Due to the presence of singularities in the solution of the elliptic problems on non smooth domains the rate of convergence of the numerical methods degrades. We present an exponentially accurate nonconforming hp/spectral element method for elliptic problems using parallel computers. We consider an elliptic boundary value problem where the differential operator satisfies Babuska-Brezzi inf-sup condition on a curvilinear polygon whose sides are piecewise analytic and assume the boundary conditions are of mixed Neumann and Dirichlet type.

A geometric mesh is used at the corners. In a neighbourhood of the corners modified polar coordinates (τ_k, θ_k) are used, where $\tau_k = \ln r_k$ and (r_k, θ_k) are polar coordinates with origin at the vertex A_k. Away from sectoral neighbourhoods of the corners a global coordinate system is used consisting of (x_1, x_2) coordinates. Differentiability estimates are derived with respect to these new variables and a stability estimate is proved.

With this mesh we seek a solution which minimizes the sum of the squares of a weighted squared norm of the residuals in the partial differential equation and the sum of the squares of the residuals in the boundary conditions in fractional sobolev norms and enforce continuity by adding a term which measures the sum of the squares of the jump in the function and its derivatives in fractional sobolev norms to the functional being minimized. These computations are done using modified polar coordinates in sectoral neighbourhoods of the corners and a global coordinate system elsewhere in the domain.

The spectral element functions are nonconforming. The element function at corner most elements is represented by a constant. The element functions are a sum of tensor products of polynomials of degree W in their respective modified coordinates in all the other elements of sectoral neighbourhood of the corners. The remaining quadrilateral elements are mapped to the unit square S and the element function is represented as a sum of tensor products of polynomials of degree W in ξ and η, the transformed variables.

The method is essentially a least-squares method and a solution can be obtained using the preconditioned conjugate gradient technique for solving the normal equations. The vector composed of the values of the spectral element functions at the Gauss-Lobatto- Legendre points is divided into two sub vectors – one consisting of values of the spectral element functions at the vertices of the domain constitute the set of common boundary values U_B, and the other consisting of the remaining values which we denote by U_I. Since the dimension of the set of common boundary values is so small a nearly exact approximation to the Schur Complement matrix can be computed. A decoupled block diagonal preconditioner is proposed for the matrix in the normal equations such that the condition number of the preconditioned system is $O((\ln W)^2)$. Moreover the preconditioner is a block diagonal matrix such that each diagonal block corresponds to a different element, and so can be easily inverted.

Let W be proportional to M, the number of layers in the geometrical mesh. Then the method requires $O(W \ln W)$ iterations of the preconditioned conjugate gradient method to obtain the solution to exponential accuracy and requires $O(W^4 \ln W)$ operations on the parallel computer with $O(W)$ processors. Once we have obtained our approximate solution consisting of nonconforming spectral element functions we can make a correction to it so that the corrected solution is conforming and is and exponentially accurate approximation to the actual solution in the H^1 norm over the whole domain.

The method works for non self adjoint problems too. The method is asymptotically faster than the $h-p$ finite element method by a factor of $O(W)$. Computational results for scalar problems and the equations of elasticity are provided to validate the error estimates and estimates for computational complexity which have been obtained.

In Chapter 2 the problem is defined, and discretization, and local transformations are described. The differentiability estimates in modified polar coordinates are obtained and prove the stability theorem on which the method is based. In Chapter 3 the numerical scheme which is based on the stability estimate, is described and error estimates are obtained. In Chapter 4 we examine the issues of parallelization and preconditioning. Finally in Chapter 5 computational results are provided.