \sqrt{\pi a} f_r \left(\frac{a}{R}\right), \tag{4.50}$$

where a is the crack length, R the radius of the vessel and $f_r(a/h)$ is a non-dimensional function. In case of a thin walled cylinder, $\sigma^\infty = pR/2h$, where p is the applied pressure. Once $K_I^r = K_{Ic}$ is satisfied, the crack starts growing radially and leaks the vessel. It is assumed that the aspect ratio a/c of the crack doesnot change while this happens. Once the crack becomes a through crack the possibility of its extending sideways along the wall surface arises. The pressure p_1 at this instant may be lower than p due to pressure drops taking place as a result of leakage. However, under certain circumstances (like a waterhammer effect or any other sort of dynamic loading) it is conceivable that $p_1 > p$. For crack growth along the wall,

$$K_I^a = \frac{pR}{2h} \sqrt{\pi c} f_a \left(\frac{c}{R}\right). \tag{4.51}$$

The crack will grow sideways once $K_I^a = K_{Ic}$ assuming that the stress intensity factors in both the cases are same. A relation that provides boundary between leak and break behaviour is

$$K_I^a = K_I^r, \tag{4.52}$$

using which we can plot curves of c/h against a/h with different values of p/p_1 (Fig. 4.5. Values of a/h to the left of these curves are expected to cause leak while values to the

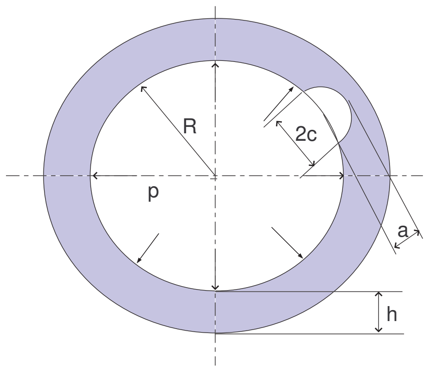

Figure 4.4: Geometry for leak before break configuration.

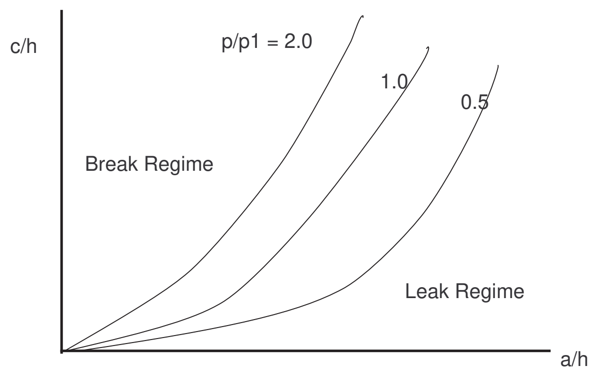

Figure 4.5: Curves of c/h vs a/h from a typical leak before break calculation.

right are expected to cause break.

The stress intensity factors K_I^a and K_I^r can also be related to that of a semielliptical edge crack for which the expression is

$$K_I = \frac{1.12\sigma^\infty(\pi a)^{1/2}}{I_2}\left[\sin^2\phi + \left(\frac{a}{c}\right)^2\cos^2\phi\right]^{1/4}. \tag{4.53}$$

Here, I_2 is the elliptic integral whose values are given in Table 4.1 and ϕ is the angle along the semi-ellipse. At $\phi = 90^o$,

$$K_I = K_I^a = \frac{1.12\sigma^\infty(\pi a)^{1/2}}{I_2}, \tag{4.54}$$

a/c	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I_2	1.000	1.016	1.1051	1.097	1.151	1.211	1.277	1.345	1.418	1.493	$\pi/2.$

Table 4.1: Values of the elliptic integral I_2

and at $\phi = 0^o$,

$$K_I = K_I^r = \frac{1.12\sigma^\infty(\pi a)^{1/2}}{I_2}\left(\frac{a}{c}\right)^{1/2}. \tag{4.55}$$

For a very shallow crack, $a/c \to 0$ and we can see from Table 4.1 that $I_2 \simeq 1$. This implies that a shallow semielliptical crack behaves like an edge crack.