Course material on ME 728: Fracture and Fatigue

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Chapter 1

Course contents and policies

1.1 Tentative list of topics

1.1.1 Introductory topics

1. Introduction to fracture based design.
2. Elements of the theory of elasticity
3. Introduction to complex variable theory
4. Methods for solving problems in linear elasticity using complex variable theory.

1.1.2 Linear elastic fracture mechanics

1. Fracture modes
2. Stress intensity factors in Modes I, II and III.
3. General methods of deriving stress intensity factors.

4. Fracture criteria
5. Energetics of cracked bodies
6. Plastic zone and fracture toughness
7. Concept of small scale yielding
8. Path independent integrals in fracture

1.1.3 Elasto-plastic fracture mechanics

1. Introduction to the theory of plasticity
2. Elasto plastic crack tip fields
3. Engineering estimation techniques in elastoplastic fracture
4. Concepts of J-dominance, K-dominance and R curves.

1.1.4 Experimental fracture mechanics

To be taught by Prof. Prashant Kumar.

1. Introduction to experimental techniques in fracture.
2. Hands-on and demonstration experiments.

1.1.5 Dynamic fracture mechanics

1. Introduction to elastodynamics
2. Stationary crack subjected to time varying loads

3. Moving steady state crack tip fields

4. Crack branching

5. Crack arrest and dynamic fracture toughness

1.1.6 Fatigue

1.1.7 Micromechanics of Fracture

1.2 Course policies

This is an advanced elective course on fracture and the major objective of this course is to impart to you an appreciation of the importance of fracture mechanics in modern engineering pratice. Fracture mechanics is still an evolving area and currently research is going on in diverse directions. The course will try to give you an idea about the classical as well as evolving concepts in fracture mechanics.

The books to be followed are 'Advanced Fracture Mechanics' by M. F. Kanninen and C. H. Popelar (Oxford University Press, New York, 1985) and 'Fatigue of Materials' by S. Suresh (Cambridge University Press, Cambridge 1991). You can also use 'Elements of Fracture Mechanics' by Prashant Kumar (wheeler Publishing, New Delhi, 1999). In addition, for understanding certain introductory topics you will need to consult textbooks on those particular topics. Moreover, we will sometimes refer to research papers which you can photocopy from the library. Classnotes will be put on the web at least once a week. The web address at which these will be available will be announced in the class.

Evaluation will be tentatively based on the following formula: Quizzes (10 Mid-sem exam performance (20 examination (20 from a list of topics supplied and will be done in groups of three students. It will be on an advanced topic and the students are advised to start working on the term paper by mid-February. Gradation will be based on how well you grasp concepts and your ability to apply them thoughtfully. Emphasis will be given on fundamental understanding rather than rote learning. All examinations (but not the quizzes) will be open book and without strict time limits. The quizzes will be based on assignment problems that will be given in the class from time to time and will be closed book and of 15 mins duration.

Chapter 2

Introduction

2.1 Fracture based design

Traditionally engineering design has been done through an approach based on the theory of elasticity. The philosophy is simple: design so that the calculated stresses at no point in the component exceed the yield stress of the material and then to be doubly sure, use a large factor of safety to reduce the calculated stresses.

Stress concentrators however, need special attention. According to a formula due to Inglis,

$$\sigma = \bar{\sigma}\left[1 + \sqrt{\frac{L}{R}}\right], \tag{2.1}$$

where, L is the half-length of a hole and R is the radius of curvature at the point of interest. For a circular hole $L = R$ and we get the well-known result that stress concentration factor at the equatorial point of a circular hole is 3. For a sharp crack, $R = 0$ and so the stress at the tip of this crack is ∞!!

Modern approaches to design recognise the fact that all bodies contain flaws and defects. Some of these defects are so small that they cannot be deteceted by the best non-destructive testing method that we have at our disposal. Also, bodies most often fail by breaking than by yielding. These realisations led to the development of fracture mechanics. The key questions that we ask in fracture mechanics are:

- What makes a small flaw grow into a crack?
- How long does it take for this flaw to graduate into a crack?
- How fast will the crack grow through the body?
- Will it grow in the same way in all materials?
- Can it be stopped or prevented from growing?

2.2 Idea of fracture as atomic bond breaking

Consider an array of atoms arranged as shown in Fig.1 with an interatomic equilibrium spacing of b. We intend to calculate the stress σ required to break an array of bonds in order to cause the nucleation of a crack. For this purpose, we assume an interatomic force-separation law of the form (Fig.2):

$$\sigma(x) = \left(\frac{E\gamma}{b}\right)^{1/2} \sin\left[\left(\frac{Eb}{\gamma}\right)^{1/2}\left(\frac{x}{b}\right)\right]. \tag{2.2}$$

Here, E is the Young's modulus of the material, b is the equilibrium interatomic spacing, γ is the surface energy per unit area and x is the displacement of an atom from the equilibrium position. While the form of this equation (the sine function) is arbitrary, it satisfies certain physically necessary conditions, namely,

1. The area under the curve is equal to 2γ as two new surfaces are produced.
2. The slope of the function at $x \simeq 0$ is equal to E, the Young's modulus of the material.

3. The peak value of the function is $\sigma_{max} = \frac{1}{2}\sqrt{\frac{E\gamma}{b}}$.

For most materials, a rough order of magnitude study indicates that $\sigma_{max} \simeq E/6$ which is way too high. This paradox was resolved by Griffith by resorting to simple energy arguements.

2.3 Griffith's approach

Consider an uncracked slab of thickness B, subjected to a remote uniaxial stress σ that nucleates a crack of length $2a$ at constant end displacements (Fig.3). This is possible if the stress is increased to σ and then held constant. The objective is to find the value of σ at which this will happen.

Let the external work done on the slad be dW and this work is spent in changing the internal energy by an amount dU and creating the new crack surfaces. If the surface energy is denoted by γ, then the total energy expended in creating two new crack surfaces of length $2a$ is $A = 4\gamma aB$. Hence, in the incremental sense,

$$dW = dU + \gamma dA. \tag{2.3}$$

Since the end displacements are zero, $dW = 0$. To calculate dU, we resort to (without proof) a solution for the opening of a crack under remote loads given by Westergaard (Fig.4). It simply states that the displacement $v(x)$ is given by:

$$v(x) = \frac{2\sigma}{E}\sqrt{a^2 - x^2}, \tag{2.4}$$

the crack being between $(-a, a)$. So, the energy expended in opening the crack is given by,

$$U = 4B\int_0^a \frac{1}{2}\sigma v(x)dx = \frac{\pi a^2\sigma^2}{E}B. \tag{2.5}$$

Using this in eq. 2.3, we get,

$$\sigma_f = \sqrt{\frac{2}{\pi}\frac{E\gamma}{a}}. \tag{2.6}$$

This gives the stress required to nucleate a crack in a slab under uniaxial loading.

2.3.1 Drawbacks of the Griffith approach

The simple approach of Griffith's has a number of serious drawbacks. It is applicable only to perfectly brittle materials and is difficult to extend to a multi-dimensional framework.

Griffith applied his theory to study fracture in glass fibres, which are extremely brittle materials. In fact, fracture resistance in most materials exceed γ by several orders of magnitudes. This is due to the fact that the energy supplied is expended not only in creating the surfaces but also in causing plastic deformation in front of the crack tip. Orowan suggested replacing γ by $2\gamma + \gamma_p$ in eq. 2.6, where γ_p is the plastic energy expended per unit crack growth.

Modern fracture mechanics however, prefers the expression

$$\frac{dW}{dA} - \frac{dU}{dA} = \mathcal{G}, \tag{2.7}$$

where $\mathcal{G}$ is known as the strain energy release rate or the crack driving force. One of the main objectives of modern fracture mechanics is to relate the quantity $\mathcal{G}$, which is a global parameter for the body in question, to the local stresses and strains prevailing at the region near the tip of the crack.