SYNOPSIS

Piezoelectric materials have tremendous potential for application in smart structures, as the same material can be used as sensors and actuators. Piezoelectric materials develop an electric potential with the application of mechanical pressure. This is known as direct piezoelectric effect. In the indirect or converse effect, piezoelectric materials undergo mechanical strain when an electric field is applied. These materials, due to the inherent microstructural details, have very interesting temporal response characteristics. These are manifested through the polarization-electric field ($P-E$) hysteresis curve, typical of ferroelectric materials. Piezoelectric materials also demonstrate strain-electric field ($\epsilon - E$) hysteresis through a butterfly loop. However, the theoretical models used for representing this behavior have been primarily ad-hoc and not based on sound thermodynamic reasoning. More rigorous models, based on micromechanics, are difficult to incorporate in structural application studies.

The current study has proposed an effective, thermodynamically consistent hysteresis model which can be easily incorporated in structural applications. Electro-thermo-elastic formulation of a piezoelectric material, to model the hysteresis effects requires understanding of dissipative models, electrostatics and continuum mechanics. The electro-magnetic phenomena associated with the motion of free charges and/or dipoles in dielectric materials are generally neglected in the formulation. The electro-thermo-elastic state of the piezoelectric material medium with hysteresis effects is obtained by using the conservation laws, first and second laws of thermodynamics and evolution models. The equilibrium equations are obtained by using conservation laws (charge, mass, linear momentum, and angular momentum). The constitutive relations have been developed by applying conservation of energy (first law of thermodynamics) and rate of entropy production (second law of thermodynamics) incorporating a thermodynamic potential. The method of local state is used to model the hysteresis

at the macroscopic level. Using the method of local state, the thermodynamic potential is taken as a function of an internal variable, to represent the phenomenon happening at the microscopic level which causes the hysteresis effects, along with the other observable variables (deformation gradients, electric field and temperature). It has been shown from the formulation that the interaction of polarization and electric field results in rendering the stress tensor non-symmetric and introduces a nonlinear force term in the equilibrium equation. Further, nonlinear higher order terms have been included in the constitutive relation to get a better fit of hysteresis and butterfly loops.

This general electro-thermo-elastic formulation can be used to study the dynamic response of smart structural members, with dissipation due to hysteresis effects, and to model the control mechanisms. A layer-by-layer finite element formulation has been developed, by using the electro-thermo-elastic constitutive relations, suitable for handling the static and dynamic problems corresponding to smart composite plates including the nonlinear effects.

The finite element formulation is validated by solving static benchmark problems available in the literature. Subsequently, nonlinear analysis has been carried out and validated with the experimental data. Further, in static analysis, the effect of piezo lay-up and ply-orientation of the core on the effective response of the composite plates has been studied. The study considers four types of piezo lay-up patterns on either side of the plate. The study of ply-orientation considers three different ply orientations.

The dynamic analysis of smart composite plates has been carried out by using modal analysis. Dissipation losses due to polarization-electric field hysteresis effects have been also included in the analysis. From the study, the following major observations have been made.

1. Electro-thermo-elastic constitutive relations have been developed based on a

consistent thermodynamic formulation by including polarization-electric field hysteresis effects. These relations are validated with the experimental results available in the literature. Theoretically formulated hysteresis and butterfly loops correlate well with the experimental data. Further, it was found that polarization-electric field interaction inherently makes the constitutive relations nonlinear and introduces a nonlinear force term in the equilibrium equations.

2. Influence of amplitude of electric field, frequency of oscillation of electric field and applied mechanical load, on the hysteresis loop have been studied. It is found that increasing the amplitude of electric field increases the area of hysteresis loop. Remnant polarization and coercive electric field are also found to be functions of electric field.

3. When the electric field is above the coercive field, corresponding to saturation polarization, the area under the hysteresis loop increases with increase in frequency of electric field indicating increase in the dissipation energy. On the other hand, if the amplitude of electric field is below the coercive field corresponding to saturation polarization, the dissipation energy decreases with increase in frequency.

4. Layer-by-layer finite element model has been developed and validated for the analysis of the composite plates with segmented/continuous piezo layers for static and dynamic cases. Polarization-electric field hysteresis and nonlinear effects due to polarization-electric field interactions have also been included in the formulation. A special $frontal$ type direct solver has been developed to efficiently solve large systems of equations in static analysis. In the dynamic analysis skyline storage is used for mass matrix.

5. Influence of piezo lay-ups and ply-orientation on the deformation of the smart composite plate has been studied. From the analysis, it has been observed that

for the continuous piezo layer, the longitudinal deflection is more than that for other configurations. Width-wise strips show more transverse bending and twisting as compared to other piezo lay-up configurations. Further, for the given piezo-configuration and actuation voltage, the composite plate with $[30_2/0]_s$ ply orientation gives maximum twist and longitudinal bending.

6. Nonlinearity due to polarization-electric field interaction in the static analysis shows softening effect in the case of longitudinal bending and twisting in the thin plates. Whereas, in the case of transverse bending it shows a slight stiffening effect.

7. It can be concluded from the static analysis that the desired actuation shape can be obtained for a specified length-to-thickness ratio of the plate, and for the given voltage, by controlling the ply-orientation of the core or changing the geometric orientation of piezo lay-ups.

8. Dynamic analysis of a smart composite plate has been carried out to study the dissipation phenomenon due to polarization-electric field hysteresis under free vibration condition. The response behavior shows a friction type damping initially; subsequently for small amplitudes the response decays in an exponential manner, similar to viscous damping.

9. The hysteresis induced dissipation is found to be significant. This observation can enable the use of piezo layer as active dampers.