SYNOPSIS

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Laminated composites are heterogeneous in nature at the microscale, and it is generally not easy to analyse the exact configuration of individual fibers and matrix in a composite. Hence, for practical applications the effective or smeared properties of composite are used. The effective properties of composite materials depend on a wide variety of parameters at the microlevel, for example, the material stiffness of fiber and matrix, fiber volume ratio, local changes in fiber orientation, intralamina voids etc. The variations in these parameters are statistical in nature, therefore, the measured effective mechanical properties of a composite material displays significant scatter. Hence, the effective properties should be modeled probabilistically. These parameters generally vary spatially in a lamina and are modeled as random fields. However, based on experimental data available in literature the spatial variations are found to be very small (within 1%). Hence, in the present analysis all random parameters are modeled as random variables. A simple micromechanical analysis is carried out in this study to determine the scatter in the effective macrolevel properties, when the scatter in the microlevel parameters is given. The variations in the effective properties ultimately lead to variations in the response of the structure. Mechanical problems of such composite structures are generally solved using stochastic finite element methods (SFEM). The present work applies the first order perturbation technique based stochastic finite element method.

The objective of this study is to investigate the statistics of the critical buckling load for laminated plates having random material stiffness. In order to obtain the statistics of the buckling load for undamaged (or perfect) systems, a generalized stochastic linear buckling formulation based on three dimensional (3D) finite element methods is developed. Generally the buckling analysis is performed by assuming a uniform prebuckled stress state, which ignores the actual distribution of inplane stresses and it is assumed that stress in the plate would be uniform throughout the domain. In the present study, an emphasis is given to first solve a linear elasto-static problem to get accurate prebuckling stresses and then these stresses are used to solve a generalized eigenvalue problem for the critical buckling load and the associated mode shape. Results obtained, by either accounting for the variation of inplane stresses in the prebuckling state or by ignoring this variation and using the uniform stress assumption, are compared with Timoshenko solutions. The present analysis gives completely different mean buckling behaviour (with difference in mean buckling load upto 80%) for SSSS and SCSC plates when compared to Timoshenko solutions. This is because the present analysis considers all six stresses in the prebuckled state while conventionally $\bar{\sigma}_{11}= 1$ is taken as the only non-zero stress in the prebuckled state. Many new interesting phenomena on the mean buckling load and associated modes have come out of this study which was not reported in the literature earlier. The statistics of the buckling strength is determined by considering uncertainties in the effective material properties of composite laminates which are related to the known scatter in the microlevel constituent parameters. A first-order second-moment based perturbation method is introduced to study the statistics of buckling load of undamaged structures. The statistics of buckling load for composite plates is also validated with the closed form solutions. In most cases the coefficient of variation (COV) of the buckling load is found to be between 6-10% for 5% variation in the microlevel properties.

Another issue of this study is to address different plate models used in literature for the analysis of laminated composite plates. It has been observed that most of the analyses are based on two dimensional (2D) plate theories such as Kirchhoff Love (KL), Reissner–Mindlin (RM) or higher order shear deformation theory (HSDT). The replacement of the 3D elasticity problem by 2D plate models leads to error between the actual 3D solutions and the solutions obtained by 2D plate models. Various factors such as boundary conditions, boundary layers, corner and line singularities etc. also affect the convergence of a given model as the thickness $d \rightarrow 0$. One way to reduce this error is to use a more accurate model where we expand $u_i(x_1, x_2, x_3)$ in terms of higher-order polynomials of x_3. In the present work, layerwise theories are developed by representing transverse variation of the displacement components in terms of higher-order piecewise polynomial functions for each layer of laminate. The present layerwise models are very accurate in predicting both local (prebuckled stresses) and global (buckling load) quantities of interest. Another issue with the equivalent 2D plate theories is that these theories predict continuous strains and hence discontinuous transverse stresses at the interface of neighboring lamina. However, in the present layerwise theory, strains are discontinuous thereby allowing for the possibility of continuous transverse stresses at interface between neighboring laminae. Layerwise displacement fields also provide a better kinematic representation of the moderate to severe cross-sectional warping associated with the deformation of thick laminates.

In reality, composites inherently contain imperfections or voids inside it. The imperfections or voids in composites may be a result of manufacturing error or a consequence of low velocity impact during its service life. Such impacts may produce fiber-break, fiber-matrix debonds, matrix cracks and delaminations that can considerably affect the behaviour of composite structures under inplane load. Response of such a damaged component is normally modeled using damage variables. The existence of damage makes the material behave nonlinearly, i.e. the load carrying capability depends on whether the crack faces are in tension or compression. Due to this material nonlinearity, a linear buckling analysis is not valid. In this case, the instability analysis has to be based on a complete nonlinear formulation. The effect of different forms of damage on the buckling strength of damaged laminated system has not been addressed earlier in the literature. The main objective of this part of study is to analyse the load carrying capabilities and the corresponding deformation of damaged composite system with existing static damage under inplane loading. In order to evaluate the buckling behavior of a damaged plate, a nonlinear 3D finite element method based on updated

Lagrangian formulation is employed along with continuum damage mechanics. In order to capture the different forms of damage, a meso-scale model is used where a composite laminate is treated as a stacked series of orthotropic layers and isotropic interfaces. The damage models are presented separately for elementary ply and interface to capture the different modes of damage at both ply and interface levels. A layer-wise plate model is adopted for the modeling of both elementary ply and interface layers of the laminate separately. The opening and closing behavior of damage modes, as the load increases and reaches the limit point, is also discussed. Some interesting phenomena like local wrinkling behaviour of the damaged region is observed which was not reported in the literature earlier. A new approach for approximating the statistics of the buckling strength of damaged system is introduced. The effect of scatter in the effective material properties along with the scatter in the intensity of different form of static damage on the load carrying capability of composite system (and the corresponding transverse deflection) is studied.

The thesis is organized in seven chapters with each one addressing a specific aspect of the problem. Chapter 1 contains an overview of past research related to the present work and objectives of the present work. The second chapter describes the development of equilibrium equations for linear elasticity in three dimensions and constitutive equations for orthotropic material, transversely isotropic, and isotropic material. This chapter also includes an introduction to a very general plate theory that can be used for both thin and thick plates. The third chapter deals with the weak formulation to three dimensional linear stochastic buckling analysis of the undamaged system. The finite element method is also introduced with detailed discussion on the implementation of different plate models. The fourth chapter covers the solution of the linear system of equations that result from the finite element formulation presented in Chapter 3. It contains the statistics (mean and variance) of the buckling load for undamaged laminated plates with and without cutout. The fifth chapter describes the development of an updated Lagrangian formulation of a three dimensional nonlinear finite element for buckling analysis of damaged system. A detailed discussion on continuum damage mechanics for the modeling of static damage in composite is presented. This chapter also contains the numerical solution technique for the damaged nonlinear systems. The sixth chapter deals with solution of the nonlinear system of equations that result from the finite element formulation presented in Chapter 5. It covers the effect of different modes of static damage on the mean buckling strength and corresponding deflection patterns of the damaged laminated plates. It also discusses the statistics of buckling load due to the randomness in the material stiffness and damage indicators for damaged composite plates. The seventh chapter discusses some of the results and trends from the examples presented in Chapters 4 and 6. It also discusses some of the problems with the proposed method and some recommendations for future work in the area of probabilistic stability analysis of structures.