Session 2003-04 **PHY 452** Semester II

Electromagnetic Theory I

PROBLEM SET II

Try and investigate all aspects of the problems posed below. Calculate numerical results wherever possible.

1. (a) Consider a (infinite) one-dimensional lattice with point charges $\pm e$ placed alternately, each at a distance a from the next one. Calculate the electrostatic energy per unit length of the system. Take $a \sim 1$ Å and evaluate this numerically. What would happen if all the charges had the same sign?

   (b) Try and extend the above calculation to a two-dimensional square lattice with alternate $\pm e$ charges.

2. A spherical conductor of radius a placed in an electric field along the z-axis has a charge distribution $\sigma = \sigma_0 \cos\theta$ on the surface.

   (a) Relate the constant σ_0 to the total charge on the conductor.

   (b) Calculate the electrostatic energy of the system.

   (c) If the electric field is switched-off, how does the energy of the system change?

   (d) Relate the last answer to the principle of energy conservation.

3. Calculate the capacitance of the following arrangements:

   (a) A spherical conductor (radius a) placed in a dielectric medium of permittivity ϵ.

   (b) A parallel-plate capacitor with a dielectric medium of permittivity ϵ between the plates.

   (c) A parallel-plate capacitor with a two layers of dielectric, with different permittivities ϵ_1 and ϵ_2 between the plates.

   (d) A (long) cylindrical capacitor with a dielectric medium of permittivity ϵ between the cylinders.

4. A long metallic wire of circular cross-section (radius r_0) is strung on poles at a height h above the ground.

   (a) Assuming that the sag of the wire and the curvature of the Earth can be safely neglected, calculate the capacitance of the system.

   (b) If the wire picks up a uniform line charge λ, calculate the force per unit length attracting the wire to the Earth.

(c) If the wire sags a bit, discuss qualitatively how the above results would change.

(d) How are the results affected by the curvature of the Earth?

5. (a) Consider a point dipole $\vec{\mu}$ of mass m placed in the electrostatic field $\vec{E}$ due to a point charge Q at the origin. Find the force on this dipole and hence set up the equation of motion. Try to solve this equation to find the trajectory.

   (b) Consider a point dipole of moment $\vec{\mu}$ at the point $\vec{x}_0$. Show that the potential due to this is the same as that of a (monopole) charge density

$$\rho(\vec{x}) = -\vec{\mu}.\vec{\nabla}\delta^3(\vec{x} - \vec{x}_0) \ .$$

   (c) Calculate the mutual electrostatic energy of two dipoles $\vec{\mu}_1$ and $\vec{\mu}_2$ placed at positions $\vec{x}_1$ and $\vec{x}_2$.

6. (a) Consider an uncharged sphere (radius a) of a linear, isotropic, homogeneous dielectric material, placed in a vacuum. Assume that a constant polarisation $\vec{p}$ has been induced inside the sphere. Find the electrostatic potential and field (i) at the centre of the sphere, and (ii) at a point $\vec{x}$ outside the sphere.

   (b) Calculate the electrostatic energy of the above system.

   (c) Show that Thomson's theorem is valid in a dielectric medium of permittivity ϵ.

7. (a) The $2p^{\pm}$ states of hydrogen correspond to the charge density

$$\rho(\vec{x}) = \frac{1}{64\pi} r^2 e^{-r} \sin^2\theta$$

   is an appropriate set of units. Find out its monopole, dipole and quadrupole moments.

   (b) An electron is in a stable circular orbit around a proton, as described by Bohr's atomic theory. Place the atom in a uniform electric field $\vec{E}$ and determine the new trajectory.

   (c) A uniformly-charged closed flexible string of length ℓ and total charge Q is in equilibrium around an immovable positive charge Q. What is the shape of the string in the equilibrium condition? Find the dipole moment of the system about the fixed charge. The entire system is then placed in a constant electric field $\vec{E}$. Find the new shape of the string and determine its new dipole moment about the fixed charge.

8. A molecule of water can be (crudely) modelled as an oxygen ion of negative charge $-2e$ between two hydrogen ions of positive charge $+e$ each, with the angle $\theta_{\widehat{H^+O^-H^+}} \simeq 108^o$. Assume that the ions are point charges and the inter-ionic distance is a.

(a) Calculate the dipole moment of this molecule in terms of e and a, about the (heavy) oxygen atom as origin.

(b) Use this result to calculate the polarisation and hence the dielectric constant of water at room temperature in terms of e and a. Assume water is a linear, homogeneous, isotropic dielectric in which the molecules are completely free to rotate.

(c) Find out the experimental result for the dielectric constant of water at room temperature and hence determine the size of the water molecule.

9. A hollow cube has conducting walls defined by siz planes $x = 0, a$, $y = 0, a$ and $z = 0, a$. The walls $z = 0, a$ are kept at a constant potential ϕ_0. The other walls are grounded.

   (a) Calculate the potential $\phi(x, y, z)$ at any point inside the cube.

   (b) Calculate the potential at the centre of the cube correct to 3 significant digits.

   (c) Compare the numerical value of potential at the centre with the average value of potential over the six walls.

   (d) Calculate the surface-charge density σ on the wall at $z = a$.

10. A sphere of radius a and dielectric constant ϵ is placed in vacuum (air) in a constant electric field $\vec{E}$.

    (a) Solve Laplace's equation in the region exterior to the sphere and calculate the electric field.

    (b) Solve Laplace's equation in the region interior to the sphere and calculate the electric field.

    (c) Determine the surface-density of charge on the surface of the sphere.

    (d) What happens if the electric field is switched off? Consider both macroscopic and microscopic aspects.